

# One Theory or Two?

## Walras's Critique of Ricardo\*

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### 1. Introduction

There are essentially two views about the development of economic theory. According to the one which nowadays appears to be almost universally accepted, the history of economic theory can be conceived of as a one-way avenue leading from primitive conceptualizations of the demand and supply approach to all sorts of economic phenomena to ever more sophisticated ones, merely leaving behind errors of reasoning and unnecessarily restrictive assumptions. This view is occasionally dubbed the 'Whig' history of economic thought. According to the alternative view, the history of our subject is not characterised by a linear development. Theories which once have dominated the discussion rather tend to get abandoned because of internal and external reasons and are replaced by fundamentally different ones: a theory may be 'submerged and forgotten' at some stage, as one commentator remarked perceptively; it need not, but may come back at a later stage, especially when new formulations of the theory succeeded in overcoming the difficulties encountered by its earlier versions.

In this paper an attempt will be made to render support to the discontinuity thesis in terms of an important example from the history of our subject: the abandonment of the 'classical' approach to the theory of value and distribution by what is now known as the 'neoclassical' approach. More specifically, we shall deal with Léon Walras's 'Exposition and Refutation of the English Theory', by which Walras meant the theory of the classical economists, paying special attention to David Ricardo's contribution, in Lessons 38-40 of Part VII of Léon Walras's *Elements of Pure Economics* ([1871] 1954). To the best of our knowledge these lessons have never received the attention they deserve. Holding an early variant of the Whig point of view, the strategy of

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Walras's criticism was obvious. Three kinds of criticisms were levelled at the classical authors: (i) Walras accused them of having committed 'fundamental errors'; (ii) to the extent to which their argument can be said to have been correct, he took it to cover but special cases of a more general analysis; and, closely related with it, (iii) Ricardo and his followers were criticised for having failed to develop, and indeed to see the very possibility of developing, 'a unified general theory to determine the prices of all productive services in the same way' (ibid., p. 416).<sup>1</sup> Such a unified general theory, Walras contended, has been elaborated by himself by generalising the principle of 'scarcity', which the classical economists had limited to natural resources, to *all* goods and factors of production alike.<sup>2</sup>

In this paper we shall scrutinize Walras's objections. In particular, we shall ask whether Ricardo's analysis can at best be interpreted as a special case of Walras's own analysis, as the Lausanne economist maintained. Is there only one theory or are there two? We shall see that while some of Walras's criticisms are correct, his main premise cannot be sustained: there is a distinct 'classical' approach to the theory of value and distribution which is fundamentally different from Walras's neoclassical one. This fact will be shown to be reflected in some remarkable differences between what Ricardo wrote as opposed to what Walras interpreted him to have written.<sup>3</sup> Moreover, since Walras based his criticism on a rational reconstruction of Ricardo's argument we must closely inspect that reconstruction and compare it with more recent ones. We shall in particular refer to the re-formulations of Ricardo's rent theory by Kaldor (1955-6), Samuelson (1959) and Pasinetti (1960) and especially to Sraffa's interpretation of the classical economists and

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<sup>1</sup> In the following all isolated pages given refer to the Jaffé edition of the *Elements*.

<sup>2</sup> For a succinct discussion of the structure of Walras's mature comprehensive model, see Walker (1996, chapter 8).

<sup>3</sup> In this paper we are not concerned with Walras's criticism of Mill. Apart from a few side remarks we shall therefore set aside this aspect of his discussion. This appears to be justified by the fact that in important respects Mill parted company with Ricardo's doctrine and anticipated later marginalism; on this, see Kurz (1999a).

Ricardo (Sraffa, 1951, 1960), and ask whether the difficulties and contradictions of Walras's interpretation are avoided in the available alternative ones.<sup>4</sup>

The composition of the main part of the paper follows closely the structure of Walras's criticism of the classical economists. Section 2 sets the stage by providing a short summary account of what we consider to be some of the main differences between the classical approach and the Walrasian variant of the neoclassical approach to the long-period theory of value and distribution. This provides the foil against which the following argument is developed. Section 3 deals with Lesson 38 which is devoted to the classical theory of value. The subject of Lesson 39, Ricardo's theory of rent, will be discussed in Section 4. Finally, Section 5 turns to Walras's disquisition on the classical theory of wages and interest in Lesson 40. Section 6 contains some concluding remarks.<sup>5</sup>

## **2. 'Classical' vs. 'neoclassical' theory of value and distribution**

Following Sraffa, the working hypothesis of this paper is that Ricardo advocated an approach to the theory of value and distribution that differs fundamentally from the neoclassical approach and thus also Walras's.<sup>6</sup> The difference becomes clear from a rational reconstruction of the sets of data, or independent variables, from which the two theories typically begin their reasoning. Notwithstanding several differences between different authors, in the interpretation under consideration the classical economists and Ricardo can be said to have approached the problem of value and distribution on the basis of the following givens:

- (R1) the set of technical alternatives available to cost-minimizing producers;

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<sup>4</sup> For a comprehensive discussion of the 'classical' approach to the theory of value and distribution, see Kurz and Salvadori (1995, 1998a, 1998b).

<sup>5</sup> There is a striking similarity of the criticisms levelled at the classical theory of value and distribution, and especially the theory of rent, by Walras and Wicksteed (1894). As is well known, Walras came close to accusing Wicksteed of plagiarism (cf. pp. 490-92). On this, see Jaffé (1964).

<sup>6</sup> For a detailed discussion of the differences between the two kinds of theories, see, for example, Kurz and Salvadori (1995, ch. 1, 1998a, ch. 1).

- (R2) the size and composition of the social product, reflecting the needs and wants of the different classes of society and the requirements of reproduction and capital accumulation;
- (R3) the ruling real wage rate for common labour; and
- (R4) the quantities of the different qualities of land available and the known stocks of depletable resources, such as mineral deposits.

Scrutiny shows that the data from which Walras typically started in his fully developed general equilibrium analysis are on the contrary:

- (W1) the set of technical alternatives available to cost-minimizing producers;
- (W2) the preferences of consumers; and
- (W3) the initial endowments of the economy with all productive resources, including 'capital goods proper', and the distribution of property rights among individual agents.

A crucial difference between the two authors appears to be methodological. In the theory of value and distribution, Ricardo, like the Physiocrats and the other classical economists, referred to data and magnitudes that can, in principle, be observed, measured or calculated. These authors refrained from having recourse to any non-observable, non-measurable or non-calculable magnitudes, or concepts which they considered metaphysical, in determining the general rate of profit and relative prices. They did so at the cost of what we might nowadays call 'implicit theorizing' – in the sense that in the theory of value and distribution they took as data or independent variables what in other parts of their analyses are regarded as unknowns or dependent variables. Thus, when determining the rate of profit and relative prices in given conditions they assumed the size and composition of the social product to be given, whereas when discussing socio-economic development and structural change the latter are naturally treated as variables. Ricardo and the other classical authors studied the long-run impact of socio-economic development and structural change on income distribution and relative prices essentially through comparisons of successive long-period positions of the economy. These were defined in terms of changes in the sets of data (R1)-(R4). For example, when, in a 'progressive' state of affairs, the total amount of corn to be produced rises, other things being equal, then datum (R2) would change, with the possible consequence that extensive or intensive diminishing returns might make themselves felt and lead to the emergence of differential rents. In this case, with the real wage rate constant, the general rate of profit would be bound to fall.

Another crucial feature that distinguishes the Ricardian approach to the theory of value and distribution from the Walrasian one is that in the former the wage rate is considered an independent variable. Closely related to this is the fact that in Ricardo, other than in Walras, there are no initial endowments of capital goods. Therefore, in Ricardo the rate of profit is not explained in terms of the relative 'scarcity' of a factor 'capital'. It is this *asymmetric* treatment of the distributive variables, with profits as a dependent residual, which indicates an important difference between classical and neoclassical theory.

We want to show in the following that the above distinction between two kinds of theories of value and distribution in terms of the different sets of data from which they start makes perfect sense. This view is implicitly corroborated by some earlier neoclassical economists, especially Walras, who had difficulties to come to grips with the analytical structure of the classical theory of value and distribution. These authors were inclined to interpret the latter as a special case of their own theory. There is an obvious way of deciding this claim. If the classical theory were a special case, it would start from the *same* set of data (W1)-W(3) but impose special restrictions on this set. Yet the neoclassical critics of classical theory, most notably Walras, were not able to demonstrate this. Somehow they themselves appear to have felt that the special case interpretation was not fully compatible with the evidence under discussion. As we shall see, this is reflected in the different kind of criticism, put forward, *inter alia*, by Jevons and Walras, that the classical authors attempted to determine two unknowns from a single equation, meaning that they attempted to determine both the rate of profit and total output in terms of a single equation. This accusation of underdeterminacy is clearly at odds with the special case interpretation. As will be shown, given the different analytical structure of the classical theory there is no problem of underdeterminacy. Raised by the neoclassical critics, the alleged problem of underdeterminacy may rather be interpreted as indicating that there are two different theories of value and not just one.

### **3. Walras on the Ricardian theory of value**

Walras introduced Lesson 38 with a compliment to the classical authors: 'The efforts of the English School to develop a theory of rent, wages and interest were far more sustained and thorough than those of the various French schools that came into existence after the Physiocrats' (§ 342, p. 398). Next he recalled the Ricardian distinction between (i) 'commodities, the value of which is determined by their scarcity alone', because no 'labour can increase the quantity of such goods, and therefore their value ... varies with the varying wealth and inclinations of those who are desirous to

possess them'; and (ii) commodities that 'are procured by labour ... and ... may be multiplied, not in one country alone, but in many, almost without any assignable limit, if we are disposed to bestow the labour necessary to obtain them' (*Works* I, p. 12). In addition he quoted a statement by J. S. Mill who had contended with regard to commodities of the second class that 'there needs be no limit to the multiplication of the products' (Mill, 1909, p. 444).

### *Two 'fundamental errors'*

Walras saw 'two fundamental errors which must be refuted' (p. 399). First, 'There are no products that can be multiplied without limit' because 'all things constituting social wealth consist of land or personal faculties or the products of the services of land and personal faculties.' However, 'land exists in limited quantities only. If that is also true of human faculties, how can products be multiplied without limit?' (ibid.). Secondly, there is no 'value of costs of production, which, having itself been determined, determines in turn the selling prices of products.' The causality is rather said to be 'the other way round' (ibid.).

As to the first 'fundamental error', Ricardo cannot be accused of having committed it: he spoke explicitly of commodities that 'may be multiplied, not in one country alone, but in many, *almost* without any assignable limit'. A proper target of Walras's criticism was Mill's statement, not Ricardo's. Ricardo was well aware of the fact that the quantities of the (reproducible) commodities effectually demanded generally have an impact on prices (and income distribution). The whole point of his theory of rent was indeed the dependence of the cost and thus the price of corn on the quantity of corn produced. In a letter to Malthus dated 9 October 1820 he wrote: 'You say demand and supply regulates value – this, I think, is saying nothing, ... it is supply which regulates value – and supply is itself controlled by comparative cost of production' (*Works* VIII, p. 279).<sup>7</sup> Demand may affect prices in the long run only in so far it affects the data (R1)-(R4), especially (R2).

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<sup>7</sup> See also his letter to Malthus of 24 November of the same year: 'I shall not dispute another proposition in your letter "No wealth["] you say "can exist unless the demand, or the estimation in which the commodity is held exceeds the cost of production." I have never disputed this. I do not dispute either the influence of demand on the price of corn and on the price of all other things, but supply follows at its heels, and soon takes the power of regulating price in his own hands, and in regulating it he is determined by cost of production. I

In Ricardo's view demand and supply regulate the 'market' prices of commodities, whereas the normal or 'natural' prices are the prices which obtain in a cost-minimizing system of production, given the data (R1)-(R4). It also deserves to be noted that the case on which Ricardo focused attention is one in which the impact of the scarcity of land on relative prices is somewhat concealed: his theory of rent focused mainly on the case of extensive diminishing returns, and thus extensive rent, which – as regards prices, wages and profits – allowed him to concentrate on the technical conditions of production on marginal land. Ricardo was also aware of, and discussed, the case of intensive diminishing returns, and thus intensive rent. However, when analysing the relationship between wages and profits, the problem that concerned him most, he set aside the problem of rent. He justified this premise as follows: 'By getting rid of rent, ... the distribution between capitalist and labourer becomes a much more simple consideration' (*Works* VIII, p. 194). We may add that his approach could also derive some justification from the finding in the modern theory of production that even in the case of intensive diminishing returns there is a fictitious technique which can be obtained from the data of the problem, in which land does not appear (see Guichard, 1982).

As to the second 'fundamental error', it should be stressed that Ricardo took only the real wage rate as given and determined the general rate of profit and the rents of land endogenously. Walras failed to see that Ricardo advocated a genuinely different theory. Apparently, it did not even occur to Walras that there could be a theory that is fundamentally different from, and not just a special (and incoherently formulated) version of, his demand and supply theory. This explains why he found nothing wrong with assessing the contribution of the classical economists in terms of his own theory. We will come back to this in Section 5 below.

### *Three categories of products*

As we have seen, one of Walras's main criticisms concerned the particular causality the classical economists were said to entertain, which is said to have run from the prices of the 'productive services' to the prices of the products.<sup>8</sup> If it could instead be shown that

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acknowledge the intervals on which you so exclusively dwell, but still they are only intervals' (*ibid.*, p. 302).

<sup>8</sup> As we have just seen this is not fully correct, because all distributive variables other than wages are treated as unknowns, to be determined together with relative product prices.

in the real world the reverse causality prevails, as Walras contended, then his own theory, based, as it were, on utility and the principle of the relative scarcity of goods, could be claimed to be not only more rigorous, but also more relevant than Ricardo's. Yet, in an attempt to distance himself from Ricardo, Walras let himself be carried away and actually contradicted one of his main tenets, namely, that both kinds of prices are determined *simultaneously* and *symmetrically* in terms of the demand for and the supply of the respective products and services.

Walras devoted four more sections (§§ 345-8) to a discussion of the issue under consideration and distinguished three classes of products. The *first* class of products is the one on which all agree: 'the case of productive services which have passed out of existence [after having been used], for example, Ricardo's "rare statues and pictures, scarce books and wines"' (§ 345, p. 400). He expounded: 'The value of such products, as both Ricardo and Mill admit, is the result of the law of offer and demand alone' (ibid.). The *second* class concerns products produced by some 'specific productive services' (§ 346). This set includes Ricardo's example of 'wines of a peculiar quality, which can be made only from grapes grown on a particular soil, of which there is a very limited quantity'. Also with respect to this class there is not much difference of opinion, analytically (cf. § 346, p. 401). However, in Walras's view this set is much larger than the classical economists were inclined to think: 'Had Ricardo and Mill been a little more methodical in their classification, they would have given examples of personal services which are no less specific than the land-services they mentioned, like the personal services of living artists, singers, eminent doctors and great surgeons' (ibid.). It appears to have escaped Walras's attention that in Smith we find several references to the concept of 'talent' and the remuneration paid for it, and that Ricardo was in agreement with Smith except whenever he explicitly said otherwise (see his Preface to the *Principles*).<sup>9</sup> What is intriguing is that Walras mentioned 'personal services' together with 'land-services', but interestingly did *not* mention the services of 'proper capitals'. In fact, in a long-period framework no capital good proper, or produced (and reproducible) means of production, can be the source of some 'specific productive service' and yield its owner a scarcity rent: in the long run the self-seeking behaviour of producers will result in proportions of the quantities of the different capitals goods proper such that a uniform 'rate of net income', Walras's term for rate of profit, obtains. Hence, implicitly Walras showed some awareness that 'proper capital' cannot be dealt with in the same

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<sup>9</sup> For a discussion of the classical theory of wage differentials, see Kurz and Salvadori (1995, ch. 11).



way as the primary factors of production, land and labour.<sup>10</sup> Yet he missed the opportunity to clarify this aspect of his doctrine by way of contrast with Ricardo's.

Section 347 is devoted to a *third* class of products: those produced by 'unspecialised productive services', which 'have competition to fear.' Echoing Ricardo's view, he added that this, 'admittedly, is the most frequent case' (p. 401). In this case

A rise in the prices of unspecialised services will attract to production other similar services which exist in more or less large quantities. If the prices of the products [of unspecialised services] rise, the prices of the productive services will also rise, *but only temporarily*; for these will increase in quantity and hence the quantity of their products will also increase. The end result will be a *slight rise* in the price of both productive services in general and of products in general (§ 347, pp. 401-2; emphases added).

The first part of the passage just quoted taken alone might be interpreted as indicating a general agreement between Walras and Ricardo: an increase in the output of a commodity which is produced by non-specific services may increase the price of that commodity and of the services used in its production *only temporarily*, that is, in the long run these changes will be annihilated. However, with no further explanation Walras surprisingly continued that the 'end result' would be a 'slight' rise in both kinds of prices. Why? The only possible interpretation of this conclusion we can think of would have to be in terms of non-specific services of land and labour, whose available amounts are both given, constant and scarce. That is, there is a change in the data (R1)-(R4): a change in the size or composition of the social product has occurred, and this change is such as to affect the extensive or the intensive margin of some qualities of land. Walras seems to believe that any change must have this effect. Now, in Ricardo we certainly encounter the assumption that the quantities of the different qualities of land are given and (almost) constant, and, depending on the levels of production of the various commodities, some of these qualities of land may be scarce. Yet, as we have already seen, Ricardo, whose main concern was the relationship between the wages of

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<sup>10</sup> The need to differentiate is also expressed in Walras's distinction between land and personal faculties, which are said to be 'natural wealth', and capital goods proper (and 'income goods', i.e. consumption goods), which are said to be 'artificial wealth' (p. 399). Artificial wealth can be created and its size and composition adjusted as required by the circumstances.

labour and the profits of capital, was keen 'to get rid of rent' by focusing attention on (non-intensively cultivated) marginal land. That is, although a change in the levels of production of commodities that are (directly or indirectly) produced by means of unspecialised qualities of land may, in principle, affect which quality of land is marginal, and thus will affect costs of production, prices, the rate of profit and the rents paid to the proprietors of intramarginal lands, in many cases such a change will not have this effect and in other cases the effect will be small. In these cases it is possible to analyze the relationship between prices, profits, and wages by abstracting from these possible effects.

As regards common labour, according to Ricardo there is no presumption that in the long run it may also be considered a scarce factor of production. (Things may obviously be different with regard to certain special talents). According to the classical authors from Smith to Ricardo, the required size of the common work force is essentially generated alongside the accumulation process. In other words, the size of the workforce is taken to be compatible with the data (R1)-(R4). Therefore, while due to an abundance or a 'scarcity of hands', to use Ricardo's expression, in the short run wages may fall below or rise above their normal or 'natural' level, in the long run a sufficient amount of labour will be forthcoming and make actual wages follow the trend of their historically and socially determined normal level, which must not be mistaken to imply a constant real wage rate.

Interestingly, in Walras we encounter a point of view which, according to Morishima (1977, p. 5), resembles that of Ricardo. In his analysis of 'economic progress' (as opposed to 'technical progress'), that is, an accumulation of capital and a growth of population with an unchanging set of technical alternatives in Lesson 36, he arrived at the conclusion: '*In a progressive economy, the price of labour (wages) remaining substantially unchanged, the price of land-services (rent) will rise appreciably and the rate of net income will fall appreciably*' (pp. 390-1; emphasis in the original). Note in particular that according to Walras in the long run any tendency of the wage rate to rise as capital accumulates and the demand for labour increases is effectively offset by an expansion of the supply of labour. Walras assumed in fact: 'Population ... does increase, for such an increase is implicit in our definition of progress; and thus additional labour, naturally proportional [?] to the additional future output, is assured' (p. 386). This makes Walras's above objection to Ricardo's long-period theory that not only land, but also human faculties exist in limited quantities (p. 399), all the more puzzling. In fact, had Ricardo known Walras's above statement, he might have received it with a certain satisfaction: at least it did not in any obvious way contradict his assumption of a *given* real wage rate when dealing with the problem of value and distribution, and indeed

went much farther than he, Ricardo, ever deemed it sensible, by postulating the long-term constancy of the real wage rate.

In § 348 Walras summarized his argument why, 'in reality, there is no absolute antithesis between the two cases distinguished by Ricardo and Mill' (§ 348, p. 402). This necessitated in his view to explain *all* prices, including the prices of the productive services, indiscriminately in terms of demand and supply.

The differences between Walras and Ricardo thus concern first and foremost the scope and content of the theory of value and distribution. Both authors were interested in investigating the *long-period* properties of an economic system, characterised by a uniform rate of interest (profit) and uniform rates of remuneration of all primary factors of production. However, the data in terms of which they attempted to achieve this aim differ significantly. These differences reflect both differences in scope and content. Ricardo was mainly concerned with that class of commodities, whose long-period prices can be determined in terms of their conditions of production and the state of income distribution, i.e. the level of the real wage rate(s). Correspondingly, he based his explanation as much as possible on observable magnitudes, that is, magnitudes that can be counted, weighed or measured, or, for short: 'objective' factors. Walras, on the other hand, felt prompted to attribute great importance to the class of commodities in whose price determination demand, rather than cost, plays a crucial role. Hence, his emphasis on the role of 'utility', that is, a non-observable magnitude. However, since Walras contended that his analysis was generally superior to Ricardo's and therefore also superior with regard to the case of commodities that are producible and reproducible, albeit perhaps at rising unit cost, we shall in the following deal only with this case. This brings us to Walras's criticism of Ricardo's theory of extensive and intensive rent (see *Works I*, ch. 2).

#### **4. Walras on the Ricardian theory of rent**

Lesson 39 was devoted to an 'exposition and refutation' of the Ricardian theory of rent. Walras stressed that this is 'a mathematical theory which must be expressed and discussed mathematically' (§ 352, p. 405). §§ 352-3 are devoted to a geometrical exposition in which each (incremental) investment involves an *amount* of £ 1,000 (which may be considered the unit of account in money terms in which the analysis is conducted). Walras objected that in Ricardo's presentation of extensive rent (see *Works I*, pp. 70-1) it is not clear what is meant by 'equal amounts of capital and labour': 'Ricardo does not state expressly in what terms these employments of capital are

evaluated or what their value is; but in the second part he explicitly supposes that they are evaluated in terms of money ['numéraire'] and that their value is £1,000 each' (§ 352, p. 405).<sup>11</sup> In § 354 Walras then criticised Ricardo that instead of proceeding in terms of increments of capital worth £1,000, he should have argued in terms of infinitesimals and should have supposed 'that every time the capital used is increased by an infinitely small quantity, the rate of yield must decrease by an infinitely small quantity' (§ 354, p. 408). Walras illustrated his argument geometrically and then, in § 355, complemented it by an algebraic formulation.

We shall begin by comparing Walras's reconstruction of Ricardo's theory of rent with Kaldor's more recent and very influential interpretation (Kaldor, 1955-6). There are three main differences. First, in his diagrammatic illustrations Walras put 'Capital Employed' on the horizontal axis<sup>12</sup>, whereas Kaldor put 'Labour'. In the literature subsequent to Kaldor we find also the expression 'Labour-cum-capital' to indicate that whilst the unit of measurement is a unit of labour, the measure refers to total capital advanced, which includes not only the labour paid the given real wage, but also the 'seed capital' used by one worker. Second, Walras drew a diagram for each quality of land, whereas Kaldor drew a single diagram for the whole corn sector (where 'corn' is taken to represent a whole 'complex of agricultural products'). Third, the curves drawn by Walras represent the derivative (or the increment) of 'the excess *per hectare* of the total number of units of product over the number of units necessary for the payment of wages [on each kind of land, respectively]' (§ 355, p. 409), whereas the curve drawn by Kaldor gives the marginal productivity of labour (reflecting the amount of capital employed). Let us consider these three differences in turn.

As regards the *first* difference, recall Ricardo's first two consecutive attempts to simplify the problem of distribution (see Sraffa, 1951). His initial step consisted of getting rid of rent in terms of the theory of extensive rent in the *Essay on Profits* (see

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<sup>11</sup> In some of the more recent literature on Ricardo the '£ 1,000' mentioned by Ricardo are interpreted just as the unit of account in terms of which the analysis is carried out, taking it for granted that the sum represents a certain amount of 'corn'. We shall come back to this below.

<sup>12</sup> Actually, these are the words used in Fig. 31 (p. 406), whereas in Fig. 32 (p. 407) Walras used 'Successive Employments of Capital' and in Fig. 33 (p. 409) 'Employments of Capital'. But these changes do not appear to indicate any change in substance.

*Works IV*); this allowed Ricardo to focus attention on marginal (in the sense of no-rent) land. In Sraffa's interpretation (Sraffa, 1951, p. xxxi), the second step consisted of trying to get rid of the problem of value by assuming the so-called 'corn model'. The assumptions underlying this model are:

- (i) There is only one type of agricultural product, called 'corn'.
- (ii) Corn is the only wage-good.
- (iii) Capital in agriculture consists entirely of the wage-bill, i.e. corn is produced by labour and land only.

In this case the rate of profit in corn production can be ascertained directly as a ratio of two quantities of corn – that of the surplus product to that of the wage-bill advanced – without any need of having recourse to prices. With corn entering (directly or indirectly) the production of all other commodities (as the only wage good and possibly also as an input) the prices of these commodities would have to adjust such that the same competitive rate of return could be earned in their production.

As can be shown, in an economy satisfying assumptions (i)-(iii) there is no difficulty to construct production functions and plot marginal productivity of capital schedules for each quality of land. It is even possible to construct a production function and the corresponding marginal productivity of capital schedule for agriculture as a whole, even if land is diversified in quality.<sup>13</sup> However, in the case in which corn enters into the production of corn not only as a wage good, but also, as it is natural to assume, as a means of production (seed), it is no longer possible to plot the conventional marginal productivity schedules or to construct an aggregate production function for agriculture as a whole, unless one is willing to replace assumption (iii) by the following assumption:

- (iv) Capital consists of seed corn and wages, and the seed corn input is strictly proportional to the labour input.<sup>14</sup>

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<sup>13</sup> For a demonstration, see Freni (1991), whose results on this point are reported by Kurz and Salvadori (1992, pp. 230-5). This shows that Morishima's claim to the contrary (see Morishima, 1989, p. 103) is wrong. Obviously, the function need not be continuously differentiable.

<sup>14</sup> If neither assumption (iii) nor assumption (iv) hold, then neither a production function for each quality of land nor a production function for the whole

Hence, both Walras's and Kaldor's construction are correct if and only if either of the following sets of highly restrictive assumptions apply: (i), (ii) and (iii); or (i), (ii) and (iv).<sup>15</sup>

The *second* difference mentioned above is thus also dealt with. Once it is clear that a presentation in terms of the marginal productivity of capital is admissible only when there is a single commodity ('corn') and corn-inputs are proportional to labour-inputs, it is indeed possible to work in terms of a single curve representing agricultural (corn) production in the economy as a whole.<sup>16</sup>

The *third* difference reflects Walras's deviation from, or, as Stigler (1941) argued some time ago, misunderstanding of, Ricardo's treatment of the wages of labour.<sup>17</sup> In Ricardo wages are included in the capital advanced at the beginning of the uniform period of production, that is, they were taken to be paid *ante factum*: they form an integral part of

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economy can be built up. Freni (1991) has provided an example (the reader unable to read Italian can consult Kurz and Salvadori, 1995, p. 313) in which there is only one quality of land (so that there is no question of the existence of a production function for each quality of land or for the agricultural sector as a whole), one product (corn), one quality of labour, and corn does enter into the production of itself but not in proportion to labour inputs (i.e., seed-capital is not proportional to the wage-bill). In this example, for a given rate of profit (interest), a given amount of land and a given amount of corn to be produced over and above the amount required as an input, there are three possible solutions.

<sup>15</sup> The same applies to the construction by Samuelson (1959) and the one by Pasinetti (1960), which is similar to Kaldor's.

<sup>16</sup> Interestingly enough, Walras had an 'imperative need' (p. 411) for restating Ricardo's reasoning in terms of infinitesimals with respect to amounts of capital and amounts of product, but not in terms of qualities of land, which are finite in number in his exposition. An exposition in terms of one curve allows also 'infinitesimal' differences among qualities of land, which, as a consequence can be uncountably infinite in number.

<sup>17</sup> Stigler (*ibid.*, p. 251) wrote: 'We may note that Walras does not understand the true nature of the English dose of capital-and-labour. He subtracts labour costs from the product, whereas in the classical theory the composite dose of capital-and-labour was treated as a unit (and fundamentally, as a dose of *capital*).'

the dose of capital-cum-labour. Walras reckoned wages instead as a part of the net product, that is, took them to be paid *post factum*. This fact is relevant also because, for Ricardo, wages are a large part of the capital advanced by the capitalist. Hence wage-goods, and especially corn, are 'necessaries' both from the point of view of the single worker and from the point of view of the productive system as a whole. Below we shall see that, strangely enough, in Walras's interpretation of Ricardo's theory of rent corn does not enter directly (seed) or indirectly into its own production. Therefore Walras's reasoning cannot have been based on either of the two sets of assumptions specified above. The 'capital' employed in corn production in Walras's argument cannot be corn, but must be some other commodity or bundle of commodities. Alas, this is left in the dark, so that it remains unclear on which foundation his 'rigorous formulation' of the English theory of rent (p. 411) rests.

*Walras's formalisation of classical rent theory*

Walras's algebraic argument can be summarized as follows. Let  $h_i$  be the excess product per hectare of land of quality  $i$  over the payment of wages,  $x_i$  the 'amount of capital' in terms of the numéraire (and exclusive of the wages of labour) employed per hectare on land of that quality, and  $t$  the 'rate of interest charges expressed in terms of [physical] units of product' (p. 409), then the rent per hectare of land of quality  $i$ ,  $r_i$ , is given by

$$r_i = h_i - x_i t, \quad (i = 1, 2, \dots, s) \quad (1)$$

where  $s$  is the number of the different qualities of land available, each of which is in given supply  $n_i$ , and where  $h_i$  is assumed to depend exclusively on  $x_i$ , that is,

$$h_i = F_i(x_i). \quad (i = 1, 2, \dots, s) \quad (2)$$

Walras stressed that in (long-period) equilibrium each quality of cultivated land must earn the same physical return per unit of capital employed,  $t$ . In the case in which qualities 1 to  $m$  are cultivated ( $m \leq s$ ), we have

$$t = F_1'(x_1) = F_2'(x_2) = \dots = F_m'(x_m). \quad (3)$$

Counting the number of equations and unknowns in (1)-(3), Walras observed that there are only  $3m$  equations but  $3m+1$  unknowns: the unknowns are  $r_1, \dots, r_m$ ;  $h_1, \dots, h_m$ ;  $x_1, \dots, x_m$ ; and  $t$ . Hence there is a degree of freedom. How did Ricardo close the system?

Walras's answer was:

Another equation is needed. We can, *without deviating in any way from a faithful interpretation of Ricardo's theory*, write the

following equation which is analogous to those given in §§ 242 and 248:

$$n_1x_1 + n_2x_2 + n_3x_3 + \dots = X. \quad (4)$$

According to Ricardo, it seems that in every economy there is a certain amount of capital ... *At any given moment, the amount of capital is determinate.* Let us call such a determinate amount  $X$ , and let us distribute it among the different kinds of land in such a way that the rate of yield is the same on all lands (*ibid.*, p. 410; emphasis added).

Walras thus interpreted Ricardo as closing the system in terms of a given 'quantity' of social capital.

From what has been said above it follows that this interpretation cannot be sustained. First, with heterogeneous capital goods, it is not clear what a given 'quantity' of capital in terms of numeraire is supposed to mean, independently of relative prices. Walras's suggested closure only makes sense, if there is a single capital good, a case which both Ricardo and Walras relegated to the realm of fiction and to which they attributed at most a heuristic value in economic analysis.<sup>18</sup> Moreover, to take  $X$  as given is by no means necessitated by the desire to get a determinate system. This becomes clear when we give a closer look to the exact role played by Walras's above closure. In order to be able to determine the rents of land and the rate of profit, given the real wage rate, what we need to specify is the *amount* of total corn production (see (R2)). This can be done in several ways. In a model in which corn is the only capital good, there is no harm in fixing total corn production in terms of the amount of corn capital employed in the growing of corn. This is indeed the assumption needed in order for Walras's reasoning to make sense. In this case equation (4) would provide the required information. However, there are more direct, and less ambiguous, ways to specify the size of corn

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<sup>18</sup> In dealing with the problem of 'capitalization' Walras in fact assumed that the economy's endowment of capital is specified in terms of a vector of given amounts of capital goods proper. However, as he noted in the fourth edition of the *Eléments*, an arbitrarily given composition of the initial capital stock, flukes apart, will generally not be compatible with a long-period equilibrium, that is, 'it is probable that there would be no equality of rates of net income' (p. 308); see also the discussion in Kurz and Salvadori (1995, pp. 439-41).



production, one of which is, of course, giving the overall level of corn output. Alternatively, the level of corn output exclusive of the wages bill could be given, and so on. The latter alternative would imply an equation like

$$n_1h_1 + n_2h_2 + n_3h_3 + \dots = H, \quad (4')$$

where H is the excess product of corn over the payment of wages. Clearly, equation (4') would do the job equally well as equation (4), and in addition, it may be contended, is more faithful to Ricardo.

*The dubious assumption of a given 'quantity of capital'*

The fact that Walras's interpretation is inconclusive follows also from a critical scrutiny of his claim that equation (4) is 'analogous' to the equation given in §§ 242 and 248. While the two are formally similar – *logically* they are very different. In contradistinction to equation (4), the allegedly 'analogous' equation in §§ 242 and 248 is not just an equation fixing the total amount of corn produced, but an *equilibrium* equation in which E (which plays the role played by X above) is the 'algebraic sum of the individual excesses of income over consumption' (§ 242, p. 275). E is therefore not a given, but a magnitude to be determined endogenously: it has the role of relating investment to saving and of rendering the two equal to one another. In short, it refers to a savings-investment equilibrium. The reference is not to a given endowment of capital, but to 'new capital goods' (p. 281).

Going back to Lesson 39, Walras then proceeded to determine t for a given X by first solving equations (3) for given values of t, which gave him the  $x_i$ 's corresponding to the different values of t, or  $x_i = \varphi_i(t)$ . He stressed: 'The lands for which  $F'(0) < t$  will not be cultivated; only those for which  $F'(0) > t$  will be brought under cultivation' (§ 355, pp. 410-11). Replacing the  $x_i$ 's by the  $\varphi_i(t)$ 's in equation (4) provided him with one equation for the only unknown t. Alternatively, it is easily checked that replacing the  $h_i$ 's by the  $F_i(\varphi_i(t))$ 's in equation (4') gives once again one equation for the only unknown t. Once t is determined, the other variables can be ascertained. Walras concluded:

Thus in final analysis rent depends on the capital of a country, and is determined without regard to wages, interest or the prices of products. This is the essence of the English theory of rent (p. 411).

This conclusion can be criticised both externally and internally. The external criticism is, of course, that Ricardo took the levels of normal output instead of the 'capital of a country' as the independent variable. This involved considering the size and composition of social capital as an endogenous variable that is taken to be fully

adjusted to the other data, such that a uniform rate of profit on and normal levels of utilization of capital obtain. As regards the internal criticism, assume first that the wage rate happens to be at another level. This would change equations (2) and *a fortiori* the rate of interest. Second, as we have seen, the only cases in which Walras's exposition would be correct are those in which there is only a single commodity (corn) which either does not enter into its own production, apart from the wages paid to workers, or, if it does, enters into them in proportion to labour. In this case it is trivially true that the prices of products do not appear in the equations. In cases with more than one commodity this is obviously no longer true. Since, as we shall see, in Walras's interpretation of Ricardo's rent theory there are at least two commodities to be taken into consideration in the production of corn, namely corn (output) and a capital good proper which is different from corn, the problem of relative prices cannot be avoided.

*The need for reasoning in terms of infinitesimals*

Walras introduced § 356 in the following terms: 'The need for restating Ricardo's reasoning in terms of infinitesimals is so imperative that a number of authors have succumbed to it even though they continued to use ordinary language. Hence the rigorous formulation which we have just given to this reasoning is the *true formulation of the English theory of rent*' (p. 411; emphasis added).<sup>19</sup> This formulation is then taken to point out 'defects in exposition and deduction resulting from the cruder modes of expression which were used by Ricardo and Mill' (ibid.). Walras maintained that

Mill's first theorem, which is in essence based on the assumption that the worst land yields no rent, is intrinsically erroneous and formally contradicts the second theorem. ... It is only necessary to inspect Fig. 33 to perceive at once that the worst lands under cultivation do, in general, yield a rent, except in the unusual case of a discontinuous productivity curve which cuts the horizontal line (representing the rate of production) only at its starting-point (§ 356, p. 411).

As a matter of fact, we do not need an argument 'in terms of infinitesimals' to see that when there is intensive rent the 'worst land' may yield a rent. For this purpose compare Walras's Figures 32 and 33. Walras emphasized the fact that when  $t = OP$  in Fig. 33, land 3 is not cultivated, land 2 is the 'worst land', but nevertheless gets a rent which is equal to the area  $QT_2y_2$ . However, if we look at Fig. 32, we see that when  $t = OL$ , once

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<sup>19</sup> Similarly, Wicksteed (1894, p. 48).

again land 3 is not cultivated, once again land 2 is the 'worst land', nevertheless once again it gets a rent which is equal to the rectangle below the segment  $t_2y_2'$  and above the straight line  $Lt_1'y_1''$ .

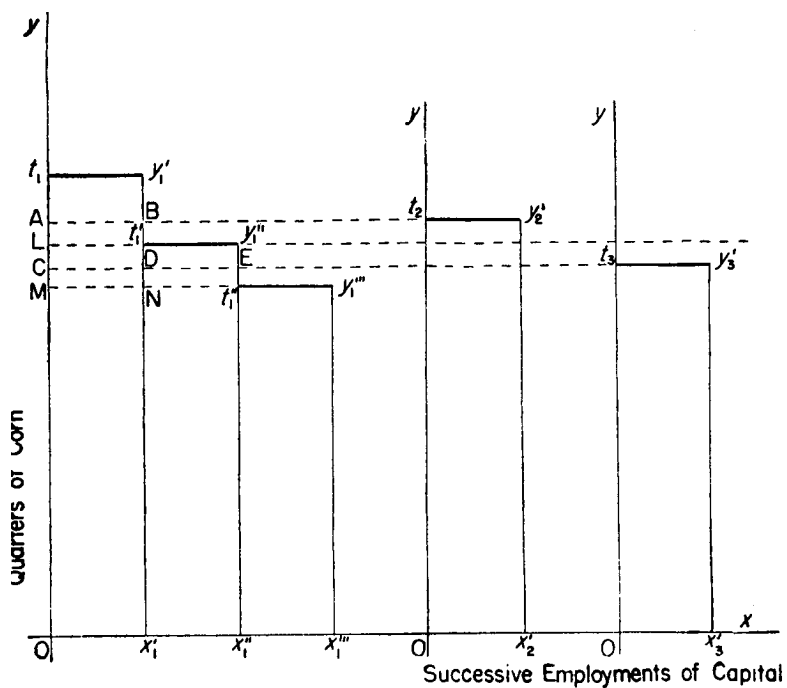


FIG. 32<sup>(6)</sup>

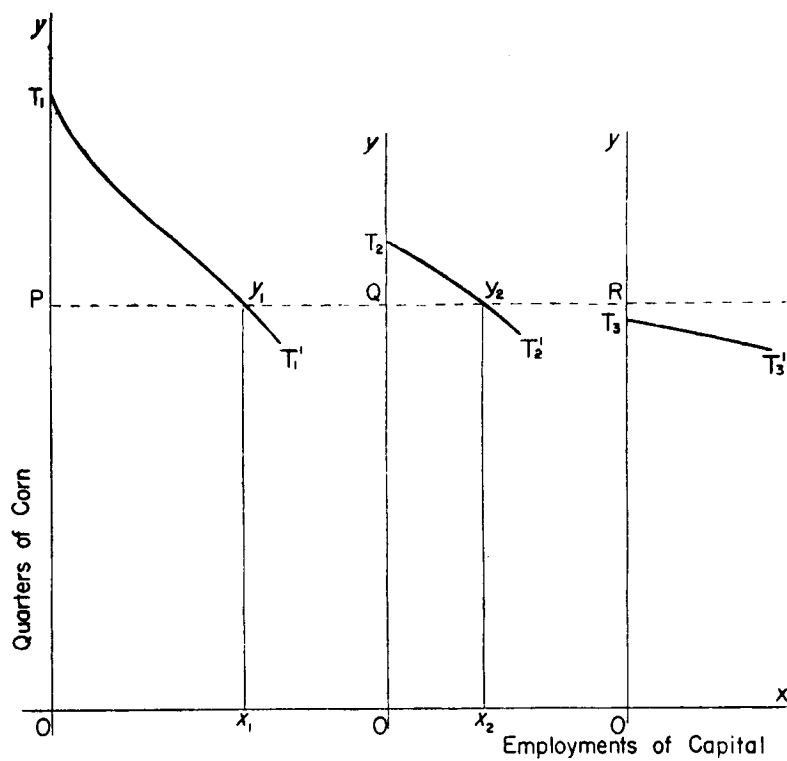


FIG. 33<sup>(6)</sup>

*A reformulation of the classical theory of rent*

The last sections of Lesson 29 (§§ 357-362) are devoted to what Walras considered his main criticisms of the Ricardian theory of rent. Before we shall address these sections it is useful to complete our discussion of Ricardo's consecutive attempts to simplify the theory of value and distribution. In the above we have already mentioned his first two steps – (i) getting rid of rent by focusing on marginal land; and (ii) getting rid of the problem of value in terms of the 'corn model'. This discussion turned out to be helpful in understanding the singularly restrictive assumptions implied in order for a Walras, Kaldor, Pasinetti or Samuelson to be justified in drawing marginal productivity of capital schedules and production functions.

Against Ricardo's basic principle, derived from the 'corn model', that 'it is the profits of the farmer that regulate the profits of all other trades' (*Works* VI, p. 104), Malthus objected that there is no industry in which the product is exactly of the same kind as the capital advanced. Ricardo did not dispute this, and, in a *third* step, in the *Principles* attempted to cope with the problem of heterogeneous goods in terms of a theory of value according to which the exchange values of commodities are regulated by the quantities of labour needed directly and indirectly in their production. Yet, Ricardo soon realised that what was to become known as the 'labour theory of value' cannot generally be sustained, because of differences in the proportions in which direct labour and means of production are employed in different industries. According to Sraffa the search for an 'invariable measure of value' may be considered the *fourth* and final step in Ricardo's efforts to grope his way towards a coherent theory of distribution.

Today we are possessed of a powerful analytical scheme capable of dealing with all the intricacies in the theory of distribution and relative prices the classical economists (and Walras) failed to master satisfactorily. Thus, in the modern formulation of this theory the strong assumptions adopted by Ricardo are abandoned (see, for instance, Kurz and Salvadori, 1995, ch. 10, and the literature referred to there). We shall now investigate §§ 357-362 of Walras's *Elements* against the background of modern classical analysis.

Walras was keen to establish the price equation

$$p_b = b_t p_t + b_p p_p + b_{p'} p_{p'} + b_{p''} p_{p''} + \dots + b_k p_k + b_{k'} p_{k'} + b_{k''} p_{k''} + \dots \quad (5)$$

where  $p_b$  is the price of the product ( $b$  is the abbreviation of the French word for corn, *blé*), the  $b$ 's are technical coefficients of production,  $p_t$  is the rate of rent in terms of the numeraire, the  $p_p$ 's are the prices of 'personal services', i.e. wages, and the  $p_k$ 's are the prices of 'capital services', i.e. interest charges. The  $b$ 's are variables, because there is a choice of technique, and all the  $p$ 's need to be determined by the theory. It is interesting to note that this equation is precisely one of the equations one encounters in the modern

theory of rent of classical derivation. For instance, in Kurz and Salvadori (1995, p. 298) we find the following model:

$$\mathbf{x}^T(\mathbf{B} - \mathbf{A}) \geq \mathbf{d}^T, \quad (6a)$$

$$\mathbf{x}^T(\mathbf{B} - \mathbf{A})\mathbf{p} = \mathbf{d}^T\mathbf{p}, \quad (6b)$$

$$\mathbf{x}^T\mathbf{C} \leq \mathbf{t}^T, \quad (6c)$$

$$\mathbf{x}^T\mathbf{C}\mathbf{q} = \mathbf{t}^T\mathbf{q}, \quad (6d)$$

$$\mathbf{B}\mathbf{p} \leq (1+r)\mathbf{A}\mathbf{p} + \mathbf{C}\mathbf{q} + w\mathbf{l}, \quad (6e)$$

$$\mathbf{x}^T\mathbf{B}\mathbf{p} = \mathbf{x}^T[(1+r)\mathbf{A}\mathbf{p} + \mathbf{C}\mathbf{q} + w\mathbf{l}], \quad (6f)$$

$$\mathbf{x} \geq 0, \mathbf{p} \geq 0, w \geq 0, \mathbf{q} \geq 0, \mathbf{u}^T\mathbf{p} = 1, \quad (6g)$$

where  $\mathbf{B}$  is the  $m \times n$  output matrix,  $\mathbf{A}$  is the  $m \times n$  matrix of producible inputs,  $\mathbf{C}$  is the  $m \times s$  land input matrix,  $\mathbf{l}$  is the  $m$ -vector of labour inputs,<sup>20</sup>  $\mathbf{d}$  is the  $n$ -vector of the levels of net (effectual) demand,  $\mathbf{t}$  is the  $s$ -vector of the existing amounts of the  $s$  different qualities of land,  $\mathbf{x}$  is the  $m$ -vector of the process intensities,  $\mathbf{p}$  is the  $n$ -vector of prices,  $\mathbf{q}$  is the  $s$ -vector of rent rates,  $r$  is the competitive rate of profit (interest), and  $w$  is the wage rate. In agreement with Walras's procedure, which differed from Ricardo's, wages are assumed to be paid *post factum*, that is, out of the net product. Inequality (6a) states that net demand can be met. Equation (6b) gives the rule of free goods with regard to products: commodities that are overproduced will fetch a zero price. Inequality (6c) states that the amounts of the different qualities of land needed in production are not larger than the amounts in given supply. Equation (6d) gives the rule of free goods with regard to land: the rent of a quality of land that is not fully employed is zero. Inequality (6e) is the no extra profit condition. Equation (6f) implies that processes incurring extra costs will not be operated. The last of the expressions of (6g) fixes the numeraire in terms of which wages, prices and rents are expressed; the meaning of the other conditions is obvious.

As was just stated, only some of the above weak inequalities will be satisfied as equations. In this way modern classical economics takes account of the fact that the

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<sup>20</sup> For simplicity it is assumed that there is only one quality of labour; for models with heterogeneous labour, see Kurz and Salvadori (1995, ch. 11).

technical coefficients (Walras's  $b$ 's) may vary because there is a choice of technique.<sup>21</sup> Moreover, the presence of the rate of interest in system (6) must not prevent one from recognizing that Walras's above equation is exactly one of the weak inequalities (6e) which is solved as an equation, because Walras's  $p_k$ 's are interest charges and not prices: they equal exactly  $(1 + r)$  times the price in the framework of system (6), in which fixed capital is treated as a joint product. The only differences are that in formula (5) only one quality of land is involved in production, whereas in the weak inequalities (6e) more than one quality of land can be used as an input; and that system (6) allows for joint production and, therefore, also for the treatment of old machines left at the end of each period as economically different goods from the machines which entered production at the beginning of the period.<sup>22</sup> However, Ricardo's assumption that at most one quality of land figures as an input in each process of production was criticized by Walras as unnecessarily restrictive.<sup>23</sup> It therefore cannot be a source of disagreement

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<sup>21</sup> Actually Walras assumed that the  $b_p$ 's and the  $b_k$ 's are not only variable but 'decreasing functions of'  $b_t$  (§ 359, p. 413). This is of course not generally true: it is very well possible that a larger output per hectare is obtained by using less (or even nothing) of some input(s) and more of some other input(s) or some positive amount of some input(s) not used at all at the smaller level of production.

<sup>22</sup> This way of treating fixed capital can be traced back to the classical economists (see Sraffa, 1960, pp. 94-5); it was also adopted by John von Neumann. For a critical remark on Walras's treatment of fixed capital, see below.

<sup>23</sup> According to Walras 'the English theory of rent is based on the further assumption that no more than one kind of land-service ever enters into [any given branch of] production. This hypothesis is no more appropriate to agriculture than to industry. For example, the wheat in Ricardo's illustration does not fit into this hypothesis, since the production of wheat requires fertiliser as a raw material which is derived from cattle that graze on grass-lands, and these grass-lands are different from wheat land' (p. 417). This elicits three comments. First, Walras's conclusion applies more to his own formalisation of Ricardian rent theory than to Ricardo's theory itself. Secondly, while it is true that in Ricardo in each process of production at most one quality of land was taken to be a *direct* input, Ricardo did not, of course, rule out that other qualities of land may indirectly enter into the production. Thirdly, the purely technical question of which lands enter directly or indirectly into the production of a

between us and Walras. The real disagreement between Walras on the one hand and us (following Ricardo) on the other concerns the way in which he attempted to determine the rate of profit: he conceived of it as reflecting the scarcity of a factor called 'capital proper'. However, as we have seen, he himself occasionally showed signs of awareness that this was perhaps not possible in a long-period framework in which the natural assumption is that the quantities of the different capital goods have had time to adjust such that a uniform rate of profit obtains. The disagreement concerns once again the substance of the theory as it is reflected in the data from which to start: in model (6) there is no equation or inequality relating to the available 'quantity of capital' (or that of labour); instead one of the distributive variables (the real wage rate or, alternatively, as in our formalisation, the rate of profit) is considered as given. As we have seen, Walras advocated a different approach, which, however, ran into difficulties as Walras by the time of the fourth edition of the *Elements* began to recognise.<sup>24</sup> But let us return to a discussion of Lesson 39.

*Walras's misunderstandings of Ricardo's theory of rent*

In § 352 Walras remarked that in the quoted passage on extensive rent 'Ricardo does not state expressly in what terms these employments of capital are evaluated or what their value is; but in the second part he explicitly supposes that they are evaluated in terms of money ['numeraire'] and that their value is £1,000 each' (p. 405). In his formalisation of Ricardo's theory Walras instead measured the outputs  $h_i$ , the rates of rent  $r_i$ , and 'the rate of interest charges expressed in terms of [physical] units of product'  $t$  in terms of product. He therefore felt the need to come back to this issue, especially because it was at the centre of his criticism. In § 357 he restated equation (1) when all variables are expressed in terms of numeraire (the  $i$ 's are dropped for the sake of simplicity):

$$\frac{p_t}{p_b} = h - x \frac{i}{p_b}$$

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commodity has nothing to do with the question of whether a particular quality of land yields its proprietor a rent. Focusing on marginal (i.e. no-rent) land therefore does not imply that Ricardo turned a blind eye to interdependences in production.

<sup>24</sup> A discussion of these difficulties is beyond the scope of this paper. See therefore the summary account in Kurz and Salvadori (1995, chs. 1 and 14)

where  $i$  is here 'the rate of net income in terms of numeraire'<sup>25</sup> and therefore  $r = p_t/p_b$  and  $t = i/p_b$ .

In our interpretation, with the doses of £1,000 Ricardo intended to refer to the units of account, that is, units of corn, in terms of which he conducted his analysis, and the fact that they are expressed as if they were 'in money terms' does not affect the substance of the argument. Walras, on the contrary, inferred from the fact that they are expressed in money, i.e. numeraire: 'Since each application always amounts to £1,000, it follows that the prices of the capital goods in question are determinate and constant' (§ 359, p. 414). Here Walras is confronted with elements of his interpretation and 'correction' of the classical analysis that contradict each other. As we have argued above, the construction of production function(s) followed by Walras (and several other commentators, including, for example, Kaldor) is possible if and only if the assumptions of the 'corn model' hold, that is, if and only if the 'agricultural' sector produces only *one* commodity which is either not used in production or is used in strict proportion to the amount of labour employed and no other produced commodity is used in production. On the contrary, Walras first introduced a production function for each quality of land and insisted to measure the non land input in money, and then used this fact to argue that relative prices need to be constant. This interpretation comes as a surprise also because Ricardo was very clear about the fact that in general (i.e. setting aside the case of the 'corn model') the concept of 'capital' is a difficult one. In a letter to McCulloch he wrote: 'I would ask what means you have of ascertaining the equal value of capitals? ... These capitals are not the same in kind – what will employ one set of workmen, is not precisely the same as will employ another set ...' (*Works IX*, pp. 359-60; see also his letter to Torrens, *Works IV*, pp. 393-4). Walras continued: 'This hypothesis [of the constancy of input prices] has important consequences' (§ 360, p. 414). In his opinion 'It led Ricardo to base the existence, the origin and the growth of rent on the increasing dearness of products. Indeed, in his view, cost of production determines selling price' (*ibid.*). How is it possible for the price of the corn output to rise, while the price of corn as an input remains the same? Walras overlooked that corn is both needed as seed and to feed workers: he missed the important aspect of circular flow of production in Ricardo.

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<sup>25</sup> The definition of the rate of net income  $i$  is given in § 233 (pp. 268-9) and from it we see immediately that  $i$  is a pure number. Nevertheless, when referring to the rate of net income, Walras insisted to add: 'in terms of numeraire'.



Finally, Walras's main argument to show that the price of corn is bound to rise is totally untenable. He first argued that (note that  $h$  in the above formula equals  $1/b_t$  in the following one)

$$p_b = b_t(p_t + xi) = \frac{p_t}{r} = \frac{i}{t},$$

then he added: 'if we ignore variations in  $i$ , this last ratio will increase indefinitely as  $t$  decreases, which is the basis of the theory' (§ 360, p. 415). How can one ignore variations in  $i$ ? How is it possible that an increase in the price of corn, *given* the prices of the capital items *and given*  $i$ , could be said to be 'the basis of the theory' of the English economists? Walras summarized:

Thus, the English theory can only determine the price of land-services and demonstrate its residual character on the twofold assumption that the prices of personal capital, the prices of capital goods proper and the rate of net income are predetermined and constant, and that, therefore, the prices of the services of personal capital and capital goods proper are also predetermined and constant (§ 361, p. 415).

Unfortunately, he refrained from substantiating his surprising claim in terms of some evidence from Ricardo's *Principles*. In fact, no such evidence can be provided. In Ricardo the falling tendency of the rate of interest is inextricably intertwined with the theory of rent. To assume that the rents of land go up, but the rate of profit stays constant, misses the whole point of Ricardo's theory of distribution. It is astonishing how Walras could go so much astray. He continued:

We may ask ... why the English School determines rent by the quantities of labour and capital-services employed, rather than wages and interest by the quantities of land-services employed; or why this school does not try to formulate a unified general theory to determine the prices of all productive services in the same way (p. 416).

This is indeed the crucial question: Is it possible in a long-period framework of the analysis to generalize the principle of rent to an explanation of all kinds of income in the same way and thus interpret the wages of labour as well as the profits of capital as scarcity prices of the respective factors of production, labour and capital? Walras thought that this was indeed possible. In his concluding words of Lesson 39:

Thus, all that remains of Ricardo's theory after a rigorous critical analysis is that rent is not a component part, but a result, of the price of products. But the same thing can be said of wages and interest. Hence, rent, wages, interest, the prices of products, and the coefficients of production are all unknowns within the same problem; they must always be determined together and not independently of one another (p. 418).

The crucial question is: How can this be accomplished? In terms of which approach and using which data can the technique adopted, the distributive variables and relative prices be consistently determined? Walras and neoclassical economists in general assume that the supplies of labour and 'capital' *must* be among the givens also in a long-period framework. Classical economists argue on the contrary that in the long period either the wage rate or the rate of profit must be treated as an independent variable, because the 'capital endowment' cannot be considered a datum in long-period analysis. As is analytically well established since the debate on capital theory in the 1960s and early 1970s, the intuition of the old classical economists was perfectly sound.<sup>26</sup>

## **5. Walras on the Ricardian theory of wages and profits**

In Lesson 40 Walras dealt with the classical theory of wages and profits. The lesson is almost exclusively devoted to a criticism of John Stuart Mill and especially his wage fund doctrine. These parts need not concern us here. There is only one section which is somewhat related to Ricardo's way of thinking and which deserves to be commented upon. We quote § 368 in full:

Let  $P$  be the aggregate price received for the products of an enterprise; let  $S$ ,  $I$  and  $F$  be respectively the wages, interest charges and rent laid out by the entrepreneurs, in the course of production, to pay for the services of personal faculties, capital and land. Let us recall now that, according to the English School, the selling price of products is determined by their costs of production, that is to say, it is equal to the cost of the productive services employed. Thus we have the equation

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<sup>26</sup> See, for example, Garegnani (1990) and Kurz and Salvadori (1995, ch. 14).

$$P = S + I + F, \quad [(7)]$$

and P is determined for us. It remains only to determine S, I and F. Surely, if it is not the price of the products that determines the price of productive services, but the price of productive services that determines the price of the products, we must be told what determines the price of the services. That is precisely what the English economists try to do. To this end, they construct a theory of rent according to which rent is not included in the expenses of production, thus changing the above equation to

$$P = S + I.$$

Having done this, they determine S directly by the theory of wages. Then, finally, they tell us that "the amount of interest or profit is the excess of the aggregate price received for the products over the wages expended on their production", in other words, that it is determined by the equation

$$I = P - S.$$

It is clear now that the English economists are completely baffled by the problem of price determination; for it is impossible for I to determine P at the same time that P determines I. In the language of mathematics one equation cannot be used to determine two unknowns. This objection is raised without any reference to our position on the manner in which the English School eliminates rent before setting out to determine wages.

Before we enter into a discussion of this criticism two observations should be made. First, essentially the same objection was put forward by William Stanley Jevons.<sup>27</sup>

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<sup>27</sup> Jevons wrote: 'Another part of the current doctrines of Economics determines the rate of profit of capitalists in a very simple manner. The whole produce of industry must be divided into the portions paid as rent, taxes, profits and wages. We may exclude taxes as exceptional, and not very important. Rent also may be eliminated, for it is essentially variable, and is reduced to zero in the case of the poorest land cultivated. We thus arrive at the simple equation –

$$\text{Produce} = \text{profit} + \text{wages}.$$

Secondly, the claim that Ricardo tried to determine two unknowns with a single equation is perhaps an expression in these authors that comes closest to admitting that there *is* a distinct classical approach which is fundamentally different from the neoclassical one. It is interesting to notice that both Walras and Jevons interpreted Ricardo as having treated the real wage rate as given in the theory of value and distribution, or rather, in the words of Walras, as having determined it 'directly by the theory of wages'. In other words, both Jevons and Walras appear to have admitted that Ricardo attempted to determine the rate of profit and relative prices in terms of something like datum (R3). This leaves two unknowns: the rate of interest (profit) and the overall size of the product. Now it is not true that Ricardo was 'completely baffled' by the problem of price determination, as Walras contended. It is rather Walras who might be said to have been completely baffled by the way Ricardo approached the task: he took the overall size of the product as given (see (R2)). For Walras, to whom economics *was* demand and supply theory, this analytical option did not exist – it was in fact unimaginable to him. He rather took it for granted that the size of the product had to be determined on the basis of the available amounts of productive resources, that is, in terms of datum (W3).

Once this is seen, it becomes clear that the critique of Walras and Jevons misses its target. The critique was explicitly refuted by the Russian mathematical economist Vladimir K. Dmitriev ([1898] 1974, pp. 51 ssq.), who showed that on the basis of (R1)-(R4) the remaining distributive variable(s) and relative prices can be consistently determined.<sup>28</sup> In the following we shall provide an argument that is logically identical

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A plain result also is drawn from the formula; for we are told that if wages rise profits must fall, and *vice versa*. But such a doctrine is radically fallacious; *it involves the attempt to determine two unknown quantities from one equation*. I grant that if the produce be a fixed amount, then if wages rise profits must fall, and *vice versa*. Something might perhaps be made of this doctrine if Ricardo's theory of a natural rate of wages, that which is just sufficient to support the labourer, held true. But I altogether question the existence of any such rate' ([1871], 1911, pp. 268-9; emphasis in the original).

<sup>28</sup> A few years before Dmitriev Knut Wicksell ([1893] 1954) had defended Ricardo against his critics. In Wicksell's view 'the way in which Ricardo develops his argument ... is a model of strictly logical reasoning about a subject which seems, at first glance, to admit of so little precision'; and 'Ricardo's theory of value is, one finds, developed with a high degree of consistency and strictness' (ibid., pp. 34 and 40). He added: 'Since, according to Ricardo, wages

to Dmitriev's, but which refers also to other parts of the *Elements*. In this way we intend to throw some additional light on certain aspects of Walras's thought which do not always seem to have been properly understood (see also Kurz and Salvadori, 1995, pp. 25-6).

Equation (7) is nothing else than equations (4) in § 203 of the *Elements* (p. 240), which are here presented using matrix notation:

$$\mathbf{p} = \mathbf{C}\mathbf{q} + \mathbf{E}\mathbf{w} + \mathbf{A}\mathbf{y},$$

where  $\mathbf{p}$  is the vector of prices of output (Walras assumes that the first element of  $\mathbf{p}$  equals unity since the first commodity acts as the numeraire),  $\mathbf{C}$  is the matrix of the production coefficients of land inputs of the operated processes,  $\mathbf{q}$  is the vector of prices of land services (i.e., the rent rates),  $\mathbf{E}$  is the matrix of the production coefficients of labour inputs of the operated processes,  $\mathbf{w}$  is the vector of prices of personal services (i.e., the wage rates),  $\mathbf{A}$  is the matrix of the production coefficients of the inputs of capital goods proper of the operated processes, and  $\mathbf{y}$  is the vector of prices of the services of capital goods proper.<sup>29</sup> If, following Ricardo, as mentioned by Walras himself, we take account only of the technology used at the margin (either extensive or intensive)<sup>30</sup> and if we assume for simplicity that there is only one quality of labour, we obtain the equation

$$\mathbf{p} = \mathbf{w}\mathbf{l} + \mathbf{A}\mathbf{y}. \tag{8}$$

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represent a magnitude fixed from the beginning, and since – as he later shows – the level of rent is also determined by independent causes, the cause of capital profit is already settled. It is neither possible nor necessary to explain capital profit in other ways, if the other assumptions are sound' (ibid., pp. 36-7). Therefore, in Wicksell's view Ricardo's system was not underdetermined. (This does not mean, of course, that Wicksell agreed with the content of Ricardo's theory; on Wicksell's theory of distribution, see Kurz (1999b).)

<sup>29</sup> As in the previous formalisation (see system (6)) and in accordance with Walras, rents and wages are taken to be paid at the end of the uniform period of production.

<sup>30</sup> For the intensive margin we have to follow the procedure provided by Guichard (1982).

Finally, assume that all capital goods proper are circulating capital, so that the depreciation charges equal the corresponding prices, and that the insurance premiums are nought, then

$$\mathbf{y} = (1 + i)\mathbf{p},$$

where  $i$  is the 'rate of net income'<sup>31</sup> and equation (8) becomes

$$\mathbf{p} = \mathbf{w}\mathbf{l} + (1 + i)\mathbf{A}\mathbf{p}, \quad (9)$$

which is a system of  $n$  equations in  $n + 1$  unknowns, since by definition the first element of  $\mathbf{p}$  equals unity, where  $n$  is the number of products, some (or all) of which could be capital goods proper.<sup>32</sup>

<sup>31</sup> Walras stressed that in equilibrium the rate of net income 'is the same for all capital goods' (§ 233, p. 269; see also §§ 238 and 249); see also Walker (1996, p. 214).

<sup>32</sup> In § 238 (similarly § 232) Walras asserts that if  $P_k$  is the price of a capital good proper, its depreciation charge and its insurance premium are respectively  $\mu_k P_k$  and  $v_k P_k$ . If the mentioned capital good is a circulating one, then  $\mu_k = 1$ ; and if the insurance premium on it is nought, then  $v_k = 0$ . These assumptions are adopted only for the sake of simplicity. If, on the contrary,  $\mathbf{M}$  is the diagonal matrix with the exogenously given depreciation charges on the main diagonal and  $\mathbf{V}$  is the diagonal matrix with the given insurance premiums on the main diagonal, then

$$\mathbf{y} = (\mathbf{M} + \mathbf{V} + i\mathbf{I})\mathbf{p},$$

and the equations (9) and (10) become

$$\mathbf{p} = \mathbf{w}\mathbf{l} + \mathbf{A}(\mathbf{M} + \mathbf{V} + i\mathbf{I})\mathbf{p} \quad (9')$$

and

$$\mathbf{p} = \mathbf{l}\mathbf{b}^T\mathbf{p} + \mathbf{A}(\mathbf{M} + \mathbf{V} + i\mathbf{I})\mathbf{p}, \quad (10')$$

respectively. Finally  $1/i$  is determined as the Perron-Frobenius eigenvalue of matrix

$$(\mathbf{I} - \mathbf{l}\mathbf{b}^T - \mathbf{A}\mathbf{M} - \mathbf{A}\mathbf{V})^{-1}\mathbf{A}$$

Taking (with Ricardo) the real wage rate as given, we get the  $n + 1$ st equation needed to determine prices *and* distribution. In fact if

$$w = \mathbf{b}^T \mathbf{p},$$

where  $\mathbf{b}$  is a given vector defining the real wage rate, and assuming (with Walras) that wages are paid post factum, equation (9) becomes

$$\mathbf{p} = \mathbf{I} \mathbf{b}^T \mathbf{p} + (1 + i) \mathbf{A} \mathbf{p}, \quad (10)$$

and if the elements of  $\mathbf{b}$  are small enough, then matrix  $\mathbf{I} - \mathbf{I} \mathbf{b}^T$  is invertible with a semipositive inverse and

$$\mathbf{p} = (1 + i)(\mathbf{I} - \mathbf{I} \mathbf{b}^T)^{-1} \mathbf{A} \mathbf{p}.$$

That is  $1/(1 + i)$  is the Perron-Frobenius eigenvalue of matrix  $(\mathbf{I} - \mathbf{I} \mathbf{b}^T)^{-1} \mathbf{A}$  and  $\mathbf{p}$  is the corresponding eigenvector whose first element equals unity.

We may thus conclude that Walras's criticism is untenable: Ricardo cannot be accused of having attempted 'to solve two unknowns with one equation'. Ricardo's system is perfectly determinate. The data, or independent variables, (R1)-(R4) from which he started his analysis of the problem of value and distribution suffice to determine the unknowns, or dependent variables, that is, the rate of profit, the rent rates and prices in terms of the given numeraire. No other data, such as utility or demand functions, are needed. In his reading of Ricardo Walras was misled by the idea that there is only a single kind of theory in economics: demand and supply theory. Assessed in terms of his own theory Ricardo's was bound to look strange. Had Walras given a closer look to Ricardo's construction he would have found out that there was no indeterminacy.

## 6. Concluding remarks

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and  $\mathbf{p}$  as the corresponding eigenvector whose first element equals unity. What prevented us from using these equations in the text is this: the modern treatment of fixed capital has clarified that a correct analysis cannot consider the depreciation charges as given independently of the rate of profit (or interest); see Kurz and Salvadori (1995, p. 210, and in general chapters 7 and 9). Therefore, Walras's approach to fixed capital cannot be sustained.

In this paper we have scrutinized Walras's criticism, in Part VII of the *Elements*, of the classical theory of value and distribution, paying special attention to Ricardo's contribution. We have shown that Walras succeeded in pointing out some weaknesses and unnecessarily restrictive assumptions in Ricardo's theory of rent and that certain aspects of his interpretation may be said to foreshadow the later ones by Nicholas Kaldor, Luigi Pasinetti and Paul A. Samuelson. However, Walras does not appear to have been aware of the fact that singularly restrictive assumptions must hold in corn production in order for marginal productivity curves of capital with regard to each quality of land to exist. In particular, the only capital good employed has to be corn, that is, an input identical with the output. That Walras did not assume this becomes clear when we turn to Walras's misinterpretations of Ricardo. First, there are misunderstandings of specific elements of Ricardo's theory. These include Ricardo's treatment of the wages of labour as a part of the capital advanced at the beginning of the period of production; instead Walras considered them as a part of the net product. Then there is a lack of understanding on Walras's part of the circular flow of production in Ricardo and especially of the fact that corn is considered a product that enters into its own production (via the wages of labour and seed capital) and, besides this, also into the production of other commodities. This implies, among other things, that the capital employed in corn production in his attempted formalisation of Ricardo's rent theory cannot be physically identical with the product. It follows that Walras was not entitled to draw marginal productivity curves of capital.

More important, Walras failed to see that the classical approach to the theory of value and distribution is fundamentally different from his own demand-and-supply approach. He treated Ricardo's theory as if it was just an early and rude version of his own elaborate neoclassical general equilibrium theory. This theory attempts to determine quantities, relative prices of goods and income distribution in terms of the following data: (W1) technical alternatives; (W2) preferences; and (W3) initial endowments of factors of production, including capital. Ricardo in his theory was said to have started essentially from the same sets of data, but to have imposed unnecessary restrictions on them and in addition to have committed logical blunders. Entirely in line with his neoclassical perspective of Ricardo, Walras believed to have been faithful to the English economist when 'closing' his model of the Ricardian theory of rent in terms of a given 'quantity of capital'. He missed the fact that the data of the classical theory are different: (R1) technical alternatives; (R2) the size and composition of the social product; (R3) the real wage rate; and (R4) the quantities of land available. He also missed the fact that in terms of these data the dependent variables – the rate of profit, the rent rates and relative prices – are fully determinate. There is no need, and indeed no possibility, to add some further givens, such as the capital endowment of the



economy or utility. Walras's objection that Ricardo tried 'to determine two unknowns with one equation', involving that his system is underdetermined, has shown to be false.

Walras's attack on Ricardo was meant to clear away the classical theory of value and distribution and establish the superiority of his own – the only and 'truly scientific theory of social wealth' (p. 428). To Walras all prices and all distributive variables were to be explained simultaneously and symmetrically in terms of demand and supply. The asymmetric treatment of the distributive variables in the classical authors, who took the real wage rate as given and determined all shares of income other than wages residually, was totally extraneous to his way of thinking. In this regard Walras's interpretation of Ricardo does not differ much from that of other neoclassical authors, for example Jevons. They showed similar difficulties to apprehend and appreciate the distinct character of the classical analysis which had gradually been 'submerged and forgotten since the advent of the marginal method' (Sraffa, 1960, p. v).

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