Walras, Léon (1834–1910)

Léon Walras was the founder of the modern theory of general economic equilibrium. He was born on 16 December 1834 in Evreux, which is in the Department of Eure in France, and christened Marie Esprit Léon. He died on 5 January 1910 in Clarens, Switzerland. His father was Antoine Auguste Walras, a secondary school administrator and an amateur economist; his mother was Louise Aline de Sainte Beuve, the daughter of an Evreux notary. After studying at the College of Caen from 1844 to 1850, he entered the Lycée of Douai, where he received the bachelier-ès-lettres in 1851 and the bachelier-ès-sciences in 1853. He entered the School of Mines of Paris in 1854, but finding the course of preparation of an engineer not to his liking, he gradually abandoned his academic studies in order to cultivate literature, philosophy and social science. Although those efforts resulted in a short story and a novel, *Francis Sauveur* (Walras, 1858), it rapidly became apparent to him that his true interests lay with social science. Accordingly, in 1858 he agreed to his father's request to devote himself to economics and promised to continue his father's investigations (Jaffé, 1965, vol. 1, pp. 1–2).

During his youth in Paris, Walras became a journalist for the *Journal des Economistes* and *La Presse* from 1859 to 1862; the author of a refutation on philosophical grounds of the normative economic doctrines of P.-J. Proudhon (Walras, 1860); an employee of the directors of the Northern Railway in 1862; and managing director of a cooperative association bank in 1865. He gave public lectures on cooperative associations in 1865; was co-editor and publisher with Léon Say of the journal *Le Travail*, a review devoted largely to the cooperative movement, from 1866 to 1868; and, during those years, gave public lectures on social topics (Walras, 1868) in which he advocated Victor Cousin's doctrine of compromise between economic classes. After the failure of the association bank in 1868, he found employment with a private bank until 1870 (Jaffé, 1965, vol. 1, pp. 3–4). During the 1860s he tried intermittently to obtain an academic appointment in France, but he lacked the necessary educational credentials, and the eleven economics positions in higher education in France were monopolized by orthodox economists who, he complained, passed their chairs on to their relatives (ibid., p. 3). His fortunes ultimately changed as a result of his participation in 1860 in an international congress on taxation in Lausanne, for that drew him to the attention of Louis Ruchonnet, a Swiss politician who secured his appointment in 1870 to an untenured professorship of economics at the Academy (subsequently University) of Lausanne in Switzerland. He was made a tenured professor there in 1871, and held that position throughout his teaching career.

Walras's personal life was initially unconventional. He and Célestine Aline Ferbach (1834–79) formed a common law union in the late 1850s. She had a son, Georges, by a previous

liaison, and she and Walras had twin daughters in 1863, one of whom died in infancy. In 1869 he married Célestine, thereby legitimizing their daughter, Marie Aline, and adopted Célestine's son. A long illness of Celestine's and the meagerness of Walras's salary made life very difficult for him for several years. His time and energy were sorely taxed not only by the need to care for his wife but by the need to supplement his salary by teaching extra classes, contributing to the *Gazette de Lausanne* and the *Bibliothèque Universelle*, and working as a consultant for La Suisse insurance company. Five years after Célestine's death in 1879, Walras married Léonide Désirée Mailly (1826–1900). The marriage was a happy one. Her annuity relieved his financial distress, and his situation was further improved in 1892 by an inheritance of 100,000 francs from his mother, which enabled him to pay debts incurred in publishing and disseminating his works, and to buy an annuity of 800 francs.

Walras's professional life was devoted to research and teaching. He frequently asserted that his research was a development of his father's, and that was true in some respects. It was under the influence of his father's classification of economic studies that Léon, as early as 1862, planned the division of his life's work into the study of pure theory, economic policies, and normative goals (Walras to Jules du Mesnil Marigny, 23 December 1862, L 81; the 'L' stands for 'letter', and, like all correspondence cited in this entry, the letter is in Jaffé, 1965), the areas of study that were ultimately set forth respectively in the *Eléments d'économie politique pure* (1874, 1877), the *Etudes d'économie social* (1896b) and the *Etudes d'économie politique appliquée* (1898). Léon adopted his father's classification of the factors of production into the services of labour, land and capital goods, regarding the source of each service as a type of capital. He adopted his father's definitions of capital as wealth that can be used more than once and of income as wealth that can be used only once, and adopted his father's vague term 'extensive utility', clarifying it by using it to mean the quantity-axis intercept of a market demand curve. The topic of utility had been treated in French thought by writers such as F. Galiani (a Neapolitan diplomat at Versailles) and E.B. de Condillac, and it was given further development under the name *rarété* by Auguste Walras, who thus bequeathed to Léon an interest in the concept of utility in relation to the value of commodities and an awareness of its dependence upon scarcity, an interest that ultimately led him to define *rareté* as marginal utility. Auguste used the word *numéraire* to mean an abstract unit of account, and Léon adapted the meaning of the word to his purposes. Auguste's philosophy of social justice and his belief in the desirability of nationalizing land were advocated by Léon throughout his adult life. Léon's major economic theories, however, were derived from his own original inspiration and from sources other than his father. Auguste's greatest contributions to Léon's development as an economist were to encourage him to study economics, to suggest that it should be a mathematical science (A.A. Walras, 1831, ch. 18; Jaffé, 1965, vol. 1, p. 493), and to give him access to a library of books on economics.

In that library was A.A. Cournot's *Recherches sur les principes mathématiques de la théorie des richesses* (1838), which Léon Walras credited with having demonstrated that economics could and should be expressed in mathematical form (Walras to Cournot, 20 March 1874, L 253; Walras to H.L. Moore, 2 January 1906, L 1614; Walras, 1905a). Cournot's work introduced Walras to the mathematical formulation of exchange between two locations, the theory of monopoly and the associated conditions for profit maximization, the analysis of how prices are repeatedly changed in a search for equilibrium in a purely competitive market, and the demonstration of the effect of large numbers of traders upon the determinacy of price, all topics that Walras developed in his own work (Walras, 1954, pp. 370–72, 434–40, 443). The first demand curve that Walras beheld was Cournot's, and he found it immensely suggestive. He was critical of it, however, because he perceived that Cournot's postulate that the quantity demanded of a commodity is a function only of its own price is inaccurate if more than two commodities are exchanged, and that Cournot did not provide a theoretical rationale for the demand function. Those perceptions, Walras observed, were the starting point for his own inquiries (Jaffé, 1965, vol. I, p. 5; 1905a).

Other ingredients that went into the composition of Walras's theories were provided by Adam Smith, John Stuart Mill, François Quesnay, A.R.J. Turgot and Jean-Baptiste Say. Smith had revealed many of the consequences of unfettered competition and had formulated the concept of normal value. Mill had provided a supplement to and reinforcement of Cournot's and Smith's analyses of competitive pricing (Walras to Ladislaus von Bortkiewicz, 27 February 1891, L 999), and also an extension and grand synthesis of classical doctrines that served Walras as a catalyst for critical studies (Walras, 1954, pp. 404–5, 411, 419, 423). Quesnay, in his *tableau économique*, had expressed the concept of a circular flow of income and of the interdependence of the various parts of the economy. Turgot had clearly delineated the idea of the simultaneous and mutually determined general equilibrium of those parts. Say (1836) had suggested the distinction between the capitalist and the entrepreneur, had portrayed the entrepreneur as an intermediary between the market for productive services and the market for outputs, and, in that analysis and in his law of markets, had adumbrated the interdependence between the incomes of the factors of production and the demand for commodities. Walras sharpened those ideas and made them a fundamental part of his general equilibrium model.

A.N. Isnard's *Traité des richesses* (1781), a book that Leon owned and that may have been in his father's library, was probably an important source of some of Walras's constructions (Jaffé 1969). Like Walras, Isnard was interested in determining equilibrium price ratios, set up a system of simultaneous equations of exchange showing the dependence of the value of each commodity upon the values of the others, stressed the necessity of having as many independent equations as unknowns, and perceived that the use of a *numéraire* rendered his system determinate. Anticipating Walras's treatment of production, Isnard assumed given ratios of the inputs in a mathematical model and expressed the costs of production in equation form. Also like

Walras, Isnard studied the allocation of capital among different uses, coming to the conclusion, as did Walras, that in equilibrium the net rate of income of different capital goods is the same.

Finally, Louis Poinsot's *Eléments de statique* (1803) exerted a powerful influence upon Walras. He first read that book when he was nineteen and kept it at his bedside for decades (Walras to Melle Dick May, 23 May 1901, L 1483). Poinsot painted a picture of the mutual interdependence of a vast number of variables, of how the dynamic forces in physical systems eventuate in an equilibrium in which each object is sustained in its path and relative position. Electrified by the implications of Poinsot's work, Walras conceived a magnificent project. He would emulate Poinsot's vision and analysis in reference to the general equilibrium of the economic universe! That he carried out that plan can be inferred from the striking similarity of the form of his work to Poinsot's, with its careful delineation of functional dependences and parameters, its sets of simultaneous equations and its equilibrium conditions.

Equipped, therefore, with ideas that he could take as building blocks and points of departure, with enough geometry and algebra to put together mathematical statements of economic relationships and conditions – his use of calculus in the *Eléments* came after the first edition – and with the explicit objective of developing a mathematical theory of general equilibrium, Walras began his scholarly activity in Lausanne in 1870. In a period of great creativity that lasted until 1878, he developed most of the foundations of the theory of general equilibrium that appeared in the first edition of the *Eléments*. Walras insisted to his publisher that the first part appear in 1874, before the second part (Walras, 1877b) was completed, because he learned in May of that year that W.S. Jevons had published a mathematical theory of utility and exchange that was similar to his own (J. d'Aulnis de Bourouill to Walras, 4 May, 1874, L 267), and he was anxious to establish the independence of his discoveries and his priority in regard to most of them. For those same reasons, he published four brilliantly original memoirs containing the heart of his theory of general equilibrium during 1874, 1875 and 1876 (Walras, 1877a), paid for the costs of publication of his books, and sent copies of them and of his articles to his many correspondents. From 1878 to 1901, Walras significantly extended, refined, and altered his system, particularly in regard to capital, money, and equilibrating mechanisms, and composed numerous essays on applied and normative economics. After 1900 he made no theoretical contributions but wrote some articles and, in late 1901 and 1902, made some inconsequential changes to the *Eléments* which were ultimately incorporated into the text of the fourth edition (1900) to produce the *édition définitive* of 1926.

Walras was an extremely conscientious teacher, but he was an uninspiring lecturer (Jaffé, 1965, vol. 2, p. 560), and the students at Lausanne were interested in careers in law, not in economics, so he failed to develop disciples among them. Moreover, he was with increasing frequency afflicted by bouts of mental exhaustion and irritability that made it difficult for him to lecture and to read and write. In 1892 he took a leave to regenerate his strength in order to be

able to continue teaching, but soon realized he would find the strain of returning to his tasks insupportable and retired in that year, being at that time 58 years of age.

Economic Theories

Walras's Subject Matter and Method

Walras recognized that there were imperfectly-competitive market structures and developed a theory of monopoly to take account of an important class of such phenomena (Walras, 1954, lesson 41). Realizing, however, that the incorporation of non-competitive elements into his general equilibrium model was beyond his powers (p. 256) and believing that a high degree of competition was 'almost universal' and deserved to be treated as the general case (Walras to Ladislaus von Bortkiewicz, 27 February 1891, L 999), he devoted most of his energies to working out a theory of 'freely competitive' markets, the aspect of his theoretical work with which this entry is concerned. Competition was most effective, he noted, in organized auction markets, and he assumed that markets are of that type (Walras, 1954, pp. 83–4), but he also regarded his analysis as applicable in a general way to less highly organized competitive markets (p. 84). The first kind of competitive model with which he dealt will be called his formal general equilibrium model. In it he assumed that preferences, the number of workers, the amounts of natural resources and technology are constant. The second kind, which Walras did not develop in any detail, expresses his ideas about economic growth. Within his formal general equilibrium model, Walras constructed the four sequential and cumulative theories (or models, as they will be interchangeably called) of exchange, production, capital formation and credit, and money and circulation. Each of them has four parts.

The first part is the structure of the market, in which Walras identified the participants and their economic characteristics, their objectives and how they try to achieve them, the types of commodities that are traded, the important institutional features and rules of the market, and the relevant supply and demand functions.

The second part is the dynamic process by which the market undergoes adjustments when it is in disequilibrium, the exposition of which Walras regarded as 'the object and proper goal of pure economics' because he realized that without a demonstration of the stability of a purely competitive system, the solutions to his equations could not be regarded as values to which the variables of his model tend (Walras to Bortkiewicz, 17 October 1889, L 927; Walras to Charles Gide, 3 November 1889, L 933). He also recognized that the dynamic functioning of markets depends on the economic agents, institutions, and conditions identified in the first part of each model, and to portray disequilibrium behaviour he accordingly discussed the activities and interactions among diverse economic agents in trade and production, the generation and elimination of profits and losses, the operation of the stock market and many other details of behaviour drawn from economic life. Thus the allegation, perpetuated by generations of

commentators (e.g. Jaffé, 1971, p. 281; 1981, pp. 252–61), that Walras devoted his attention almost exclusively to the conditions of static equilibrium in an abstract model devoid of institutional detail, economic facts and dynamic behaviour, is a misrepresentation of his work.

Walras was partially responsible for that misrepresentation, because he sometimes referred to his general equilibrium model as 'static' without qualification, and contrasted it with what he called 'the dynamic point of view', by which he sometimes meant the view taken in considering economic growth (Walras, 1954, p. 318). On the other hand, he also stated on many occasions that a dynamic theory is contained in his formal model of general equilibrium, and his usage will be followed in this essay. The 'static theory of exchange', he wrote, 'may be defined as the exposition of the equilibrium formula'. The 'dynamic theory', in contrast, which Walras claimed to have been the first to explore, is 'the demonstration of the attainment of that equilibrium through the play of the raising and lowering of prices' (Walras [1895], 1965, vol. 2, p. 630). Similarly, in responding to Irving Fisher's criticism that he had not considered time, Walras pointed out that that was true only of his exposition of the conditions of static equilibrium, and that his theory of production, for example, is given a dynamic treatment in lesson 20 (Walras, 1889) of the *Eléments* (Walras to Fisher, 28 July 1892, L 1064). In his first model of the dynamics of production, which he espoused from 1873 to 1899, disequilibrium production and use of consumer commodities and new capital goods occur during the course of the equilibrating process and are part of it. That he never abandoned most of the language describing this model explains why the 1926 edition of the *Eléments* can be cited in reference to it. In the second model, introduced in 1899 (Walras, 1899, p. 103) and inserted into the *Eléments* in 1900, he excluded those disequilibrium phenomena. In both models, and in all his work, he described the equilibrating process as one of *tâtonnement*, which means 'groping', to emphasize that the equilibrium magnitudes of prices and quantities are not known by the participants during the disequilibrium phase but are found by repeated experiments. His use of the word in all contexts, followed in this essay, was made possible by its neutrality with respect to the presence or absence of disequilibrium transactions or production.

The third part of each of Walras's models is its conditions of equilibrium, and the fourth is its comparative statics.

Theory of Exchange

Walras was concerned in this theory with the determination of the equilibrium prices of commodities and the quantities of commodities exchanged. Setting forth the structure of exchange markets, he assumed that the preferences of the traders and the aggregate amounts of the commodities they hold in each market are given. Until constructing his fourth model, he abstracted from the use of money and assumed that commodities are exchanged directly for each other. The participants in exchange markets include brokers, agents, professional traders, retailers, wholesalers, the owners of the factors of production in their capacities as demanders of

commodities, and entrepreneurs, who supply and demand commodities. The supply and demand functions are reciprocally related (Walras, 1954, pp. 96–7). Given a trader's demand curve for A, its price times the related number of units he wants to buy is his supply of B expressed as a function of the price of A in terms of B. Observing what happens to the areas of the rectangles under the demand curve for A as its price rises, Walras deduced that the quantity supplied of B initially rises and then falls. In the same way, a trader's supply of A can be derived from his demand for B. Walras summed the individual demand and supply curves respectively in the market for A to obtain the market curves, and similarly for B. It will be seen that he adapted and extended this analysis of the dependence of the supply of one commodity upon the demand for another when he took up the question of multi-commodity exchange. Walras also assumed that, as in the 19th-century bourse, in each market the rule is enforced that disequilibrium transactions are not allowed (Walras, 1880a, p. 461; 1880b, p. 78; 1954, p. 85).

To explain demand and infuse his model of exchange (Walras, 1869/1870) with purposive action, Walras developed a theory of preferences shortly before 1872 in which he assumed that traders want to maximize utility, that utilities are independent and additive, and that the marginal utility of a commodity is a decreasing function of the quantity acquired or consumed. Nevertheless, he was floundering in his attempts to relate utility to market behaviour, so he appealed for help to Antoine Paul Piccard, a professor of industrial mechanics at the Academy of Lausanne, who responded in 1872 by developing a model of utility maximization and deriving the individual demand function within it (Jaffé, 1965, vol. 1, pp. 308–11), thus meriting a part of the credit that has previously been given to Walras for that achievement. Everything then fell into place for Walras, and he proceeded to develop the view of economizing and maximizing behaviour that he imprinted on Continental neoclassical economics, extending the technique shown in the model to obtain the equilibrium conditions of the participants in a multi-commodity system (Walras 1954, lesson 12), and making utility maximization the driving force in each of his models.

The dynamic behaviour of Walras's exchange model is an automatic *tâtonnement* process in the sense that the path of the price is the unplanned outcome of market forces, but the process does not result from unconstrained human nature. It depends upon the rules, institutions and conditions devised and enforced by market authorities and by government (Walras, 1880a, 1880b; [1895], vol. 2, p. 632; and see 1954, p. 474). A price is initially cried at random (*crié au hasard*) (Walras 1877b, p. 127; 1954, p. 169), presumably by any of the traders or by a price-setter, and the participants subsequently follow the Walrasian pricing rule, namely that the price is changed in the same direction as the sign of the market excess demand for the commodity. Since preferences are constant, and since the rule against disequilibrium transactions ensures that the asset distribution remains unchanged during the equilibrating process, the initial supply and demand functions and, consequently, the equilibrium values of the system, are not affected by the disequilibrium behaviour of the traders. In an isolated market for two commodities the

tâtonnement process therefore leads to the price and quantity that are the solutions to the market supply and demand equations that depend upon the traders' initial utility functions and initial asset holdings, whereupon the equilibrium amounts of each commodity are exchanged (Walras, 1954, p. 106, lessons 6, 9).

Markets are not isolated, however, in recognition of which Walras introduced the central feature of his contribution to economic science, namely an account, in his theory of exchange and in his other models, of the interrelationships among the markets for different commodities. If a trader has a commodity that he wants to trade for several others, the amount that he offers in any market is related to the amounts that he offers in the other markets, so the amount that he wishes to purchase or sell of any commodity is seen to be a function not only of his preferences, his income and the price of that commodity but, in principle, of the price of every other commodity; consequently, the market supply and demand quantities and the price in any market are dependent in part upon the prices in other markets (lesson 12). Moreover, Walras perceived that because a trader's demand for any commodity implies the offer of commodities in exchange for it, the sum of the values of a trader's planned purchases must equal the sum of the values of his planned sales. That relation is one way of stating the individual budget equation. By summing those equations, Walras also perceived that the sum of the net quantities demanded by all traders in a market times the price of the commodity, summed over all markets. He therefore saw that the sum of the individual positive and negative excess demand quantities in all markets is identically zero, a relation that has come to be known as Walras's Law (p. 170, and § 118, 210, 244). He was able to identify that fundamental statement of the way that markets are interrelated in part because the device of the *numéraire*, a commodity in terms of which the values of all commodities are expressed (p. 161), made clear to him, as it had to Isnard, that there is exactly the right number of excess demands: in a system with n commodities, there are only n–1 independent market equations involving n–1 price ratios, but also only n–1 unknowns, because the price of the *numéraire*, the nth commodity, in terms of itself is unity (pp. 161–2, 241).

The interdependence of markets, Walras explained, gives rise to the major problem of general equilibrium analysis, which is whether a system of markets that is initially in disequilibrium will tend towards a position of equilibrium. As the price is, for example, adjusted downwards in the market for one commodity when there is initially a negative excess demand for it, the excess demands for other commodities will be affected. Do their markets then become closer to equilibrium or further from it? When their prices change in response to the altered conditions, the equilibrium in the first market will be disrupted. Will its subsequent adjustment aid or impede the equilibrating process taking place in other markets? Walras claimed that he had shown that the answer to those questions is that the market system converges to general equilibrium as a result of the Walrasian pricing rule (pp. 172, 179–80).

Walras then specified the conditions that prevail in the static equilibrium of exchange of a multi-market system. The ratio of the *raretés*, or marginal utilities, of any two commodities is

equal to the ratio of their prices, and the price of any commodity in terms of another commodity is equal to the ratio of the prices of those two commodities in terms of any third commodity (p. 157). Those conditions are satisfied when the quantities supplied and demanded of each commodity are equal (p. 172).

Finally, Walras briefly examined some features of the comparative statics of the exchange model (pp. 147–9). He shifted the utility curves for a commodity and determined that its equilibrium price changes in the same direction as the shift in the curves. He then successively increased and decreased the traders' endowments of a commodity and determined that its equilibrium price successively decreases and increases.

Theory of Production

In this model Walras was concerned with the determination of the equilibrium prices of productive services and the equilibrium rates of output of the quantities of consumer commodities. Setting forth the structure of the model, Walras first identified the market for productive services, in which he assumed that the amounts of economic resources and their services are given. The demanders of productive services are the entrepreneurs. Their ultimate aim is to maximize utility, which they achieve through maximizing profits. In their capacities as managers of firms, they combine productive services and materials in proportions that are determined by what Walras called the technical coefficients of production. The coefficients, which he assumed to be fixed in his formal general equilibrium theory, indicate the amount of each of the inputs that is used to make a unit of output. With fixed coefficients and given prices of the productive services, the average cost is constant as the firm's output varies. If any of those prices change, the average cost curve shifts in the same direction. The suppliers of productive services are workers, who own personal faculties; landlords, who own natural resources; and capitalists, who own capital goods or provide capital funds (Walras, 1877b, p. 218; 1954, pp. 214–15). Their aim is to maximize utility, which motivates them to demand income from the entrepreneurs. Capital goods are specific items of real capital; capital funds, raised by the sale of shares on the bourse, constitute fluid and mobile purchasing power which can be used to acquire economic resources to construct different kinds of particular physical capital (Walras, 1954, pp. 270, 311). Walras referred to the three sources of services as different types of capital because they all endure through time and produce a flow of services, but in this entry the unqualified word 'capital' or the term 'capital goods' will mean durable, man-made, inanimate instruments of production.

Walras then identified the market for consumer commodities. The suppliers of these are the entrepreneurs. The demanders are the workers, landlords, and capitalists acting in their roles as consumers, motivated in their purchases by the desire to maximize utility. They pay for them with the incomes that they have been paid by the entrepreneurs. Walras assumed that in the markets for productive services and consumer goods all resources are highly mobile, and that

entrepreneurs have good knowledge of the profitability or unprofitability of producing any particular commodity.

The *tâtonnement* in the production model is principally the outcome of the actions of entrepreneurs, who are purely intermediaries that buy productive services, semi-finished goods and raw materials, and sell services and finished goods to consumers (lesson 21, and pp. 426–7; Walker, 1986). The payment that they receive in disequilibrium for their entrepreneurial activity is profit, which Walras defined on a per unit basis as the price of output minus its average cost. An entrepreneur may undertake the functions of other factors of production – he may also, for example, be a capitalist or a manager of the firm – and ordinarily he has to do so, but his role as buyer and seller is a distinct one (Walras, 1954, p. 222). One aspect of the *tâtonnement* occurs in the production and sale of consumer goods. It converges to equilibrium as entrepreneurs increase or decrease their output (p. 247) because the change in the output of a product has a direct effect on its price that is unidirectional, whereas the unidirectional changes that it induces in the output of other products has only indirect effects on its price, and because the latter more or less cancel each other out by being some in one direction and some in another (p. 246). 'The [resulting] system of new quantities of manufactured products and of new selling prices is thus closer to equilibrium than the old one; and we have only to continue the process of groping to approach still more closely to equilibrium' (p. 246). The other aspect of the *tâtonnement* occurs in the market for productive services and is a process of groping for the equilibrium amounts of resources employed in different industries. Disequilibrium quantities of productive services are hired and used during the process, and disequilibrium quantities of consumer goods are produced (Walras, 1877b, pp. 254, 264; 1889, pp. 234–5, 240–1, 249–50, 280; 1896a, pp. 235, 240–1, 249–50, 280). As long as the quantities supplied and demanded are unequal, prices and rates of production continue to be changed.

The *tâtonnements* in the markets for productive services and for consumer goods are interrelated. If the consumers' demand for a commodity increases, the quantities demanded and supplied become equal at a high price because the supply is initially highly inelastic. The price of the product then exceeds its cost of production, so the entrepreneurs in the industry make profits. Attracted by the prospect of doing the same, other entrepreneurs enter it, and existing firms increase their output. As the demands for productive services increase, their prices are bid up, which raises the average cost of production. As the supply of output function changes so that more output would be offered at each possible price, the price in the exchange market for the commodity is lowered by the entrepreneurs in an effort to dispose of the entire flow of output. Thus the average cost rises and the price falls (Walras, 1954, p. 253). If demand decreases for a commodity, its price falls below the average cost of production and the entrepreneurs make losses. This leads some of them to leave the industry and some of them to diminish the output of their firms. The prices of the productive services fall as the demand for them decreases, which

lowers the average cost of production. As less output is offered, its price tends to be forced up. Thus the average cost falls and the price rises (p. 253).

Walras concluded that whether the demand for a commodity increases or decreases, the average cost of production and the price of the commodity become equal, whereupon equilibrium is reached and the *tâtonnement* ends. It follows that in the equilibrium of production the entrepreneur obtains neither profit nor loss (Walras, 1877b, p. 232; 1954, p. 225). This is Walras's concept of the zero-profit entrepreneur. The equilibrium, Walras stressed, is a theoretical notion. It is the normal state of the market in the sense that it is the one to which the variables perpetually and automatically tend in a regime of free competition (Walras, 1954, p. 224). Since it implicitly contains the equilibrium of exchange (p. 224), it is characterized by the additional conditions that the quantities supplied and demanded of each consumer commodity are equal and that the quantities supplied and demanded of each productive service are equal. A stable circular flow is established in which the total cost equals the total revenue in each firm, the incomes received from the entrepreneurs by the owners of the factors of production equal the revenues earned by the firms, and those incomes are spent on consumer goods by the owners of the factors of production. Walras's theory of production therefore showed how input and output markets are linked together.

Walras then considered variations in some of the parameters of the production model. If the marginal utility curves for a commodity shift up, he reasoned, its price in terms of the *numéraire* increases. If the marginal utility curves shift down, the opposite occurs. If the quantity of a product or service possessed by the holders changes, its price changes in the opposite direction (p. 260).

Theory of Capital Formation and Credit

In this theory Walras first examined the determination of the prices of land and personal faculties, as distinct from the prices of their services. The aggregate supply of land is perfectly inelastic, and its price is simply its gross income divided by the rate of net income (pp. 270, 309). The number of workers is a given condition so far as economic science is concerned, and the price of a worker is equal to his gross income minus the cost of replacing and insuring him, divided by the rate of net income. Workers are not bought and sold, however, so their prices are virtual (p. 271). Since the prices of the services of capital goods are explained in Walras's production model, and since he assumed a fixed relationship between capital goods and the amounts of their services, his theory of capital is concerned with the determination of the amounts and prices of capital goods themselves and the determination of their rate of net income. Capital is formed by capitalists saving funds and, most commonly, lending them to entrepreneurs (p. 270), who purchase capital goods, earn revenue from their use and repay the loans (Walras, 1883, p. 113; 1954, p. 290, §§ 190, 208, 235). Walras's identification of that process explains why he inserted the word 'credit' into the name of his capital-goods model, but obviously he did

not develop a true theory of credit within it, because he did not introduce credit extended by banks. Some capitalists prefer to own capital goods, so Walras assumed occasionally that they acquire them directly in physical form (Walras, 1954, p. 289), and assumed frequently that they acquire them through buying stock certificates (p. 289). In each case, the physical capital is used by entrepreneurs, so 'the demand for new capital goods comes from entrepreneurs who manufacture products and not from capitalists who create savings' (p. 270). The entrepreneurs purchase the particular kinds of capital goods that are profitable, with the result that the kinds that are produced and used reflect the structure of demand for consumer commodities (pp. 225, 276, 303; 1871, p. 36).

The net saving of the capitalists equals aggregate income minus aggregate consumption, minus the depreciation and insurance costs of capital goods. A positive, zero or negative excess of aggregate income over aggregate consumption was introduced by Walras into his capital-goods model in 1900 through the concept of a fictional commodity (E) constituted of perpetual annuity shares, a concept that he added without eliminating his references to the purchasing of stock certificates and commercial paper. Apart from the latter, it appears that Walras therefore wished to express aggregate savings as a single homogenous quantity – the demand for E – whereas he treated investment as the construction of heterogenous capital goods and viewed it as reaching equilibrium through adjustments in the markets for those different capital goods (pp. 275–6, 308). Each perpetual share pays one unit of *numéraire* per year, and its price, determined by supply and demand, is the reciprocal of the rate of interest. If people want an additional amount of interest income, they provide savings through purchasing new perpetual shares, and the *numéraire*-capital that they pay for them is used by entrepreneurs to buy productive services and materials that are transformed into new capital goods (274–6, 309); and if net saving is positive, the economy grows.

The *tâtonnement* in the capital-goods market is a disequilibrium-production process (Walras, 1877b, pp. 295–7, 300, 304; 1889, pp. 280, 284–94; 1896a, pp. 280, 284–94; 1954, pp. 287–91, ∮ ∮ 258–60), one aspect of which takes place in the stock market and the other in the course of the production of capital goods. In the stock market, which is the market for new capital goods, each capitalist attempts to maximize utility by saving and acquiring more stocks that have relatively high yields and less of those with lower yields (Walras, 1954, p. 289), with the result that the total value of new capital goods and the excess of income over consumption both move in the same direction as prices. It follows, Walras maintained, that the tendency of the change in prices to destroy the equality between the total value of new capital goods and the excess of income over consumption is weaker than the tendency of the change in the rate of net income to bring the total value of new capital goods and the excess of income over consumption into equality with each other. 'Thus the system involving the new rate of net income and the new prices will be closer to equilibrium than the old system; and it is only necessary to continue the process of groping for the system to move still more closely to equilibrium' (p. 288).

In the production of capital goods, entrepreneurs acquire more capital goods that yield relatively high returns, and diminish their use of capital goods that yield lower returns. As a consequence, during the *tâtonnement* the net rate of return on all capital goods tends toward equality (p. 273). If profits are being made from the production of capital goods in an industry, new entrepreneurs enter it, and those already in it increase their rate of production. As a result they drive up the prices of productive services, which causes the average cost to rise towards equality with the price of the capital good. If losses are incurred, entrepreneurs diminish production. As a result, they drive down the prices of productive services, which causes the average cost to fall toward equality with the price of the capital good (pp. 292–3). It is probable, Walras maintained, that the effects of changes in the output of a new capital good that tend to cause equality between its average cost and its price will be stronger than the contrary effect of interrelated changes in the output of other capital goods, so the process converges to equilibrium (p. 293).

The equilibrium conditions in the formation of new capital goods are expressed in the lengthy analysis that Walras called the theorem on the maximum utility of new capital goods, which he regarded as crowning and confirming his entire theoretical system (Walras to H.S. Foxwell, 16 December 1888, L 859; see Walker, 1984b). He initially assumed that new capital goods do not require amortization or insurance, but then he made the realistic assumption that capital goods wear out and are subject to accidents. The net rate of income to a particular capital good is then given by the gross income it earns minus amortization and insurance costs, divided by the price of the capital good. In equilibrium each trader maximizes utility by holding the quantities of capital goods that make the ratio of the marginal utility of each capital good to its price equal for all his capital goods, a single price for any type of capital good prevails, and because of the adjustment of yields and capital good prices, the net rate of income derived from every capital good is the same (Walras, 1889, p. 306; and see 1954, p. 305). That rate is the rate of interest, and its equilibrium value equates aggregate saving and investment (Walras, 1889, p. 280; 1954, pp. 276, 281, 300, 305). Through this analysis Walras believed he had seen behind the veil of money or *numéraire* and discovered the real determinants of that rate. It is manifested in the banking system, he argued, but it is determined in the stock market. It is the ratio of net profit to the price of a share of stock, which in equilibrium equals the common ratio of the net price of capital services to the price of the good that yields them (Walras, 1954, p. 290). In equilibrium the prices of well-maintained old capital goods are equal to the prices of the same kinds of new capital goods, so the equilibrium prices of all capital goods are equal to the ratios of their net incomes to the rate of net income. Finally, the price of every capital good equals its average cost (pp. 301–5, 309).

Walras then turned to the comparative statics of the capital goods market (lesson 28). If the price that has to be paid for the services of a capital good increases or decreases as a result of a parametric change, the price of the capital good itself increases or decreases. Its price also varies

inversely with the rate of depreciation and with the rate of the insurance premium. If the rate of net income changes, the prices of all capital goods change in the same direction (pp. 309–10). If the utility curves for net income shift up or down, the rate of net income changes in the opposite direction. If the quantity of net income varies, the rate of net income varies in the same direction. If utility functions and the quantity of net income both vary in such a way that the marginal utilities remain unchanged, the rate of net income also remains unchanged (p. 307).

Theory of Money and Circulation

Walras wanted to design the structure of this model on 'exactly the same terms and in precisely the same way' as in the models of exchange, production and capital formation (p. 42). He therefore described the functions of money and the formation of money prices on the assumption that there is no uncertainty in equilibrium, and consequently that the dates and monetary value of future purchases and sales are known (pp. 317–18). Money is one type of circulating capital; the other is circulating physical capital. Replacing his initial formulation in terms of an equation of exchange that had anticipated Irving Fisher's (Walras, 1877b, pp. 180–81), Walras asserted that circulating physical capital yields utility from its 'service of availability' – that is, by being readily available – and money provides, by proxy, the same service of availability as the commodity that it is destined to purchase and yields the same utility as that service. All economic agents try to hold utility-maximizing amounts of money and circulating physical capital (Walras, 1954, pp. 320–1). The latter, held by consumers and entrepreneurs, is acquired with money, so the essential concern of Walras's model of circulating capital reduces to the demand for and supply of money and its price: an individual has utility functions for the services of availability of commodities and perpetual net income, 'not *in kind*, but *in money*' (p. 320). Entrepreneurs and some consumers have net demands for cash balances because of the non-synchronization of payments and receipts (pp. 319, 321; 1886, pp. 40–44); that is, income received between the present and the date of a future payment is for many firms and people insufficient to provide enough cash for it. Savers make some of their balances available as loans through buying commercial paper or common stocks or perpetual annuity shares or through deposits in banks (Walras, 1954, pp. 318–20). The aggregate gross supply of money is the total stock issued by the monetary authority in the case of a fiat money economy, and is the amount of circulating coin in the case of a commodity-money economy (pp. 321–4).

If money is the *numéraire*, which is in fact the case, its price is unity and the price of its service is the rate of interest (pp. 320, 327). In one passage the latter was defined by Walras as the price of the service of cash balances held for the acquisition of fixed capital, and the rate of discount as the price of the service of cash balances held for the acquisition of circulating physical capital (p. 332), but he did not develop that analysis nor modify the rest of his exposition in the light of it. Accordingly, Walras's position was that, given the flows of receipts and purchases, the individual gross and net demand for cash balances and the individual net

supply of them are functions of the rate of interest. The sum of the individual net demands for money is the aggregate demand function, and the sum of the individual net supplies of money is the aggregate supply function (pp. 320–21).

The *tâtonnement* in the money market explains how the rate of interest and the equilibrium aggregate net quantities of cash balances supplied and demanded are determined (pp. 325, 327). The rate of interest changes according to the Walrasian pricing rule. When the excess quantity demanded of cash balances is positive, the rise in the rate decreases the quantity demanded of cash balances by consumers and entrepreneurs by increasing the cost of the service of availability that money provides, and also decreases the quantity demanded by entrepreneurs by causing a fall in profits and hence in the desired rate of production. The rise in the rate of interest also causes the net quantity of cash balances that savers want to supply to increase. If the desired supply of cash exceeds the desired demand at the current rate of interest, the opposite effects occur (p. 333). The *tâtonnement* continues until the equilibrium price of the service of availability of money is found – namely, the price that equates the net and therefore the gross quantities supplied and demanded of cash balances – whereupon the money market reaches equilibrium (Walras, 1889, pp. 379–81; 1899, p. 96; 1900, pp. 297–319; 1954, pp. 315–33). The equilibrium prices of all commodities in terms of money are given by its role as *numéraire* and by the workings of the entire model that determine the ratio of exchange between each commodity and the *numéraire*. In general equilibrium, the price of all money held by different individuals for different purposes is the same (Walras, 1954, p. 326). Moreover, because an underlying influence upon the rate of interest on money is the value productivity of physical capital, an influence exerted through variations in the volume of funds invested, the equilibrium rate on money is the same as the equilibrium rate of net income determined in the market for capital. There is therefore equality in the rate of net income from all capital goods and real and monetary circulating capital (p. 323).

Walras then considered the comparative statics of the model. He changed the utility functions for the service of money and deduced that the *rareté* or value of the service of money changes in the same direction. He changed the quantity of money and deduced that the *rareté* or value of the service of money changes in the opposite direction, and that all prices change in the same direction without any alterations in relative prices (p. 333). The remainder of Walras's discussion of monetary comparative statics is concerned with a commodity-money economy, taking up such topics as the mining of precious metals and the melting of coins.

The Elimination of Irrevocable Disequilibrium Behaviour

A problem of internal consistency in Walras's disequilibrium-production model is that the production of consumer goods and capital goods at disequilibrium prices changes some of the conditions that he postulated are constant in constructing the equations of general equilibrium, namely the stocks of goods in exchange and the amounts of capital goods and services. The

consequences of those changes spread throughout Walras's model. With each different distribution of the stocks of consumer goods produced during the course of the disequilibrium-production *tâtonnement*, the traders' net supply and demand functions change. No disequilibrium transactions occur in exchange, but each batch of commodities traded in each particular market while the economy is undergoing adjustments in production ordinarily has a different price, and frequently the equilibrating process in a particular market is interrupted by the appearance of new stocks. Each different disequilibrium rate of production and sales of capital goods changes their prices and the amounts of capital-good services that are offered and their prices, and therefore changes average costs, profits and the rate of net income. Disequilibrium hiring of productive services results in the payment of disequilibrium incomes, which changes the demand for consumer goods and, adding its impact to that of the varying rate of interest, changes the supply of saving. The result of these and other disequilibrium phenomena is that the model proceeds along a path of growth that has no relation to the values determined by Walras's equation system.

Walras's solution to this problem was to eliminate disequilibrium production by devising a *tâtonnement* of *ex ante* behaviour in which the intentions of economic agents are reconciled. He assumed that at any quoted price the suppliers and demanders of productive services and capital goods write down their offers on pieces of paper. These markers are *good for* the indicated amounts if the market quantities supplied and demanded are equal, just as a bond carries a pledge, and so Walras called them *bons*, or pledges. Although he did not describe the mechanism of ascertaining the market supply and demand quantities, he asserted that if the aggregate desired quantities are not equal, another price is cried in accordance with the Walrasian rule, and the participants revise their pledges (Walras, 1899, p. 103; 1900, pp. 215, 260; 1954, pp. 242, 282). Walras's model of exchange already contained pledging behaviour, inasmuch as the traders' offers are pledges to trade particular amounts provided that the price that is cried is one at which the quantities supplied and demanded are equal, although the traders, being face to face in each particular market, have no need to write their offers down. To call the entire formal general equilibrium model that Walras presented in 1900 his *pledges model* is therefore consistent with the character of its various parts. Since in it Walras eliminated disequilibrium production and acquisition of consumer goods and new capital goods, and since he had never permitted disequilibrium transactions in exchange, none of the parameters of his system of equations of general equilibrium undergoes endogenously induced changes during the *tâtonnement* phase of behaviour. Walras believed that the equilibrium is therefore the one given by the solutions to that system, and that his new version of *tâtonnement* in production converges to it for the same general reasons as the old one.

Another problem of consistency in Walras's work arises from his assumption that there is aggregate net saving and investment. On the one hand, his equations are constructed on the assumption that the amount of capital services is the fixed amount with which the economy,

initially in disequilibrium, embarks upon its equilibrating path, an amount that is a fixed proportion of the constant amount of capital goods in use during the *tâtonnement* process. On the other hand, the use of net additions to the stock of capital goods, even if there is no production during the *tâtonnement* process, changes the capital-goods and services component of the economic resources parameter, with ramifications that alter some of the other nominal parameters and all the variables of the model, thus preventing the solutions of Walras's equations from materializing as its equilibrium values.

Walras's solution to this problem was to assume that new capital goods are not used. This led him to define three phases of the economy. First, there is 'the phase of preliminary gropings towards the establishment of equilibrium in principle' (Walras, 1954, p. 319). Second, there is 'the static phase in which equilibrium is effectively established *ab ovo* as regards the quantity of productive services and products made available during the period considered, under the stipulated conditions, and without any changes in the data of the problem' (p. 319). This means that the economy 'remains [for the time being] *static* because of the fact that the new capital goods play no part in the economy until later in a period subsequent to the one under consideration' (p. 283). Third, there is 'a dynamic phase in which equilibrium is constantly being disturbed by changes in the data and is constantly being re-established' (p. 319). The new capital goods, both fixed and circulating, Walras wrote, 'are made available during the second phase' but 'are not put to use until the third phase'. When they are used, however, 'the first change in the data of our problem' occurs (p. 319), and the economy begins to grow, tracing out a moving equilibrium to which Walras's formal general equilibrium model is seen as a prelude.

Economic Growth

Walras did not develop a complete model of economic growth, but he examined two aspects of the topic. One was the possible dynamization of his formal general equilibrium model in the sense of introducing endogenous variations in its parameters. He was led to speculate about that subject by the consideration that the equilibrium identified in his formal theory is never reached in reality because *tâtonnement* takes time, and consequently parameters such as preferences and the amount of labour change before equilibrium is reached (p. 380; and see p. 224). In order to take account of this situation:

> we need only suppose the data of the problem, viz. the quantities possessed, the utility or want curves, etc., to vary as a function of *time*. The *fixed* equilibrium will then be transformed into a *variable* or *moving* equilibrium, which re-establishes itself automatically as soon as it is disturbed (p. 318).

Walras did not analyse that moving equilibrium or 'continuous market' (p. 380), but he briefly explained that when the parameters become variables, annual production and consumption change continually with them. Money, workers and goods are used up and are produced. Net

new capital goods come into existence and are put to use, and circulating capital is borrowed by entrepreneurs from the capitalists in the form of short-term loans of money that mature immediately after the sale of the products (p. 319).

The other aspect of growth that Walras examined was 'the laws of the variation of prices in a progressive economy' (p. 382), or some of the features of alternative paths of economic growth. For this task he first defined economic progress as the substitution of capital services in place of land services in given production functions (p. 383). The substitution implies variable coefficients of production, and to introduce these Walras used the theory of marginal productivity. He did not originate that theory nor claim to have done so. In fact, Hermann Amstein, a mathematician at Lausanne, developed it in 1877 (Amstein to Walras, 6 January 1877, L 364; translated by Walker, 1983, pp. 205–6). Walras did not understand or use Amstein's work, however, and the major credit for the theory of marginal productivity that first appeared in the *Eléments* in 1896 (Appendix III) must be given to Enrico Barone (1895). Walras then defined technical progress as changes in production functions, including the introduction of entirely new processes, but he did not analyse it, beyond concluding that it contributes, along with economic progress, to ensuring that output increases without limit in a progressive economy (Walras, 1954, p. 387). He also examined, in a highly general way, how the prices of products and services vary with different amounts of capital and magnitudes of the population (pp. 389–91). His principal conclusion was that the rate of net income falls as the stock of capital grows, the proximate causes of the process being rising rents and falling prices of capital services.

Economic Policies

Walras was greatly interested in the economic problems of his day and in socio-economic reform. His normative convictions, derived from his father's philosophy of society and justice, were a mixture of conventional 19th-century liberalism and the doctrine of state interventionism (Walras, 1896b, pp. 3–263). Like many writers, each with different views, he bestowed the title of 'natural law' upon the principles of justice that he considered desirable, and so he might be called a natural-law philosopher or casuist. Nevertheless, he was not a natural-law economist. He did not believe that behind observable facts is a structure of economic laws that are divinely ordained, or that are peculiarly in tune with the structure of the universe and human aspirations, or that are ceaselessly at work so that violations of them can only result in friction. Nor did he construct his economic model with the conscious intention of expressing his normative views. Sharply distinguishing normative and positive economics, he designed his theories for the purpose of understanding economic reality (Walker, 1984a) and presented his normative work explicitly as such and carefully segregated it from his economic theories (Walras, 1896b, 1898).

Walras's policy recommendations ranged over natural monopolies, which he believed should be nationalized; prices, which he believed should be stabilized by a monetary authority; bimetallism, which he believed had both advantages and disadvantages; the stock market, which

he believed should be regulated by the state in order to improve its organization and ensure its integrity; taxes, which he believed were unjust and confiscatory and should be abolished; and land, which he believed should be purchased by the state and rented to private users, thereby providing it with revenue (Walras, 1905b, pp. 272–3). Arguing that his advocacy of nationalization of land and natural monpolies was based upon a scientific analysis, Walras called himself a 'scientific socialist'.

Evaluation

The first major criticism of Walras's work relates to the structure and dynamic behaviour of his pledges model. In it, in disequilibrium, entrepreneurs consult the market aggregates of pledges to buy and sell and move from industry to industry and plan to create firms or to expand or contract their existing firms without actually hiring or spending or producing at all. Owners of productive services are imagined to offer their services repeatedly at disequilibrium prices without actually earning any income or consuming any goods or services. The entire system of interrelated markets is imagined to go through complex costless processes of information acquisition, price changes and changes in the demand and supply quantities that are pledged, all without anyone being allowed to agree to a single actual transaction or act of production or consumption, until the equilibrium set of prices has been found. An additional aura of unreality is imparted to this scheme by Walras abstracting from money throughout his theories of exchange, production and capital formation, and then by introducing it in such a way that it does not change their characteristics (Walras, 1954, pp. 319–24). In particular, by excluding uncertainty he eliminated consideration of much of the behaviour associated with money and portrayed it as making no difference in the properties of his model. That is not true of the role of money in the economy, a fact of which his writings on monetary policy reveal him to have been perfectly cognizant. Moreover, his concept of fictional perpetual annuity shares is a superfluity that further detracts from the verisimilitude of his models of capital formation and money. He should instead have elaborated upon the behaviour related to some of the major financial assets in which people actually invest. These various assumptions result in a model of a purely hypothetical economy that is so radically unlike any past or present economy as to fail to be useful as even a highly abstract analysis of economic behaviour. The reason for this situation is that the pledges model is designed to be consistent with certain mathematical conditions – that is, with the solutions of a set of equations – rather than being a set of assumptions and mathematical conditions designed to explain economic behaviour. It is a pity that Walras and most subsequent general equilibrium theorists adopted the pledges model and abandoned his robust and more realistic disequilibrium production model, for through its development lies the way to a more useful general equilibrium theory.

The second major criticism is that Walras was wrong to assert that he had proved that equilibrium exists and proved that the *tâtonnements* of his models converge to equilibrium. He

should have been chided gently for making those erroneous claims but not castigated for his failure to give the proofs, because they were beyond the technical ability of any 19th-century economist (see Weintraub, 1983; van Daal, Henderiks and Vorst, 1985).

The third major criticism is that the alleged equilibrium of Walras's formal model is logically flawed. This has already been indicated with reference to his disequilibrium-production model of *tâtonnement*, and the present criticism refers to his pledges model. To construct a model with a genuine static equilibrium, which was his intention, Walras should have assumed that the capital stock is maintained but that there are no net additions to it. He could not, incidentally, have salvaged the solutions of his equations by supposing that the net addition to the capital stock in any year is so small relative to the previously existing stock that its effects in any year are negligible, because many of the economic processes within his model are long-run phenomena. Since he assumed that there are net additions to the capital stock, the equilibrium of his pledges model is of no significance because it cannot be actualized. It is factitious, existing only transitorily while his model is held in a state of arbitrarily suspended animation by the posulate that additions to the capital stock are not used – a *deus ex machina* that interrupts the incomplete workings of its endogenous processes. The instant the postulate is removed, the 'equilibrium' is ruptured and, through a dynamic process that is the antithesis of true equilibrium, many of the nominal parameters and all the variables of the model change, in the way indicated earlier in the discussion of the consequences of the use of net new capital goods. Any stationary equilibrium that the system may eventually reach is quite unlike the solutions to Walras's equations of general equilibrium, and if net new capital goods continue to be produced, the system follows a path of growth. Proofs of the existence of static equilibrium in pledges models in which net investment occurs are therefore invalid.

The fourth major criticism is that Walras's treatment of comparative statics was unsatisfactory because he did not consider some important parametric changes and did not adequately trace the consequences of the parametric changes that he did make, and therefore failed to explore the ramifications that his general equilibrium model was intended to identify. For example, in his theory of production he did not consider alternative values of the fixed coefficients of production, nor changes in the work and income preferences of the workers, nor in their numbers; and in his formal theory of the capital market he did not make parametric changes in the stock of capital goods that exists initially.

These criticisms cannot obscure the greatness of Walras's contribution. When he began his investigations in 1868, economics on the Continent was hardly a scientific pursuit but rather a mixture of normative prescriptions, classical theories expressed alongside protectionist doctrines, and commercial law. In England it was in the state exemplified by the work of J.S. Mill – with much that could be used as a basis for future investigations, but also without a clear view of the relationships of distribution and production, limited by a cost of production theory of value, and lacking a theory of supply and demand in multiple markets. The attitude of most of Walras's

contemporaries was that, since economic behaviour involves preferences and the human will, it cannot be expressed in a rigid and deterministic set of algebraic relations. Walras changed all that, transforming economics and propelling it forward in a gigantic intellectual leap.

His contribution can be divided into two interrelated parts. One was that he constructed or refined or adapted to his purposes many of the fundamental building blocks of modern economic theory. Putting his pledges model aside, it can be seen that in this effort he accomplished an enormous amount of highly creative economic analysis, brilliantly structuring economic reality to bring many of its essential features into clear relief, in eight major original contributions to economic theory. First, he went far beyond the work of the other developers of the marginal utility theory by using it to analyse the behaviour and equilibrium in multiple markets of a variety of participants undertaking different economic functions, rather than confining the theory to the investigation of consumption and simple exchange. Second, he had clear priority in constructing the theory of exchange in multiple competitive markets. In that regard, his work was greatly in advance of his predecessors' and was replete with fruitful constructions, theorems and postulates, like the reciprocal relation of supply and demand, the device of a numéraire, the individual budget equation, Walras's Law, the theorem of equivalent distributions, and the laws of change of prices. Third, he constructed a theory of the firm and of market supply in which he appears to have developed independently the modern idea of a firm's production function, derived the equation for a firm's average cost, expressed the firm's offer of output mathematically, and aggregated the firm's supply functions to obtain the market supply in a particular industry. Fourth, he was the first to examine the question of the existence of equilibrium in a competitive multi-market system of exchange and production. Fifth, in his work on *tâtonnement* he initiated the study of the problem of the stability of competitive general equilibrium and contributed significantly to its understanding, with his most successful theorizing on the topic relating to a disequilibrium production economy. There is nothing in the literature before Walras's time nor until the time that his work was discussed by others that is even remotely like his examination of the process of convergence to equilibrium of a competitive multi-market system. Sixth, he developed a theory of the entrepreneur, of profits and of the allocation of resources that became the basis of Continental work on those topics (Pareto, 1896/7 passim; Pareto 1906/9, passim; Barone 1896, p. 145; Schumpeter [1911/26] 1961, p. 76; Schumpeter, 1954, p. 893; Walker, 1986). Seventh, Walras created a fruitful theory of capital, achieving an early formulation of the conditions for a Pareto optimum in capital markets. As in a number of his other investigations, his characteristic contribution was not to be the first to think of the problem but to be the first to structure it thoroughly and to provide a mathematical demonstration of the equilibrium conditions. Eighth, he developed a cash-balances theory of money which had great originality and has stimulated much valuable research (Marget, 1931; Marget, 1935; and see Walker, 1970, p. 696). Those eight areas of analysis were the core of

neoclassical microeconomic theory and thus constituted much of the structure of knowledge that was the starting place for 20th-century economics.

The second part of Walras's contribution was his idea of the general equilibrium of the economic system and his concrete implementation of that idea through devising a system of equations to express it. By his work Walras demonstrated that it is possible in principle to set up a system of equations to describe the functional relationships and static equilibrium of a multi-market economy in which complex economic processes occur, thus accomplishing by the mid-1870s far more than any other economist had done in regard to building a model of the economic system as a whole, and more single-handedly in that regard than any other economist in the history of the discipline. He therefore provided a substantial beginning for the mathematical analysis of the interrelationships of all parts of the economy as it has developed since the 19th century.

The two parts of Walras's contribution are complementary aspects of a single theoretical whole. Other economists had helped in fashioning the building blocks that he used, but Walras's achievement was not only to develop them but to integrate them into a comprehensive model. To do him complete justice, it is necessary to appreciate the richness of the texture of his work resulting from the many aspects of economic life that he analysed, for without them his general equilibrium theory would be an empty and sterile mathematical shell; and it is necessary to appreciate the mathematical structure that he devised for the purpose of supporting and weaving together those accounts of economic processes, for without it their significance and place in the economic system would not have been fully revealed.

Influence

Walras's work was hardly noticed in France during the twenty-five years after 1874, and as late as 1934 his centennial elicited no conference on his work there. By the 1950s, however, the French attitude had changed towards Walras, as was ultimately symbolized by the creation in 1984 of the Centre Auguste et Léon Walras at the Université Lyon II. With the English, Walras's experience was also disappointing. His initial cordiality towards W.S. Jevons, as a fellow pioneer in mathematical economics, was dissipated by Jevons's failure to recognize Walras's contributions to the theory of exchange and to the construction of a complete theory of a competitive economy, and eventually Walras, quite unreasonably, came to regard Jevons as a plagiarist of his work (Walras to M. Pantaleoni, 17 August 1889, L 909). Similarly, Walras's relations with P.H. Wicksteed began well (Wicksteed to Walras, 1 December 1884, L 619) but deteriorated sharply when Wicksteed failed to give credit to those whom Walras considered to be the true originators of the theory of marginal productivity (Jaffé, 1965, L 1220, n. 3; Walras, 1896a, pp. 490–92). Walras felt neglected by Alfred Marshall, who mentioned him only thrice in the briefest of comments in the *Principles* (Marshall, 1890, 1920) and wrote not a word about Walras's development of general equilibrium theory. Walras also came to dislike Edgeworth for

criticizing his theories of *tâtonnement,* capital goods and the entrepreneur (Walras to Gide, 3 November 1889, L 933, and 11 April 1891, L 1000; Walras to Pantaleoni, 5 January 1890, L 953). In general, Walras believed, the English had closed their minds to his theories and had become spiteful in their treatment of them (see Walker, 1970, pp. 699–70).

The extremity of the language with which Walras characterized the English was unjustified, because, although he had reason for disappointment with their neglect of his general equilibrium theory, Jevons (1879, preface) and Edgeworth (1889) had recognized valuable elements in his work, and he was the only living economist included in the first edition of Palgrave's *Dictionary of Political Economy* (Sanger, 1899). The fact is that Walras grew hypersensitive about the motives of his critics, the failure of the majority of economists to recognize the value and priority of his contributions, and the possibility of plagiarism of his ideas during the 1880s and 1890s. There had been two periods in his life, he complained, 'one during which I was a madman, and one during which everyone made my discoveries before me' (Walras, undated, in Walker, 1983, p. 203, n.54).

This account of Walras's disappointments should be balanced by a realization that his scientific labours had afforded him, 'up to a certain point, pleasures and joys like those that religion provides to the faithful' (Walras to Marie de Sainte Beuve, 15 December 1899, L 1432), and a recognition of the professional satisfactions that he increasingly experienced in the last two decades of his life. Maffeo Pantaleoni (1889), Enrico Barone ([1895], 1983, p. 186; 1896), and Vilfredo Pareto (Pareto to Walras, 15 October 1892, L 1077) contributed greatly toward giving Walras's work a secure place in Continental economics and thus ultimately in economics everywhere. In 1895 Pareto's appointment as Walras's successor to the chair of economics at Lausanne assured Walras that his doctrines would be perpetuated and developed, and the accessible literary presentations of Walras's ideas in Pareto's books (1896/7, 1906/9) began their widespread dissemination. Pareto borrowed most of the ideas of Walras that have been mentioned in this essay, using them as the basis for his contributions to the theories of general equilibrium, the monopolistic entrepreneur, capital, and production. Wilhelm Lexis, Ladislaus von Bortkiewicz and Eugen von Böhm-Bawerk gave Walras's theories serious attention. Knut Wicksell based his theory of price determination squarely upon Walras's work (J.G.K. Wicksell to Walras, 6 November 1893, L 1168), as did Karl Gustav Cassel (1903, 1918). Walras was given recognition in the United States: in 1892 he was made an honorary member of the American Economic Association, Irving Fisher praised his work (Fisher, 1892, p. 45; 1896), and H.L. Moore became his avowed disciple and explicator (Moore to Walras, 19 May 1909, enclosure to L 1747; Moore, 1929).

These manifestations of acceptance led Walras to believe he would ultimately triumph, and that enabled him to achieve a mental calmness (Walras to Marie de Sainte Beuve, 15 December 1899, L 1432; Walras to A. Aupetit, 28 May 1901, L 1485). 'Be assured of my serenity', he wrote to old friends in 1904, 'I have not the least doubt about the future of my method and even

of my doctrine; but I know that success of this sort does not become clearly apparent until after the death of the author' (Walras to G. and L. Renard, 4 June 1904, L 1574). A strong indication of what the future would hold for his theories was given by the celebration of his jubilee in 1909 by the University of Lausanne, in the course of which he was honoured as the first economist to establish the conditions of general equilibrium, thus founding the School of Lausanne (Jaffé, 1965, L 1696, n. 5). His achievements were praised in a statement signed by 15 leading French scholars, including Charles Gide, Charles Rist, Georges Renard, Alfred Bonnet, A. Aupetit and François Simiand (enclosure to L 1747), and in communications from many others (Pareto to the Dean of the Faculty of Law of the University of Lausanne, 6 June 1909, L 1755; Schumpeter to Walras, 7 June 1909, L 1756). It is now clearly apparent that his prediction of great success was accurate, because his theory of general equilibrium, as improved by his contemporaries, has been given a great deal of attention and further development in the 20th century by writers such as Henry Schultz, John von Neumann, Abraham Wald, John R. Hicks, Oscar Lange, Paul A. Samuelson, Lionel McKenzie, Gerard Debreu, Don Patinkin, Kenneth Arrow, Frank Hahn and Michio Morishima (see Weintraub, 1983, 1986), and the filiations of Walras's ideas have become so numerous and dense as to be an integral and central part of the mainstream of modern economics. Thus his twofold achievement of developing particular theories and binding them together in a model of an entire economic system has given his work an influence on economic theory that has been durable and immense. For sheer genius and intuitive power in penetrating the veil of the chaos of immediately perceived experience and divining the underlying structure of fundamental economic relationships and their extensive interdependencies and consequences, Walras has been surpassed by no one.

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See also

general equilibrium; tâtonnement and recontracting.

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