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Jules Dupuit's Contribution to Mathematical Economics*

"So soon as it is realized […] that political economy is concerned with quantities susceptible of a more or a less, it must also be recognized that it is in the realm of mathematics […]. Not only do the symbols and drawings of mathematics give body and form to abstract ideas and thereby call the senses to the aid of man's intellectual power, but its formulae take hold of these ideas, modify them and transform them, and bring to light everything that is true, right and precise in them, without forcing the mind to follow all the motions of a wheelwork the course of which has been established once for all. They are machines which, at a certain stage, can think for us, and there is as much advantage in using them as there is in using those which, in industry, labour for us".

Jules Dupuit, 1844.

* First draft - please do not quote.

0. Introduction

Serving as a civil engineer for the State of France, in charge of the construction and maintenance of roads, bridges and canals, Jules Dupuit[1] was led from engineering questions into his seminal contributions to economic theory. His articles "De la mesure de l'utilité des travaux publics" (1844) and "De l'influence des péages sur l'utilité des voies de communication" (1849) are now well-known, not only in the French-speaking world[2]. Key features of subsequent neoclassical economics have been recognized in these works, and he has been acknowledged as a pioneer of the intellectual tradition of microeconomic enquiry (Ekelund and Hébert, 1999). Moreover, major surveys of the origins and early developments of mathematical economics review his contributions (Theocharis 1993, pp. 50-55).

This paper focuses on the latter aspect, and aims to go deeper into Dupuit's involvement in the gradual progress of mathematical economics at the time, by taking into account the state of advancement of the discipline in the middle of the 19th century. The use of mathematics, including relatively advanced tools such as geometry, algebra, and calculus, was rather uncommon in economic writings, although several attempts had already been made at using them in the study of economic phenomena, as documented by Theocharis (1961, 1993) and Baloglou (1995). What may seem striking to today's readers, however, is that existing work in mathematical economics mainly concerned the study of the supply side of the market –calculations of costs of production, of profits, etc., while formal treatment of demand, utility, and, more generally, consumers' behavior was relatively infrequent. One reason for that is the widespread influence of the classical school, placing emphasis on production rather than consumption, on the economic thought of this period. At the same time, however, various intellectuals did recognize utility as a key determinant of value –so why did they not significantly inspire early mathematical economists? An intuitive explanation is that estimating profits and costs only requires knowledge of easily accessible information, notably input prices and available production techniques; by contrast, the utility of a good to a consumer is dependent on subjective factors, of qualitative rather than quantitative nature, and is therefore hard to translate into precise figures. Even a masterpiece such as Augustin

[1] Trained at the École Nationale des Ponts et Chaussées in Paris, Arsène-Jules-Etienne-Juvenal Dupuit (1804 – 1866) was chief engineer in the Sarthe region, where he was in charge of roads and waterways, then in the Marne and Maine-et-Loire regions. He was called back to Paris in 1850 to work for the municipal water system, served as inspector-general of the Corps des Ingénieurs, and was then appointed to the Conseil Général des Ponts et Chaussées.

[2] An English translation of Dupuit's 1844 article, entitled "On the Measurement of the Utility of Public Works", appeared in 1952, while only a section of the 1849 article has been translated, under the title "On Tolls and Transport Charges" (1962).

Cournot’s 1838 *Researches* contains evaluations of firms’ profits and costs, as well as profit maximization calculations, but no effort to express consumers’ viewpoint in mathematical terms. His market demand function is not derived from an assessment of consumers’ wishes or needs, on the grounds that the latter can hardly be evaluated on an objective basis:

> “one should distinguish the abstract idea of wealth or exchangeable value, which can be rigorously analyzed, from the accompanying ideas of utility, scarcity, adequacy to man’s needs and pleasures, which the term wealth, in everyday language, calls to mind: these ideas are by nature erratic and indefinite, thus one could not build any scientific theory upon them” (Cournot 1838, p. 9, our translation).

At the heart of this paper is an effort to reinterpret Dupuit’s accomplishments in the field of microeconomics, previously highlighted by Ekelund and Hébert (1999), by taking into account this general context, in order to assess how his work innovated with respect to the mathematical economics of his time. In this respect, the existing literature on Dupuit’s achievements in pre-neoclassical economic theory actually provides some useful hints, since the theoretical innovations that have been credited to him suggest that he was dealing precisely with the problem of finding a way to apply some form of mathematical reasoning to the study of utility, demand, and consumer behavior. The paper therefore asks whether he succeeded in converting an apparently qualitative concept as utility into a numerical magnitude that could be included into calculations and represented in diagrams. Insofar as this question is answered in the affirmative, it will be possible to draw the conclusion that Dupuit extended the field to which mathematics could be applied, by adding utility-related topics to the evaluation of costs of production and to profit-maximization calculations. In this sense, it can be said that his contribution opened the way to further developments in the mathematization of economics.

Yet proving that Dupuit’s contribution to the study of utility can be reckoned as a step forward in the advancement of mathematical economics, would be no evidence that the author himself also played a major role in promoting the use of mathematical tools (especially advanced ones) in economics. Admittedly, his praise of mathematics in the concluding paragraph of his 1844 article indicates that he grasped the potential usefulness of mathematics to the progress of economic theory:

> “so soon as it is realized […] that political economy is concerned with quantities susceptible of a more or a less, it must also be recognized that it is in the realm of mathematics […]. Not only do the symbols and drawings of mathematics give body and form to abstract ideas and thereby call the senses to the aid of man’s intellectual power, but its formulae take hold of these ideas, modify them and transform them, and bring to light everything that is true, right and precise in them […]. They are machines which, at a certain stage, can think for us” (Dupuit 1844, pp. 109-110).

The problem, to date never addressed in the secondary literature on Dupuit, is that this paragraph seems at odds with the approach he actually adopted, since he almost always used simple numerical examples that only required knowledge of basic arithmetic to be understood. Only in a short appendix to his 1844 article did he introduce advanced (by the standards of the early decades of the 19th century) mathematical methods, by writing demand functions in the general form $y = f(x)$, stating their essential properties, and representing them graphically in diagrams. Accordingly, another purpose of the present paper is to explain the seeming inconsistency between Dupuit's claims on the power of mathematics as a tool of economic analysis and the fact that he made a very limited use of it. Clarity on this point may be of help in assessing Dupuit's contribution to mathematical economics in full.

Our study will be based on a closer examination of Dupuit's 1844 article, although there will be a few references to the 1849 paper as well. The first section highlights Dupuit's contribution to utility theory, by examining how he came to propose an operational definition of the measure of utility. The second section goes deeper into the topic of the notion of utility and the principle of utility maximization, and the third section scrutinizes Dupuit's mathematical treatment of demand and utility, as can be found in the 1844 appendix. In the last section, we will sum up and draw some conclusions.

1. Dupuit's major achievement: quantifying the "utility" of a good to consumers

Dupuit first studied choice of technique problems, most prominently in his 1842 article "Considérations sur les frais d'entretien des routes", where he compared the costs of production of different techniques of road construction and maintenance, so as to identify the one that would minimize total expenses. His contribution was part of a wide-ranging debate, involving a number of French engineers (Grall 2002, p. 64). However, being in charge of public works such as roads and bridges, Dupuit soon realized that cost minimization was an inappropriate choice criterion. In the case of public goods provision, one should not choose the technique that is least expensive for the producer, but rather prefer the one that proves most advantageous to the whole community of users. Accordingly, he tried in subsequent works to replace choice models based on production costs evaluation with decision rules founded on an assessment of the benefit that consumers get from a good or service.

This was, however, very difficult to do, owing to the fundamental asymmetry that exists between producers and consumers: while it is relatively easy to calculate the gains that an exchange yields to a producer, because one only needs to know the difference between market price and cost of production for each unit sold, it is much harder to estimate the gain of a consumer, which appears as a subjective, qualitative rather than quantitative notion. In order to tackle this problem, Dupuit tried to create a method for quantifying consumers' advantage, by transforming it into a numerical magnitude that could be used in calculations. He first outlined his solution in the above-mentioned 1844 article, and gave further details in later papers. Let us briefly present its main features, by following the author's own reasoning closely, before commenting on its interest and novelty.

Consider a public work, for example a road. Since it is used by merchants in order to transport goods from the place where they are produced to the location where they are to be sold, it has an impact on the total cost of production of these goods, defined as "what it costs to make an article available for consumption" (Dupuit 1844, p. 94), including both manufacturing and transport costs. Hence, a new road is advantageous if it cuts the total cost of production of goods, either by lowering their transportation cost, or by bringing to the market new products, whose manufacturing costs are lower. For instance, suppose a town uses 10,000 tons of stone each year for the construction and repair of its houses (Dupuit 1844, p. 93). Each ton costs 20 francs, including 16 francs for extraction from the quarry and 4 francs for transport over a short distance, say 4 leagues. Now suppose a new means of communication is established, e.g. a canal. It may happen that the cost of transporting the stone is higher by the new route than by the old, on account of the new route being longer (say 100 leagues), but that this extra cost can be compensated by other circumstances: for example, the canal may pass by an easily worked quarry which had not formerly been exploited, so that total costs of production now amount to 15 francs, including 2 francs for extraction and 13 for transport over a long distance. Since the total cost of production of the old stone exceeds the total cost of production of the new one, the inhabitants of the town will purchase the latter instead of the former from now on.

Clearly, the canal proves to be beneficial to the inhabitants of the town, who can now obtain the same good for a lower price. However, how can this benefit be precisely measured? One might be tempted to believe that it equals 50,000 francs, i.e. the 5 francs reduction in the unit cost of production multiplied by the 10,000 tons used. However, says Dupuit, such an evaluation would underestimate the benefit, because the effect of the canal in having reduced the cost of production and thus in having yielded benefits for buyers does not stop there. Indeed, the fall in price will render the stone suitable to new uses: “in many buildings it will replace brick and timber; streets will now be paved which were not so before, and so on” (Dupuit 1844, p. 94). Consumption increases from 10,000 to, say, 30,000 tons each year.

The problem now is, how can the benefit produced by the 20,000 extra tons be measured? The engineer Henri Navier, writing on this same topic a few years before (1832), took the difference between the old cost of production and the new one (as for the first 10,000 tons) but such an evaluation is incorrect, because this additional quantity was not purchased at the old price. In order to find a solution, Dupuit searched for a subjective notion of benefit (“utility”, in his own wording), defined in such a way that, on the one hand, different quantities of the same object yield different advantages to an individual, according to the importance of the needs that they are meant to satisfy, and on the other hand, equal quantities of the same object yield different benefits to different individuals. But, how to evaluate this gain, which changes from one individual to another, and may vary for the same individual? The way people react to a given price can provide a hint. If, at the initial price of 20 francs, some individuals did buy a ton of stone, it is presumably because they believed that the benefit they could obtain from the stone would exceed the benefit obtained by keeping the 20 francs. In a certain sense, it may be said that a ton of stone yields an advantage of at least 20 francs to them. If others did not buy at the price of 20 francs, it is apparent that they did not attribute that much utility to the consumption of stone; if they buy at 15 francs, then they attribute a utility at least equal to 15 francs (but certainly lower than 20 francs), to a ton of stone. More precisely, among those who buy at 15 francs, some attach so little value to the consumption of stone that they would give up if the price were to rise by as little as 1 franc. For them, utility is therefore less than 16 franc. Others would cease to buy after a rise of 2 francs: for them, the utility of a ton of stone is between 16 and 17 francs, and so on. In short,

> “in order to know the utility of each ton consumed it would be necessary for each consumer to make known the strength of his desire in terms of the price which would make him cease consuming” (Dupuit 1844, p. 94).

The problem is how to estimate the “strength of desire” of consumers, at first sight a subjective and variable notion, difficult to quantify. Dupuit’s solution is a thought experiment, using taxation as a tool capable of revealing apparently invisible benefits:

> “it is beyond doubt that a tax can add nothing to the utility of a product; but when we look at it from the consumer’s point of view we can say that its existence brings to light undeniably that the product has a utility greater than the cost of production” (Dupuit 1844, p. 85).

A buyer pays a good on which a tax is imposed if he finds at least an equivalent utility in it:

> "for, in spite of the tax, he is at perfect liberty to buy it or not to buy it. It is not within the power of the state to make him pay, by means of the tax, anything more than the utility which he derives from this purchase" (Dupuit 1844, p. 85).

The proposed thought experiment is as follows:

> "Suppose that all those similar commodities of which we want to discover the utilities, are all subjected to a tax which rises by small steps. Each successive increase will cause a certain quantity of the commodity to disappear from consumption. This quantity, multiplied by the rate of the tax, will give its utility expressed in money. By thus letting the tax go up until there are no more consumers, and by adding together all the products of this multiplication process, we will arrive at the total utility of the goods" (Dupuit 1844, p. 96).

If a tax of 1 franc imposed on stone deprives the canal of the carriage of 7,000 tons, then we may say that the utility of this transport is 1 franc. If a new tax of 2 francs reduces traffic by another 5,000 tons, the utility they yield may be evaluated at 2 francs at most, and so on. Suppose that, by relating taxes with the amounts of traffic they cause to disappear, we obtain the following result for the 20,000 new tons carried by the canal:

7,000 tons	1 fr.	7,000 fr.
5,000	2	10,000
4,000	3	12,000
3,000	4	12,000
1,000	5	5,000
Total : 20,000		46,000

Table 1.

In other words, the advantage ("relative utility", in Dupuit's terminology) derived from the 20,000 extra tons that the canal allows to transport can be estimated at 46,000 francs. If to these 46,000 francs of utility we add the 50,000 francs corresponding to the 10,000 tons of initial consumption, we arrive at a figure of 96,000 francs for the total relative utility of this type of transport. Notice that the calculation would be identical, except for the sign, if one had to evaluate the loss to users that may be due to a rise in the price of the good, or to the imposition of an indirect tax. Then Dupuit computes the "absolute utility", equal to the sum of the relative utility and of total cost of production, which in turn, is equal to the unit cost of production, multiplied by the number of tons sold: in this case, it is 96,000 + 30,000 x 15 = 546,000 francs. Absolute and relative utility obviously coincide if costs of production are nil. Dupuit claims that his own method of measurement is very general, in view of the fact that it

> “is not peculiar to means of communication, but can be applied to everything, to any working tools whatever and to their products; so that we can say in general that the measure of the utility of a product is the tax which would prevent it being consumed” (Dupuit 1844, p. 96).

In order to fully appreciate the interest and novelty of the measurement procedure Dupuit proposed in his 1844 article, it is helpful to relate it to the state of mathematical economics at the time –specifically, to the fact that the subjective nature of concepts such as utility, satisfaction, and need, prevented most early mathematical economists from considering them in their work. In spite of these difficulties, Dupuit set out to conceive a method to provide an operational definition of consumers’ point of view that could be subject to mathematical treatment. He seems to have realized that measurability is not an intrinsic property of phenomena, but can be assigned to them, by bringing about measurement procedures, founded on theoretical or empirical arguments.

Dupuit’s definition is not meant to be a rigorous measure of “the quality which things have *of being able to satisfy men’s needs*” (Dupuit 1844, p. 89, italics in the original). It would be difficult (or even impossible) to assess human needs or wishes as such, but his method allows him at least to take account of the intensity of needs and wishes through their monetary expressions. It is based on individuals’ decisions to buy or not to buy a good, given their incomes and the price of the good: “political economy only bakes bread for those who can buy it” (Dupuit 1844, p. 89). This method has the merit of allowing for an assessment of the total quantity that a number of different individuals wish to buy at a given price, without requiring interpersonal comparisons of needs and wishes, which would be problematic:

> “it would be difficult to say whose hunger was the greater –the rich man’s, who would be willing to give a million for a kilogramme of bread, or the poor man’s, who, having nothing else to give, would risk his life for it”, Dupuit, 1844, p. 89).

Dupuit’s first and foremost contribution to mathematical economics consists in his effort to provide an operational definition that permits to include a measure of consumers’ satisfaction into economic calculations. Although he only used simple arithmetic in his reasoning, as the above cited quotations confirm, his work can be reckoned as a progress for mathematical economics, which was hardly capable of including utility-related arguments up to that time. This advance was indispensable to enhance mathematical economics at this stage of its historical development, by enlarging its field of application from the study of the profit-maximizing (or cost-minimizing) behavior of producers to the study of the utility-oriented behavior of consumers. In this sense, Dupuit's contribution constitutes a step forward with respect to Cournot's, who believed that no rigorous study of utility would ever be possible.

2. Demand, utility, and utility maximization in Dupuit

After claiming that Dupuit's contribution helped to include consumers' satisfaction into mathematical economics in some sense, it may now be interesting to go deeper into the meaning he attached to the concept of utility, in order to better assess its novelty and interest.

2.1 Utility and demand

Let us first examine the basic characteristics of the relationship between price and the quantity of objects consumed at each price, as Dupuit conceived it. He never called it "demand"; he did not label it in any particular way in the 1844 article, but as we shall see later, he used the expression "curve of consumption" in the mathematical note at the end of this article, and "law of consumption" in the 1849 paper. Be that as it may, his reasoning is based on the intuition that an individual attaches a different utility to different quantities of the same good. Take the example of stone which we referred to earlier: at a price of 20 francs per ton, buyers use it to build or to repair their houses but, if price falls to 15 francs, they may not only buy the quantity that they need for necessary construction or renovation work, but they may also increase their consumption, by using stone for other, less urgent needs, e.g. they may be willing to replace brick and timber in some buildings. It is thus clear that, in general, the consumption of each individual increases as price falls. Consequently, at the system level, the sum of the quantities consumed by many different individuals also increases as price falls.

This is, according to Dupuit, the first "general law" (Dupuit 1844, p. 103) to which the relationship between price and (aggregate) demanded quantity "remains constantly subject" (Dupuit 1844, p. 103). This law is always valid, in his opinion, despite the fact that the exact shape of the relationship between price and quantity may not be known for any commodity, and that "it can even be said that it will never be known since it depends on the volatile will of human beings; it is today no longer what it was yesterday" (Dupuit 1844, p. 103).

The second general law states that:

> "the increase in consumption due to a price fall will be the greater, the lower the initial price. If a fall in the price of an article from 100 to 95 francs brings in another thousand consumers a further fall from 95 to 90 will bring in more than a thousand" (Dupuit 1844, p. 103).

This particular aspect (which obviously concerns aggregate but not individual demand functions) depends on the income distribution that prevails in modern societies, according to an argument already found in Germain Garnier (1796, pp. 195-6) and Jean-Baptiste Say (1828-29, p. 358):

> "this property reflects the structure of society which, if it is divided into groups according to income, and these groups are placed one on top of the other starting with the poorest, has a shape similar to one of those pyramids of cannon-balls which are to be seen in parks of artillery –the lower the layer, the more balls it contains. Thus, as the price of an article falls, its use spreads to more and more consumers, quite apart from the fact that existing consumers purchase it in greater quantities" (Dupuit 1844, p. 103).

Thus Dupuit's justification of the two "laws" relies on individual evaluations (depending on the tastes, needs and available resources of each buyer), as well as on information supposedly derived from the empirical observation of the structure of society at a given moment in time. But what about "utility", to which Dupuit constantly referred, starting from the very title of his paper? Indeed, the question arises whether he envisaged the whole chain of derivations from utility maximization to demand functions, similarly to neoclassical economists, who first introduce a utility function, conditional on the needs, tastes or preferences of an individual, and a budget constraint, depending on the prevailing market conditions, summarized by current prices; then maximization of each individual's utility function subject to his or her own budget constraint gives rise to individual demand functions and ultimately to aggregate demand functions.

It does not seem possible to answer this question in the affirmative. As a matter of fact, Dupuit never thinks in terms of utility functions independently of market conditions, as a necessary presupposition of demand analysis, and never explicitly distinguishes utility considerations and budget constraints as two separate determinants of demand. His thought experiment leads him to build his demand function directly, so to speak, as the individuals he pictures would be unable to determine the maximum sacrifice they would be willing to make in order to acquire a given good without knowing prices. His measurement criterion takes into account, at the same time, the personal tastes or needs of the individual concerned and the prevailing market conditions. The lack of a separate, preliminary treatment of utility in his work is also plainly revealed by a few passages, such as "utility and everything that can be said about it derive from the law of consumption" Dupuit 1849, p. 7). Together with the absence of a clear-cut distinction between utility and demand considerations, this is probably one reason why his approach did not make much sense to some later neoclassical writers, including Léon Walras, who claimed that “Mr. Dupuit’s theory consists in an utter confusion between utility curves and demand curves” (Jaffé 1965, vol. I, p. 535, our translation).

In fact, what Dupuit calls “utility” –more precisely, “relative utility”- is a concept similar to Alfred Marshall’s consumer surplus. Both approaches provide practical ways to compare monetary with non-monetary magnitudes, so as to build money measures of a person’s satisfaction, in terms of the amount of money s/he would be willing to pay for an object rather than renounce to it. Both measures have the appeal of seeming scientific objectivity, in that they enable to calculate costs and benefits from the observed behaviour of individuals, i.e. their actions of buying or not buying the object, with a minimum of subjective judgements.

It is beyond the scope of this paper to discuss the merits and the shortcomings of consumer surplus arguments in detail. It is enough to recall the controversies that the use of measurable utility indexes has aroused, especially since the discoveries by Fisher and Pareto, that all the propositions regarding equilibrium of the consumer in competitive markets can be derived without them. Though Paul Samuelson (1974) has shown how a “money-metric utility function” can be defined in a rigorous manner, the use of consumer surplus analysis as a tool of investigation in a rigorously acceptable way comes with a cost, in that it is necessary to make stringent assumptions about consumer preferences (Chipman and Moore, 1976).

2.2 Utility or profit maximization?

If Dupuit’s writings are somewhat confused about the distinction between utility and demand, what about the principle of utility maximization? In a sense, Ekelund and Hébert’s suggestion that Dupuit contributed to place utility maximization at the heart of economic theory (2000) is well documented by his whole work. The idea that “the purchaser never pays more for the product than the value he places on its utility” (Dupuit 1844, p. 89), on which Dupuit insists in his writings, and which actually underlies most of his numerical examples, clearly hints that consumers do behave rationally, in that they always choose the alternative they value most between an object and the monetary equivalent of its price. At a time in which it was relatively uncommon to interpret consumers’ behaviour in this light, Dupuit’s emphasis on their ability to make consistent choices between different alternatives can be seen as a significant change, opening the way to later efforts to found explanations of consumer behaviour on maximization models. Despite the novelty of this insight, and its implications for subsequent theoretical developments, however, the principle of utility maximization seems to be rather underdeveloped in Dupuit’s work. The aforementioned absence of clearly distinguishable utility functions, similar to the ones neoclassical theory has accustomed us to, obviously implies that there are no utility maximization calculations that take into account the preferences of an individual over a range of different goods. More to the point, there are some ambiguities in the way Dupuit depicts consumers' behaviour. On the one hand, his idea that "no one is ever a dupe except in relation to the cost of production" (Dupuit 1844, p. 89) emphasizes buyers' ability never to pay more than what they think is the value of an object; on the other hand, it does not necessarily imply that they maximize their utility, insofar as consumers would accept to pay any price lower than the utility they attach to an object, but this price may be above the cost of production -hence even a small decrease in price would increase their satisfaction. Sometimes, Dupuit even characterizes consumers' behaviour in much more unfavourable terms, for example by evoking their "vanity" and "credulity", which have enabled merchants to "set traps" for them, and to conceive "devices for taking in dupes" (Dupuit 1844, p. 89).

By contrast, the principle of profit maximization is clearly present and recognizable in Dupuit's economic thought. An example in his 1849 article is telling in this respect. Suppose the table below gives the number of crossings over a bridge corresponding to each toll rate, with tariffs in the first column and number of crossings, i.e. demanded quantity, in the second one. The number of crossings is at its maximum, taken to be 100, when price is zero, decreases gradually as the tariff rises, and falls to zero for a toll rate of 12. The third column gives the reduction in demand due to each unit increase in tariff. Column four gives the (marginal) loss of utility due to a unit increase in the toll, calculated as the product of the reduction of crossings (column 3) and the toll rate (column 1), while column five indicates the total loss of utility, i.e. the sum of the marginal losses for each toll rate from 0 up to the level considered. The total loss corresponding to the maximum toll rate, equal to 445 in this case, obviously equals the total utility of a zero rate; hence, Dupuit re-writes this same amount in the first line of column six, which represents total utility. The following lines of column 6 are obtained by subtracting the sum of partial utilities lost, taken from column 4, from the total utility of 445. In the last column, Dupuit shows the yield of the toll for each rate.

Since demand decreases as the toll rate increases, the yield of the toll is nil for a zero rate, then grows with the toll, reaches a maximum (here, for a toll rate of 5), and falls again back to zero. Dupuit insists, both in his 1844 and in his 1849 articles, that two entirely different toll rates, one below and one above that which brings in the maximum revenue, may yield more or less the same revenue to the supplier of the good or service, while bringing about considerably different losses of utility. In this particular case, toll rates equal to 1 and 9 correspond approximately to the same yield, i.e. 80 and 81 francs, respectively, but the loss of utility is only 20 for a unit toll rate, and is equal to 346 if the toll rate increases to 9. The reason of this is that the toll is not only a burden to those who pay it, but also to those who cannot cross the bridge because they cannot afford it.

Toll rate	Number of crossings	Reduction of crossings due to rate increase	Utility			Yield of toll
			lost by rate increase	lost by toll	corresponding to toll	
0	100	0	0	445	0	0
1	80	20	20	425	20	80
2	63	17	34	391	54	126
3	50	13	39	352	93	150
4	41	9	36	316	129	164
5	33	8	40	276	169	165
6	26	7	42	234	211	156
7	20	6	42	192	253	140
8	14	6	48	144	301	112
9	9	5	45	99	346	81
10	6	3	30	69	376	60
11	3	3	33	36	409	33
12	0	3	36	0	445	0
Totals		100	445			

Table 2

On this basis, Dupuit explained how to choose a toll rate in the 0-12 range. Interestingly, he claimed that this choice will be different according to whether the bridge belongs to a private company or the government, because the purpose in view is not the same in these two cases. If the bridge is a public property, the government "will want to recover from the toll merely a fixed sum representing interest on the capital spent for construction, maintenance cost and perhaps amortization" (Dupuit 1849, p. 11). Suppose that the bridge costs 150,000 francs to build and that the figures shown in the above table are one-hundredth of the real traffic figures. Yearly proceeds of 8,000 are enough to cover interest on the capital spent for construction, at a rate of 4%, and to leave about 2,000 francs for maintenance and amortization. The government will thus choose toll rate 1, yielding a revenue of 8,000 each year. On the contrary, a private company « has only one aim, and that is to get the largest possible income from the toll » (Dupuit 1849, p. 11). Accordingly, the company will charge 5, i.e. the toll rate that corresponds to the maximum yield (165,000 francs). The differences between the results of private and public operation are shown in the table below:

Type of operation	Utility (fr.)	Crossings	Toll yield (fr.)
Public	42,500	80	8,000
Private	27,600	33	16,500
Difference	14,900	47	8,500

Table 3

The extra 8,500 francs that the private company charges the users of the bridge are profit for its shareholders, who gain just as much as users lose. It is a redistribution of wealth from consumers to producers, but from the point of view of society as a whole, it is not a loss. On the contrary, the difference of 14,900 francs is an additional utility that consumers gain in the case of public operation, and a dead loss for everybody in the case of private operation: "those who have not crossed the bridge have been deprived of a service which they value at 14,900 francs and which would have cost the company nothing" (Dupuit 1849, p. 12).

Nevertheless, it is possible to reduce the disadvantages associated with private operation, by differentiating tariffs in order to make all users of the bridge pay a toll proportionate to the utility they derive from the passage. Suppose for example "that the toll for a bridge is so arranged that every user pays half the price which would stop him from crossing" (Dupuit 1849, p. 16). In this way, nobody would be deprived of the service, and hardly any utility would be lost to society. Furthermore, he argues, the yield would at any rate be sufficient to allow the company make some profit, after covering its costs: price differentiation is more profitable for the producer than fixing a single rate at the lowest level that demand conditions allow for, which corresponds to the same quantity purchased (number of crossings in this case), but would yield a lower revenue. Hence, both producers and consumers would benefit from such a tariff scheme (it can thus be reminded in passing that Dupuit is acknowledged as one of the first advocates of price discrimination).

Of course, such a price policy is not easy to enforce, because users are not willing to declare the true utility they attach to the bridge, in the hope that they be charged less:

> "I need hardly say that I do not believe it possible that such a tariff should be applied voluntarily, because it would come up against the unsurmountable obstacle of the general public's universal dishonesty" (Dupuit 1849, p. 16).

The ability of a company manager thus consists in trying

> "to guess the needs of the consumers as well as the sacrifices they are prepared to make to satisfy these needs, and then to define the general characteristics by which consumers may be classified in the tariff schedule" (Dupuit 1849, p. 16).

There are examples of business people succeeding in structuring their tariff schedule in such a way that consumers are led to pay according to the sacrifice they are ready to make to satisfy their needs. The two or three-class tariff policies of most railway companies are ways to leave passengers to classify themselves, according to their willingness to pay:

> "there is a presumption that those who are willing to make the largest sacrifice for their journey are also those who value their comfort most and who have their carriages luxuriously appointed inside and out. So this is the treatment the company gives them. It also tries to guard against their avarice, which might induce them to travel in a lower class, by differentiating as much as possible the comfort provided for passengers" (Dupuit 1849, p. 24).

It is not against the poor that the war is being conducted, he concluded, but against the likely avarice of the rich. In short, business people have already made good progress in this direction "by just going ahead at random"; the problem Dupuit is trying to deal with "is merely one of according scientific treatment" to this question (Dupuit 1849, p. 16).

The above remarks show that Dupuit attributed a high degree of rationality to producers. He explicitly described their purpose in terms of profit maximization, and performed the necessary calculations to determine the tariff level that yields the greatest gain. He also seems to believe that most producers or merchants of goods and services have some intuitive knowledge of the relationship between price and demanded quantity, and try to use it to their own advantage. Of course, he was aware that the exact price / quantity relationship is normally unknown, but he believed that this is "no obstacle to the rational calculation of toll rates" (Dupuit 1849, p. 12). All business calculations turn on conjectures, after all:

> "if the law of consumption were fully known […] then these calculations would have to do only with perfectly determined problems soluble by the simplest arithmetic. But in the producer's uncertain world the solution depends both on his skill in guessing the needs of the consumers and on his imagination in devising a method of making them pay as much as possible" (Dupuit 1849, p. 12).

These abilities constitute what he called the "business talent" that "lets one merchant or one manufacturer make a fortune in an industry which ruins his neighbour" (Dupuit 1849, p. 26). In an uncertain environment, scientific knowledge can be of help in providing "general principles which might serve as a guide" (Dupuit 1849, p. 26). The author understood his own study as a means to improve merchants and producers' performance, by helping them to grasp the general characteristics of consumers' behavior, so that they no longer have to guess "at random", but only to work out the specific characteristics of the particular demand behaviour of their customers.

In sum, there seems to be a gap between the behaviour of producers and merchants, who are supposed capable of using all the possible means to maximize their profits, and consumers, whose utility-maximizing behaviour is depicted in much paler shades. Besides, Dupuit acknowledged that the well-being of society as a whole depends on producers' behavior: if they are able to guess consumers' needs and wishes so as to fix different prices for different degrees of their willingness to pay, everybody will gain. Thus, although Dupuit's contribution does reduce the gap between producers and consumers in the economic thinking of his time, it does not entirely eliminate it: there is still a significant degree of asymmetry between the two sides of the market. Beside the technical obstacles to the development of utility maximization calculations, one reason for this is, presumably, that the general vision underlying most economic theories of the time was still one that put greater emphasis on the production / supply rather on than the consumption / demand side of the market, as in classical economic thought. In this sense, there is much to gain by interpreting Dupuit's contribution not only in the light of later developments in neoclassical microeconomics, but also in relation to the thinking that prevailed in early 19th century economics.

3. Dupuit's line of reasoning in the appendix: mathematics

Although Dupuit mainly used numerical examples in his reasoning, he attached a note to the 1844 article, in which he presented some of the general principles of his theory, and some of their most significant implications, in mathematical form. Let us now have a closer look at this attempt, in order to compare Dupuit's mathematical arguments with the numerical examples used elsewhere in his writings, in order to assess their usefulness and relevance.

3.1 Fig. 1: a novel formulation of Dupuit's basic principle

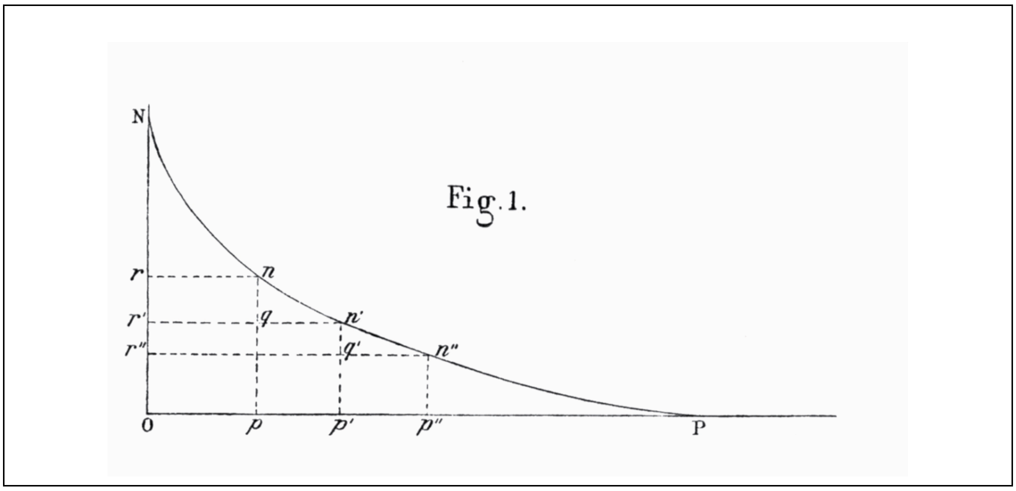
In fig. 1, the lengths *Op, Op', Op''*, represent various prices for a good, and the verticals *Or, Or', Or''*, indicate the number of articles consumed at these prices. Let *ON* represent the quantity consumed when the price is zero, and *OP* the price at which consumption falls to zero. By relating prices and quantities, it is possible to construct the curve *Nnn'n''P* –i.e. aggregate demand, or the "curve of consumption", as Dupuit calls it in the appendix. It appears as a real function defined on the positive half-line, monotonically decreasing, convex, continuous, and probably differentiable (it may be useful to recall that at the time, the existence of continuous but non-differentiable functions was not known). It follows that the first derivative must be negative and increasing, and second derivative positive.

Two of these properties are explicitly recognized by the author in the first, non-mathematical part of the article. The curve is monotonically decreasing because consumption expands when price falls, and convex because the increase in consumption due to a price fall will be the greater, the lower the initial price (see par. 2.1). These two properties also characterize the laws of consumption Dupuit used elsewhere in his articles, represented by two columns of figures showing the number of articles consumed corresponding to each market price from zero (at which consumption is largest) up to the price which causes all consumption to cease. By contrast, continuity and differentiability do not apply to the numerical examples previously used, which refer to changes in demand due to discrete changes in prices. Does the introduction of these two additional assumptions change anything, with respect to Dupuit's arithmetical arguments?

To answer this question, let us refer to what Dupuit himself suggested in a footnote in the first part of his article:

> "for convenience of exposition we have used calculated differences instead of using the differential calculus. Those who are familiar with the elements of the calculus will see later how precision may be substituted for approximation" (Dupuit 1844, p. 95).

Indeed, most of the calculations performed in the first part in order to measure the changes in the quantity purchased due to a series of changes in price, are based on discrete price variations, arbitrarily chosen by the observer. Consequently, they are necessarily approximate. Still, improvements are possible, by evaluating changes in demand due to smaller and smaller price changes. In this respect, the introduction of a continuous relationship between prices and quantities in the appendix can be seen as an attempt to make Dupuit's method more accurate, by taking into account small differences in price levels and their effects on consumption.



Dupuit uses this relationship to give visual form to his concepts of relative and absolute utility. Since *pn* represents the number of articles consumed at price *Op*, the area of the rectangle *Ornp* expresses the costs of production of these articles. The "utility" of each of the *pn* articles is at least *Op* and for almost all of them it is greater than *Op*. Indeed, by raising a perpendicular from *p'* it can be seen that for each of *p'n'* articles the utility is at least *Op'*, since they are bought at that price. Of the *pn* articles there are therefore only *qn* = *pn* – *p'n'* for which utility is really *Op* (or rather, between *Op* and *Op'*), for the others it is at least *Op'*. The same argument, applied to another price level *Op''*, leads to the conclusion that the utility of the quantity *q'n'* = *p'n'* – *p''n''* is between *Op' and Op''*, while the utility of the remaining *p''n''* articles is at least equal to *Op''*. By continuing to apply the same argument an indefinite number of times, and by taking smaller and smaller price differences each time, it can be shown that the absolute utility of the *pn* articles to the consumer is the mixtilinear trapezium *OrnP*. Relative utility can be calculated by subtracting the cost of production from absolute utility: in graphical terms, it is the mixtilinear triangle *npP*.

The above remarks are sufficient to highlight some of the advantages of this method, compared to the numerical examples used elsewhere. Consider one of the earliest criticisms against Dupuit, raised by his colleague, the engineer Louis Bordas, in a 1847 comment to the 1844 article we are examining here[3]. Among other, Bordas stressed the possibility of income effects that may make Dupuit's measure inadequate. Suppose, he suggested, that the introduction of a new technique lowers the cost of production of a pair of stockings from 6 to 3 francs. A consumer who used to buy four pairs of stockings each year, for a total expenditure of 24 francs, is now able to buy eight pairs of stockings for the same amount of money. Before the technical innovation, he would have needed 48 francs to purchase eight pairs, i.e. he would have had to cut his other expenses by 24 francs. Hence in his perception, a technical change in the production of stockings is equivalent to a 24-francs increase in his annual income. If, instead of using all his extra income to purchase stockings, he decided to buy only seven pairs, and to use the remaining 3 francs to buy some other product, his relative gain measured by Dupuit's criterion would be only 21 francs –i.e. the change in price multiplied by the quantity formerly consumed, plus the new price multiplied by the additional

[3] In fact, the first three sections of Dupuit's above mentioned 1849 article are a reply to Bordas's criticism.

quantity purchased. In other words, Dupuit's method of measurement underestimates the advantage of the consumer (Bordas 1847, pp. 77 – 78).

A possible solution may consist in considering Dupuit's measure of the advantage of consumers as an approximate rather than an exact measurement procedure, which would be acceptable insofar as the price variation is so small that the resulting change in the individual's income is negligible. Most of the numerical examples that Dupuit used in the main part of his article are based on discrete (hence, relatively large) price variations, and thus hardly permit to solve the problem; conversely, the use of real instead of integer numbers and the assumption of continuity introduced in the appendix make the study of such small changes possible. In this sense, the mathematical presentation of the note turns out to be superior to the simple arithmetic initially used by Dupuit. The author himself, however, does not seem to be aware of this, in that he does not rely on this point in his 1849 reply to Bordas's critique.

3.2 Fig 2: the effects of a change in methods of production

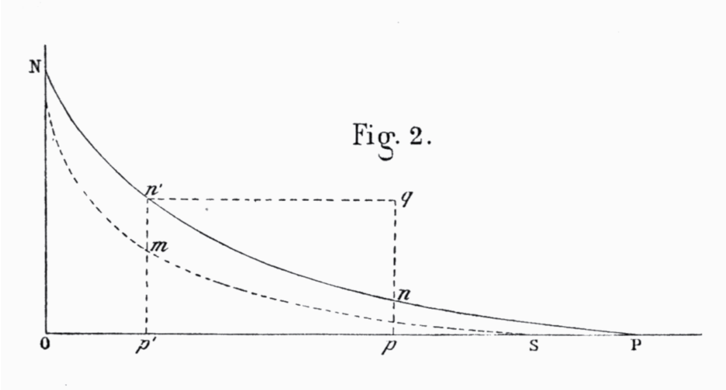
Dupuit used the general principles outlined above to illustrate some of the most important implications of his theory. To begin with, he considered the effects of an improvement in the production technique, which reduces the cost of production from *Op* to *Op'* (fig. 2). The increase in consumers' utility is represented by the surface of the mixtilinear trapezium *n'p'pn*. Navier's mistake, mentioned earlier (par. 1.1), was to overestimate the increase in utility, by taking, instead, the rectangle *n'p'pq*, i.e. the product of the price difference and the quantity purchased at the new price.

In most cases, a technical change not only reduces costs, but also brings about a modification in the quality of the product. Before giving details on how Dupuit dealt with this topic in the appendix, with the help of fig. 2, it is useful to briefly recall how he treated it in the first, non-mathematical part of the article, so as to be able later to compare the strengths and the weaknesses of the two methods. The non-mathematical argument is as follows: qualities have a value which must be taken into account in the calculation of utility. Suppose, as Navier did, that transporting goods by a newly built canal costs 0.87 francs less than by the old road. The advantage of the canal cannot be valued at 0.87, even if the transported goods come by canal in the same quantities in which they formerly came by the road, because

> "carriage by road being quicker, more reliable and less subject to loss or damage, it possesses advantages to which business men often attach a considerable value. However, it may well be that the saving of 0.87 induces the merchant to use the canal; he can buy warehouses and increase his floating capital in order to have a sufficient supply of goods on hand to protect himself against the slowness and irregularity of the

canal, and if all told the saving of 0.87 francs in transport gives him an advantage of a few centimes, he will decide in favour of the new route. But the advantage of the new route to him will only be precisely these few centimes, and if a toll of the same amount is established on the canal, then goods will no longer be moved by this route" (Dupuit 1844, p. 100).

Let us now look at the way Dupuit presented these concepts in his mathematical note. With a change in the quality of the product, the curve of consumption would move, to the right in case of an improvement in quality, and to the left in the case of deterioration. Ekelund and Hébert draw the conclusion that quality is a parameter that remains constant along a consumption curve (Ekelund and Hébert, 1999, p. 139). Fig. 2 illustrates the effects of a change for the worse in quality, moving the curve from *NP* to *NS*: in this particular case, the change in utility is represented by the difference between the two triangles *mp'S* and *npP*.



Are the two methods equivalent, or is one of them better than the other? Notice, first, that in his numerical example, Dupuit took into account a rather simple state of affairs, in which a technical change brings about both a fall in price and a deterioration in quality, while the quantity does not change. The lower quality of the product is equivalent to an increase in costs, which offsets to some extent the favorable effects of the initial price decrease –so that in this particular case, the advantage diminishes from 0.87 francs to "a few centimes".

Fig. 2 illustrates a more complex situation, in which the fall in price and in quality are accompanied by an increase in quantity. It becomes more difficult in this case to compare the two different situations; however, the graphical method gives some general guidance, by providing a visual representation of consumers' satisfaction in the two different cases, which may be of help in establishing which situation is preferable. If the equations of the two curves were known, it would also be possible to calculate the areas representing utility in the two cases, by integrating under the curves of consumption over the relevant intervals. On the contrary, numerical examples would not be of great help in this case –they provide no general guidance as to whether the previous or the novel method of production is preferable, so that it would be more difficult to draw general conclusions from them. Hence it can be said that, as this type of problem is concerned, the mathematical method outlined in the appendix to Dupuit's 1844 article provides a slightly better, and certainly a more general, analytical tool.

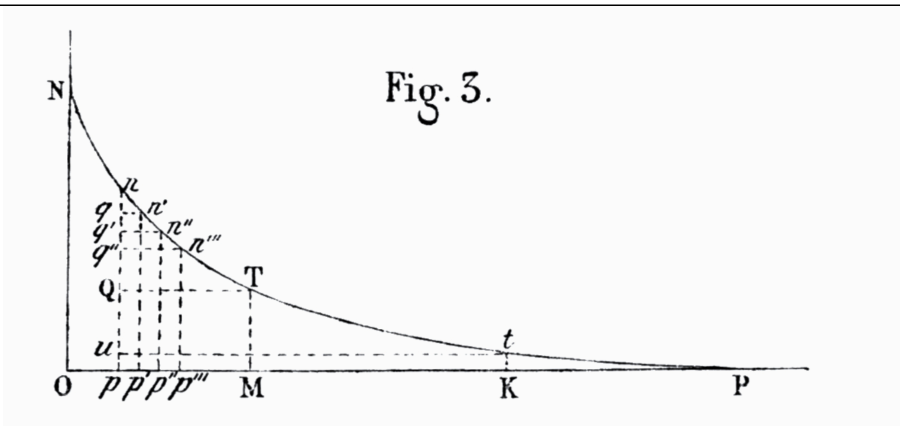
3.3 Fig. 3: how demand theory provides guidance to policy-makers

Let us now have a look at fig. 3, which illustrates the effects of the imposition of a tax on an article. His purpose is to advise the government on how to raise a given sum by means of taxation, while minimizing the loss of utility to society. Similarly to the yield of a toll (par. 2.2), the revenue the government obtains from taxes is nil for a zero tax rate, then increases as the tax rate rises, reaches a maximum, and then falls to zero again. On the other hand, the loss of utility to society increases steadily as the tax rate rises. In particular, Dupuit argued, the dead loss due to an increase in a tax increases as the square of the tax. As before, let us first briefly recall how he treated this question in the non-mathematical part of the article, so as to be able to compare the two methods afterwards. In that section, Dupuit explained this principle by taking, again, the example of stone, and supposing that, starting from a price of 15 francs, the government imposes a 5-francs tax. Since the introduction of this tax is equivalent to an increase in price from 15 to 20 francs, table 1 above shows that consumption will fall to 10,000, so that the yield of the tax will be 50,000. How to calculate the amount of utility lost? First, Dupuit calculates that, if quantity decreases in a uniform manner as price rises, that is to say 2,000 by the rise from 15 francs to 16, 2,000 by the rise from 16 to 17, etc., the average loss of utility would be 2.50 francs per ton. Since, however, each successive rise in price prevents less and less numerous consumers from buying stone, the average loss will be somewhat lower. Hence, says Dupuit, "the utility lost or gained through a change in price has for its upper limit the amount by which the quantity consumed changes, multiplied by half the change in price" (Dupuit 1844, p. 104). Specifically, if a tax of 5 francs reduces consumption from 30,000 to 10,000, the utility lost is below 20,000 x 5 x $\frac{1}{2}$ = 50,000 (in this particular case, we know from the calculations presented earlier that the loss is equal to 46,000). The second step of Dupuit's reasoning consists in saying that "the smaller the tax the nearer does this limit approach the actual figure", and thus

"it is permissible, where a tax is small relative to the cost of manufacture, to suppose a uniform rate of decrease. Thus a tax of 1 franc on a thing worth 100 francs will cause the number of consumers to fall to an extent not markedly different from a tax of 2, 3, 4, 5 or 6 francs; for the relations between the numbers 100, 101, 102, 103, 104, 105 and 106 are little different" (Dupuit 1844, p. 104).

In this case, the utility lost as result of a tax of 1 franc will be the change in quantity multiplied by $\frac{1}{2}$ of 1; raising the tax to 2 francs will double the change in quantity, so that the utility lost will be twice this number multiplied by $\frac{1}{2}$ of 2; for 3 francs, $\frac{1}{2}$ 3 x 3. It may thus be said, concludes Dupuit, that "the loss of utility is proportional to the square of the tax" (Dupuit 1844, p. 104). It is still an approximation, though a better one.

Fig. 3 provides an alternative proof of this same proposition. Let *Op* be the initial price of an article "which is cheap and consumed in large quantities" (Dupuit 1844, p. 107), and suppose a small tax of *pp'* is imposed. The government's revenue from the tax will be *pp'n'q*, and the utility lost is equal to the triangle *nqn'*. If the tax is doubled, its yield of *pp''n''q'* is smaller than the double of the rectangle *pp'n'q*, yet the loss of utility *nq'n''* is four times the loss represented by the triangle *nqn'*, since both its base and height have been doubled. Similarly, says Dupuit, if the tax is trebled, the loss of utility increases nine-fold, and so on. Therefore "*The loss of utility increases as the square of the tax*" (Dupuit, 1844, p. 107, italics in the original). Furthermore, suppose that the level *pM* is the one for which the yield for the government is at a maximum, *pMTQ*. Beyond this level, the yield diminishes and equals that given by a much lower rate of tax, while the loss of utility increases further: "*the higher the tax, the less it yields relatively*" (Dupuit 1844, p. 107). Specifically, a tax of *pP* will bring nothing into the treasury, but will do society the greatest harm.



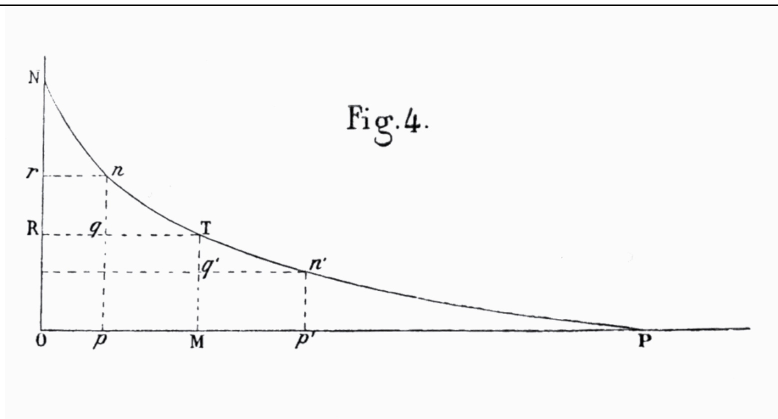
Note that Dupuit's argument concerns the elastic part of the demand curve, where the demanded quantity reacts most to changes in price. More to the point, he treats the imposition of the tax as a small increase in price, so as to be able to use a linear approximation of the demand function. His argument may be thought of as an application of Taylor's formula, which enables to replace a function in the vicinity of a given point by the straight line which is tangent to the function at that point. Under these conditions, despite the convexity of the curve, the square of the tax is a good estimate of the loss of utility due to the tax. By making it possible to take smaller variations in the tax rate (i.e. smaller than the ones that Dupuit envisaged in his numerical example), giving rise to small reductions in demanded quantity, the continuity of the demand curve makes it possible to obtain a better approximation of the total utility lost. Specifically, it is an improvement with respect to the numerical example previously used. In this sense, introducing the assumption of continuity makes Dupuit's argument on the whole more convincing than the one he used in the first part of his article, based on rougher calculations.

3.4 Fig. 4: producers' behaviour and price discrimination

This same framework may be useful to study not only the effects of taxation, but also producers' behavior –i.e. how they fix prices or tolls for a good or service they provide, say a bridge. A general way of reasoning consists in writing the curve of consumption $y = f(x)$, where the variables y and x stand for demanded quantity and price, respectively. If it is wished by means of a toll (or price) to raise a sum A representing what is needed to repay the capital advanced (as in the case of public operation, mentioned above), then "we must solve the equation $xy = A$" (Dupuit 1844, p. 107). If it is wished to raise the greatest revenue (as in the case of private operation), "we must solve the equation $\frac{dxy}{dx} = 0$" (Dupuit 1844, p. 108).

The mathematical expression of the curve of consumption and the equations that determine the toll rate according to the purpose of the provider of the service considered, permit to go deeper into the meaning and the implications of the assumption that prices and quantities are real numbers. In fig. 1-4, a price may be represented by any point between O and P, and a quantity by any point between O and N. Some of these points correspond to rational numbers, which may result from dividing the units of measure of prices or quantities into smaller and smaller parts; they may thus result from the application of Dupuit's method of assessing the changes in the quantity purchased due to a series of (small) changes in price. But other points on lines ON and OP correspond to irrational numbers, which by definition cannot be obtained in this way. They may only result from certain calculations, or from solving certain algebraic equations, such as $\frac{dxy}{dx} = 0$. They make sense in an appropriate theoretical framework, but never result from the direct measurement of a magnitude. Strictly speaking, the methodology consisting in imposing a tax on a good, increasing it gradually and measuring the reduction in demand it causes at each step, until the tax is so high that demand falls to zero, does not lead to the diagrams of the note. This conclusion would remain valid even if it were possible to estimate the changes in demand due to very small price variations. In this sense, the curves of consumption that Dupuit draws in his note represent more than a mere extension of the method of measurement he proposed in the first part of his article. Rather, they hint at a different conception of the relationship between price and the demanded quantity of a good. Instead of the direct measurement procedure that Dupuit illustrated with the device of a gradually rising fictive tax, the method he proposed in his mathematical note consists in assuming a general law that associates any price level on segment *OP* to a quantity level belonging to interval *ON,* and in indicating the main characteristics of this law, i.e. monotony, convexity, continuity, and differentiability. It is no longer a method of measurement, but an abstraction, which permits to determine the quantity corresponding to any price between *O* and *P* (and the other way round), while the former method typically takes into account integer or rational numbers only. This may be another reason why the model outlined in Dupuit's note is characterized by "precision", while the other one is a mere "approximation" (Dupuit 1844, p. 95).

Let us now consider the way Dupuit uses this method to re-prove his conclusions about the desirability of price discrimination. Suppose that the value of x derived from equation $\frac{dxy}{dx} = 0$ is *OM*, corresponding to the maximum revenue to the producer, graphically represented by rectangle *OMTR* (fig. 4). The utility of the good to consumers is the triangle *TMP*, and the loss of utility is *RTN*. If consumers can be placed in several categories each of which attributes a different utility to the same service, it is possible, by an appropriate combination of different prices, to increase the producer's revenue and to diminish the loss of utility. If from among the initial *pn* articles sold you can distinguish the consumers that would buy *pq* at price *OM*, and among the latter, those who would buy *Mq*' at *Op*', then the revenue is the sum of the three rectangles *Ornp* + *pqTM* + *Mq'n'p'*, the utility to consumers is *nqT* + *Tq'n'* + *n'p'P*, while the loss of utility shrinks to *Nrn*.



Although the use of diagrams and the continuity assumption do not lead Dupuit to new results on price discrimination, they enable him to shorten his argument, to rely on the general characteristics of the curve of consumption instead of the specific aspects of a particular numerical example, and even to make his idea clearer, by providing an image of it –thus confirming his point that "the symbols and drawings of mathematics give body and form to abstract ideas and thereby call the senses to the aid of man's intellectual power" (Dupuit 1844, p. 110).

On the whole, the mathematical note at the end of Dupuit's 1844 article does not lead to any significantly new findings, and may even be skipped on a first reading. Nonetheless, it provides a better analytical tool than the method of measurement outlined in the first part of the paper. Diagrams, functions and calculus enable to take into account any level of price and quantity in a given range of possible (real) values, and to express the essential features of the relationships between relevant magnitudes in the general form $y = f(x)$, which can then be adapted to examine each particular case. They provide a general theoretical framework for the study of economic phenomena, which enables to write equations such as $\frac{dxy}{dx} = 0$ or $xy = A$, aimed at developing principles of universal validity. Furthermore, the use of continuous and differentiable functions makes Dupuit's reasoning more precise, by taking into account smaller price and quantity variations, and in some cases, improves his proofs, notably by giving a better argument in support of Dupuit' conclusion about the loss of utility due to a tax, and by giving reasons to neglect the income effects that might have invalidated his method.

In light of the above remarks it can be said that, although Dupuit does not make the most of mathematics as a tool of investigation in his own work, since he only used it in the

1844 appendix, he seems to have grasped its potential usefulness in making economic analysis more general as well as more accurate. In this sense, the aim of his appendix seems to be precisely to hint at the opportunities for the future development of economic theory that the use of mathematics may provide:

> "in presenting, in this note, some of the principles of our science in this particular form, it was our wish to try and make clear how great would be the advantages of an alliance with mathematics, despite the anathema which economists of all times have pronounced against the latter" (Dupuit 1844, pp. 109).

4. Concluding remarks

Our attempt at reading Dupuit's 1844 article in the light of the difficulties that made the mathematical study of utility difficult at the time, has emphasized the novelty and interest of his contribution. His effort to provide an operational definition of the measure of utility, transforming it into a "quantity" that could be subject to mathematical and geometrical treatment, succeeded in extending the domain of application of mathematics. In this sense, his contribution is to be thought of as a considerable achievement, and a move ahead in the development of mathematical economics (in spite of the criticisms that have been addressed to consumer surplus arguments many years later).

On this basis, Dupuit was able to develop a fairly satisfactory analysis of market demand, useful to gain a better understanding of the market mechanism, especially of the principles determining prices. Nonetheless, we have shown that his notion of utility is still far from the neoclassical one, that it is somewhat confused with demand, and that the principle of utility maximization is underdeveloped in his writings. By contrast, the principle of profit maximization is clearly stated and well developed in Dupuit's writings –in a way that reminds, to some extent, of Cournot's *Researches*. Interestingly, despite the fact that Dupuit is essentially remembered for his seminal contribution to the study of demand and consumption, there is still a significant asymmetry between the producer and the consumer in his work. One possible reason is the strong influence of the classical school, placing emphasis on the production rather than the consumption side of the market.

Dupuit was also a pioneer in using advanced mathematics, including analytical geometry and calculus, to study utility-related questions. Only Cournot had drawn demand curves before –without, however, using them to analyse consumers' utility and satisfaction. Our examination of Dupuit's text has stressed the advantages of using advanced mathematical tools, instead of numerical examples, for discovery and proof in economics. It has been shown that the use of mathematics makes Dupuit's arguments more general, provides a broad theoretical framework capable of dealing with a number of different particular cases, improves his reasoning, and gives more accurate proofs of some of his main results. His work gives a hint of the opportunities that the application of mathematical tools may provide for the development of economic analysis, and in a sense, it prefigures the flourishing of mathematical economics a few decades later. Interestingly, Dupuit's praise of mathematics in the last paragraph of his 1844 article indicates that, at least to some extent, he was aware of the potential usefulness of mathematics as a tool of economic analysis.

Yet Dupuit used geometry and calculus only in the appendix to his 1844 article. He had used numerical examples in the first part of the article, and switched back to them in subsequent papers. Presumably, he did not change his mind as to the power of mathematics as a tool of analysis, and was still aware of the weaknesses of numerical examples, which have "the disadvantage of leaving some initial doubt about the general validity of the principles to be developed" (Dupuit 1849, p. 7). He seems to have relied on them in the hope to make himself understood in a simpler way, as well as in an effort to hint at the possible practical application of his ideas to real-life cases: indeed, he ascribed to numerical examples "the advantage of being much clearer and of showing up much better the applications of which the theory is susceptible in practice" (Dupuit 1849, p. 7). Ironically, he may have been led to this methodological choice by his very engineering background, which had provided him with the mathematical knowledge necessary to perform the calculations and to conceive the proofs outlined in the 1844 article. His duties as engineer in charge of public works may have led him to give priority to the applied part of his work, so that he eventually preferred imperfect, but practically more useful tools of analysis as numerical examples, to theoretically sounder, but apparently more difficult to communicate, mathematical investigations.

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