The agents of production are the commodities themselves
On the classical theory of production, distribution and value☆

Heinz D. Kurz *

University of Graz, Department of Economics, RESOWI-Zentrum F4, A-8010 Graz, Austria

Abstract

The paper discusses the classical theory of production, distribution and value as interpreted by Piero Sraffa in his published and unpublished work. The emphasis is on the characteristic features of the theory, including its objectivist orientation; its explanation of all property incomes in terms of the social surplus generated in production conceived of as a circular flow; and its treatment of wages either as paid out of social capital (given inventory wages) or as paid out of the surplus (a given share of wages). The latter concept is traced back to Ricardo's proportional wages.
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> The agents of production are the commodities themselves . . .. They are the food of the labourer, the tools and the machinery with which he works, and the raw materials which he works upon. (Mill, [1821] 1826, p. 165)

1. The classical approach

The analysis of the classical economists from William Petty to David Ricardo of the problems of production, distribution and value revolved around the concept of the annual *social surplus*. In the first step, this surplus was typically seen to be exclusively distributed to the propertied classes in the form of rents or profits and used for the purposes of consumption and capital accumulation. In the second step, some authors then discussed workers' participation in the surplus. The surplus refers to those quantities of the different commodities that were left over after the necessary means of production used up and the means of subsistence in the support of workers have been deducted from the gross outputs produced during a year. In this conceptualisation, the necessary real wages of labour were considered no less as indispensable inputs and thus agents of production than raw materials, tools or machines. What became known as the 'surplus interpretation' of the classical economists focuses attention on the mature classical economists' approach to how the surplus is distributed and which system of exchange values of the different commodities can be expected to emerge as the result of the gravitation of 'market' or 'actual' prices to their 'natural' or 'ordinary' levels, or 'prices of production'. In conditions of free competition, that is, the absence of significant barriers to entry and exit from all markets, prices can be taken to oscillate around levels characterized by a *uniform* rate of profits on the value of the capital advanced at the beginning of the uniform production period and a uniform rate of rent for each of the different qualities of land.

The determination of the general rate of profits, the rents of land and the corresponding system of relative prices, given the system of production in use, constitutes the analytical centrepiece of classical political economy. It was designed to lay the foundation of all other economic analysis, including the investigation of capital accumulation and technical progress; of development and growth; of social transformation and structural change; and of taxation and public debt.

Piero Sraffa deserves the credit for having rediscovered and clarified the distinct analytical structure of the classical approach to the theory of production, distribution and value. He, at the same time showed that it cannot be interpreted as an early and somewhat crude version of demand and supply analysis which became prominent with the so-called 'marginalist revolution'. Last but not least, Sraffa reformulated the classical approach by shedding the weaknesses of its earlier versions and elaborating on its strengths. Equipped with Sraffa's reconstructive and interpretive work, as it is available in his published and hitherto unpublished work, we may reconsider the classical authors' contributions. This will lead us to a deeper understanding of what they were after, and why; what they accomplished, and how; why they failed to elaborate a fully satisfactory theory revolving around the concept of social surplus, and for which reason; and finally what was needed in order to accomplish the task. In this paper, we focus on Sraffa's interpretation of the classical authors: we shall draw both on his published works (Sraffa, 1951, 1960) and on his unpublished manuscripts.[1] In a note

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[1] As it is well known, Sraffa published very little during his lifetime. What is less well known is that he left a huge amount of notes and manuscripts. Many of those which relate directly to our theme were written as early as the late 1920s. A selection from his papers and correspondence is currently being prepared for publication. Sraffa

entitled 'Principio' of November 1927 Sraffa specified the task he had taken on as follows:

> I shall begin by giving a short "estratto" of what I believe is the essence of the classical theories of value, i.e. of those which include W. Petty, Cantillon, Physiocrats, A. Smith, Ricardo and Marx. This is not the theory of any one of them, but an extract of what I think is common to them. I state it of course, not in their own words, but in modern terminology, and it will be useful when we proceed to examine their theories to understand their portata {bearing strength} from the point of view of our present inquiry. It will be a sort of "frame", a machine, into which to fit their own statements in a homogeneous pattern, so as to be able to find what is common in them and what is the difference with the later theories. (D3/12/4: 12)

The composition of the essay is the following. Section 2 deals with the scope and method of the classical theory of production, distribution and value. Section 3 turns to the central classical concept of production conceived as a circular flow and the related concept of physical real cost. Section 4 addresses briefly the tension that exists in major classical authors between concepts on the one hand and tools to deal with them on the other and how this unresolved tension made them have recourse to devices, such as the labour theory of value, that involved, to use Sraffa's word, a 'corruption' of the approach. Section 5 shows in terms of Sraffa's 'first' equations which relate to a system without surplus that relative prices of commodities can be determined exclusively on the basis of their physical real costs by means of a system of simultaneous equations. Section 6 turns to systems with a surplus and the early classical authors' attempts to ascertain the general rate of profits and some of the difficulties they encountered. Section 7 shows how these difficulties can again be mastered on the basis exclusively of the information about physical real costs in terms of what Sraffa called his 'second' equations. Section 8 addresses the problem of how the approach has to be modified in order to allow for the participation of workers in the surplus product. This leads to Ricardo's concept of proportional wages, or wages conceived as a share, and to attempts to determine the rate of profits independently of prices. It is argued that with his concepts of constant capital and organic composition of capital Marx anticipated, albeit imperfectly, two important propositions of production theory. First, in a strictly circular flow framework the maximum rate of profits is finite. Secondly, the actual rate of profits depends on two magnitudes instead of on only one: it depends not only

on the share of wages, as Ricardo had thought, but also on the maximum rate of profits, that is, the inverse of the organic composition, properly defined. Section 9 then shows that Sraffa with his construction of the Standard system and Standard commodity succeeded in demonstrating the correctness of the second proposition. The concluding Section 10 then argues that the classical approach can also deal with the empirically important problems of joint production, fixed capital, jointly utilized machines, capital utilization, and renewable and exhaustible natural resources.

2. Scope and method

The concern of the classical economists was the laws governing the emerging capitalist economy, characterized by the stratification of society into three classes: workers, land owners, and the rising class of capitalists; wage labour as the dominant form of the appropriation of other people's capacity to work; an increasingly sophisticated division of labour within and between firms; the co-ordination of economic activity via a system of interdependent markets in which transactions were mediated through money; and significant technical, organizational and institutional change. In short, they were concerned with an economic system incessantly in motion. How to analyse such a system? The ingenious device of the classical authors to see through the complexities of the modern economy consisted in distinguishing between the 'actual' values of the relevant variables – the distributive rates and prices – and their 'normal' values. The former were taken to reflect all kinds of influences, many of an accidental or temporary nature, about which no general propositions were possible, whereas the latter were conceived of as expressing the persistent, non-accidental and non-temporary factors governing the economic system, which could be systematically studied.

The method of analysis adopted by the classical economists is known as the method of long-period positions of the economy (see Garegnani, 1984). Any such position is the system of production towards which the economy is taken to gravitate as the result of the self-seeking actions of agents, thereby putting into sharp relief the fundamental forces at work. In conditions of free competition, the resulting long-period position is characterized by a uniform rate of profits (subject perhaps to persistent inter-industry differentials reflecting different levels of risk) and uniform rates of remuneration for each particular kind of primary input. Competitive conditions were taken to engender *cost-minimizing systems of production*.

The classical economists proceeded essentially in two steps. In the first step, they isolated the kinds of factors that were seen to determine income distribution and the prices supporting that distribution in a given system of production, that is, in a given place and time. The theory of production, distribution and value was designed to identify in abstracto the dominant factors at work and to analyse their interaction. In the second step, the classical authors then turned to an investigation of the causes which over time affected systematically the factors at work from within the economy and which brought about new systems of production. This was the realm of the classical analysis of capital accumulation, technical change, economic growth and socio-economic development.

It is another characteristic feature of the classical approach to profits, rents and relative prices that these are explained exclusively in terms of magnitudes that can, in principle, be

observed, measured or calculated. This *objectivist* orientation of classical economics has received its perhaps strongest expression in a famous proclamation by William Petty, who was arguably its founding father. Keen to assume what he called the "physician's" outlook, Petty in his *Political Arithmetick*, published in 1690, stressed:

> I have taken the course (as a Specimen of the Political Arithmetick I have long aimed at) to express my self in Terms of *Number, Weight* or *Measure*; to use only Arguments of Sense, and to consider only such Causes, as have visible foundations in Nature; leaving those that depend upon the mutable Minds, Opinions, Appetites and Passions of particular Men, to the Consideration of others . . .. (Petty, 1986, p. 244; emphasis in the original)

Notwithstanding their many differences, classical economists generally shared in one form or another an essentially objectivist outlook on the problem of value and distribution. This becomes clear when we turn to the concept of cost entertained by them.

3. Circular flow and physical real costs

Sraffa singled out especially two interrelated features characterizing the classical theory of production and cost. First, the classical concept of production is essentially that of a circular flow. This idea can be traced back to William Petty and Richard Cantillon and was most effectively expressed by Franois Quesnay ([1759] 1972) in the *Tableau économique* (see Aspromourgos, 1996). The classical view that commodities are produced by means of commodities is in stark contrast with the view of production as a one-way avenue leading from the services of original factors of production to consumption goods, as it was entertained, among others, by the 'Austrian' economists. Second, the classical economists held essentially a concept of physical real cost. Their starting point can be summarized by paraphrasing a famous dictum of James Mill: Man cannot create matter, man can only decompose and recompose it, change its form and move it. Production involves destruction, and the real cost of a commodity consists in the commodities destroyed in the course of its production.

We encounter the classical view in Petty who reckoned as the costs of a commodity the means of production and the means of subsistence in support of the workers necessary in order to carry out the production. Yet, as Sraffa noted, Petty was probably not the first author to have advocated such a point of view. Traces of it can also be found in the concept of 'just price' in the canonists. After Petty the new science of political economy was taken up and further developed by the Physiocrats, who adopted essentially the same point of view. The concept of physical real cost recurs in the writings of Adam Smith, David Ricardo, James Mill, Robert Torrens, and Karl Marx, to name but some of the leading representatives of the classical approach. The physical real cost approach is clearly discernible in the concept of 'capital', which Ricardo defined as 'the food and clothing consumed by the labourer, the buildings in which he works, the implements with which his labour is assisted' (*Works*, vol. I, p. 52). The perhaps clearest expression of the physical real cost approach has been put forward by James Mill in his *Elements of Political Economy*, first published in 1821. Mill insisted that, in the last instance,

The agents of production are the commodities themselves …. They are the food of the labourer, the tools and the machinery with which he works, and the raw materials which he works upon. (Mill, [1821] 1826, p. 165, emphasis added; see also Sraffa's excerpts in D3/12/9: 106–118)

The concept of agency expressed in this passage differs markedly from that entertained by marginalist economics which rose to dominance around the turn of the nineteenth century, with the Cambridge economist Alfred Marshall as one of its main heralds. At the time the concept of 'psychic cost', reflected in such notions as marginal 'utility' or 'disutility', 'abstinence', 'waiting' or 'opportunity cost', gradually replaced the earlier classical concept of material cost. Marshall in the *Principles* tried to patch over the breach by alluding to the received concept in terms of the notion of 'real cost' of production of a commodity but then specifying its content as 'the exertions of all the different kinds of labour that are directly and indirectly involved in making it; together with the abstinences or rather the waitings required for saving the capital used in making it.' (Marshall, [1890] 1970, p. 282)

Sraffa who, in the mid 1920s, in two papers had dealt a serious blow to Marshallian partial equilibrium analysis (Sraffa, 1925, 1926) was convinced that Marshall's interpretation of the classical economists as early demand and supply theorists, with the demand side in its infancy, did not stand up to close scrutiny. At the same time it had to be explained why the classical approach had been abandoned and replaced with a theory which Sraffa considered to be inherently flawed. A main reason for the decline of classical economics, as Sraffa saw it, was that the analytical tools available to the classical authors did not live up to the concepts advocated by them. The main reason for the failure of classical economics was not so much a lack of fecundity, an inability to explain salient features of the modern economic world, but deficiencies in the form in which the theory was presented. The doctrines of the classical economists from Adam Smith to David Ricardo, while essentially sound, suffered from internal contradictions and incoherences. Let us have a brief look at what Sraffa considered to be a mismatch between tools and concepts.

4. Concepts versus tools: the classical economists and the problem of value

The classical economists were convinced that the concept of physical real cost held the key to an explanation of value: the value of a commodity was closely related to the amounts of the means of production and the means of subsistence used up, or destroyed, in producing it. As Sraffa stressed, 'the sort of "costs" which determines values is the collection of material things used up in production' (D3/12/7: 106). In the case in which there is no surplus product in the economic system to be distributed as profits, interest or rent, the value of a commodity is equal to the sum total of the values of the products used up. However, this appears to be arguing in a circle, explaining value with reference to value. So how can the values of heterogeneous commodities be ascertained? The classical economists sought to cope with the problem of the heterogeneity of commodities by invoking an 'ultimate measure of value' in which the value of each and every commodity could be expressed (see Garegnani, 2004). Petty had suggested to take as such an ultimate standard

the means of subsistence of workers, 'food'. Each and every commodity, he posited, could ultimately be reduced to a well-specified quantity of food needed directly and indirectly in the support of workers producing the commodity. In this way, all commodities could be made commensurable and their values expressed in terms of the total amounts of food requested in order to feed workers across the different stages of the processing of the commodities under consideration.

Authors such as Ricardo (and Marx) did not follow Petty in this regard. Ricardo rather started from the observation that 'labour' is needed in the production of all commodities and he thus took labour to be the sought measure. As far as value theory is concerned, to him commodities represent first and foremost different amounts of direct and indirect labour 'contained' or 'embodied' in them. This then led to the labour theory of value which in its crude version claims that the exchange ratios of commodities equal the relative amounts of labour bestowed upon them. To Sraffa this implied an aberration from Petty's sound starting point: 'A. Smith and Ricardo and Marx indeed began to corrupt the old idea of cost – from food to labour. But their notion was still near enough to be in many cases equivalent' (D3/12/4: 2). Yet, he added, small errors may grow into larger ones:

> The fatal error of Smith, Ricardo, Marx has been to regard "labour" as a quantity, to be measured in hours or in kilowatts of human energy, and thus commensurated to value . . . . All trouble seems to have been caused by *small* initial errors, which have cumulated in deductions (e.g. food of worker = quantity of labour, is *nearly* true). Petty had foreseen the possibility of being misunderstood . . . . (D3/12/11: 36; similarly D3/12/4: 4)

While quantities of food consumed and of means of production used up can be measured in their own technical units, it is far from clear what is meant by a certain 'quantity of labour' and how it could be measured. Typically, the time dimension of the work performed is considered an important aspect of the sought measure. However, as Sraffa pointed out with reference to Petty and the Physiocrats, in agriculture workers have to be paid even in periods when natural conditions prevent them from performing at all or at least from performing their normal tasks, such as in winter time (see Sraffa's respective observations in D3/12/12: 8). Hence, as far as cost is concerned, what matters are real (i.e. commodity) wages paid during a given period of time, irrespective of whether and which kind of work is performed. The difficulties besetting the concept of labour are aggravated once one takes into consideration the aspects of labour intensity and heterogeneity. These have never been dealt with in a satisfactory way by those who advocated some kind of labour cost-based theory of value.[2]

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[2] Sraffa in the late 1920 also studied contemporary contributions to economic history, ethnology and anthropology in order to see what these disciplines had to say on 'primitive' societies, characterised by a small or even negligible surplus product. Was value in such societies closely connected with labour, as economists from Smith via McCulloch to Friedrich Engels had maintained? In Büchner (1901) Sraffa read that 'labour among primitive peoples is something very ill-defined' (see D3/12/9: 50); in Eldridge (1923, pp. 21, 22 and 42) that 'in India waiting is a rule', that 'time is immaterial where price is concerned', and that 'not labor-saving but material-saving devices of modern industry have the greatest vogue in China' (see D3/12/10: 18); and he noted that Hoyt (1926) provides 'striking examples' of a 'failure to accord value to time and labour even when exchange is well developed' (D3/12/9: 42).

These and other considerations prompted Sraffa to stay aloof as much as possible from the infamous concept of labour as a quantity. Indeed, during most of the first period of his constructive work (1927–1931) we see him totally avoiding the concept. The following passage, presumably written in the late 1920s, counterposes the physical real cost approach and the labour value-based interpretation of the former. It provides a further reason for Sraffa's critical attitude towards the labour theory of value:

> The difference between the "physical real costs" and the Ricardo-Marxian theory of "labour costs" is that the first does, and the latter does not, include in them the natural resources that are used up in the course of production (such as coal, iron, exaustion {sic} of land) – [Air, water, etc. are not used up: as there is an unlimited supply, no subtraction can be made from ∞]. This {is} fundamental because it does away with "human energy" and such metaphysical things. (D3/12/42: 33)

And in another document stemming from November 1927 he expounded:

> It is the *whole* process of production that must be called "human labour", and thus causes all product and all values. Marx and Ricardo used "labour" in two different senses: the above, and that of *one* of the factors of production ("hours of labour" or "quantity of labour" has a meaning only in the latter sense). It is by confusing the two senses that they got mixed up and said that value is proportional to quantity of labour (in second sense) whereas they ought to have said that it is due to human labour (in first sense: a non measurable quantity, or rather not a quantity at all). (D3/12/11: 64)[3]

Seen in this way, 'all values are "due" to labour, or to wheat or to any other thing that enters in the production of every {one} of them' (D3/12/10: 71). Yet, even setting aside the problems of labour intensity and heterogeneity, it was not at all clear how the quantities of labour embodied in a commodity could be ascertained. Whilst this was no problem in the case of the Austrian view of production as a one-way avenue leading from original factors of production via a *finite* number of stages to the final product, when 'commodities are produced by commodities ... the idea that the process of production has a beginning and an end {must be replaced} with that that it is a circular one – an idea first introduced by the Tableau économique' (D3/12/7: 2). Hence the idea that labour values could be somehow known and taken as the basis of a theory of normal prices was far from clear.

In Sraffa's view the source of much of the trouble in which the classical approach got entangled was that its advocates lacked the tool which would have effectively allowed them to tackle the problems they were faced with: the tool of *simultaneous equations* and the

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[3] In this context, it is perhaps interesting to note that in his copy of the French edition of Marx's *Theorien über den Mehrwert* – the eight volumes of the *Histoire des doctrines économiques* (Marx, 1924–1925) – which he read in the summer of 1927 Sraffa noted carefully all passages in which Marx distanced himself explicitly from an approach that proceeds exclusively in terms of commodities or 'use values'. Right at the beginning of the *Histoire*, in volume I, Marx took issue with Petty who had singled out food, not labour, as the measure of value. In the margin Sraffa placed a wrinkled line along the passage in which Marx contended that any such physical input 'n'est pas la mesure immanente des valeurs' (Marx, 1924–1925, vol. I, p. 3, fn). And then again, on the fly-leaf at the end of volume VI, we find in Sraffa's own index the entry 'Marx against physical costs 122' (Marx, 1924–1925, vol. VI).

mathematics to solve them. As Sraffa stressed in a document written around the turn of 1927, 'the fundamental force is physical real cost' which, however, is 'seen only in general equilibrium' (D3/12/42: 46), where by general equilibrium here he meant the solution of a system of simultaneous equations (see below Section 5).[4] Lacking this tool the earlier economists tried to tackle the problem in roundabout ways, the most prominent being the labour theory of value. However, as we have just seen, the route via labour values was not really a way out of the impasse in which they found themselves: it rather landed them right in that impasse again. Commodities were produced by commodities and there was no way to circumnavigate the simultaneous equations approach.[5]

5. 'First' equations: production for subsistence

What made it so difficult, if not impossible, for the classical authors to see that the theory of value and distribution could be firmly grounded in the concept of physical real cost? Given their primitive tools of analysis, they did not see that the information about the system of production in use, described in terms of the amounts of means of production used up and the quantities of the means of subsistence in support of workers, was all that was needed in order to determine *directly* the rate of profits and relative prices. Solving a set of simultaneous equations of production accomplishes the task in a straightforward manner. This was clarified by Sraffa in the winter of 1927–1928 by means of what he called his 'first' and 'second' equations, relating respectively to systems without and with a surplus and given real (or as Sraffa occasionally wrote: 'inventory') wages. To illustrate the two cases we may for simplicity start from James Mill's above broad classification of commodities in three groups: tools (*t*), raw materials (*m*), and the food of the labourer (*f*). In this section, we deal with the no-surplus economy, or production for subsistence; the following two sections then address the with-surplus case. We provide only a summary account of Sraffa's argument and do not enter into the details and precise chronology of his reasoning.[6]

In the case in which the economy is just able to reproduce what has been used up in the course of production, neither more nor less, that is, the economy is in what Sraffa called a 'self-replacing state', production in the three industries may be tabulated in the following

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[4] It had not escaped Sraffa's attention that Vilfredo Pareto (and, following him, also Francis Y. Edgeworth) had criticised earlier authors for treating as givens what had to be considered as unknowns in the theory of value: Pareto's focus were especially the wage fund theory, the labour theory of value, cost of production theories and the Austrian theory. Sraffa had carefully studied several of Pareto's contributions at an early time which is reflected in many annotations in copies of these works in his library and in several references to Pareto in his early papers; see, in particular, Pareto (1901, 1902a,b, 1906).

[5] As Sraffa noticed in the second period of his constructive work, which for the main part extended roughly from 1942 to 1946, in special circumstances the quantity of labour embodied can be seen at a glance. This is the case in Marx's scheme of simple reproduction where the total quantity of labour employed in both departments – department I producing means of production and department II means of consumption – is equal to the labour value of the net product consisting only of consumption goods. Marx's scheme can be said to foreshadow the concept of 'sub-system' or vertically integrated sector. See Sraffa (1960, Appendix A) and Pasinetti (1973).

[6] See, therefore, Garegnani (2004) and Kurz and Salvadori (2005).

way:

$$T_t \oplus M_t \oplus F_t \to T$$
$$T_m \oplus M_m \oplus F_m \to M \qquad\qquad\qquad (Q)$$
$$T_f \oplus M_f \oplus F_m \to F,$$

where T_i, M_i and F_i designate the inputs of the three commodities (employed as means of production and means of subsistence) in industry i ($i = t, m, f$), and T, M and F total outputs in the three industries; the symbol '$\oplus$' indicates that all inputs on the left hand side of '$\to$', representing production, are required to generate the output on its right hand side.[7] Adopting the terminology of the classical authors, Sraffa called these relations 'the methods of production and productive consumption' (Sraffa, 1960, p. 3). In the hypothetical case in which the economy is just viable, that is, able to reproduce itself without any surplus (or deficiency), we have $T = \sum_i T_i$, $M = \sum_i M_i$, and $F = \sum_i F_i$.

From this schema, we may directly derive the corresponding system of values which Sraffa, with explicit reference to Adam Smith and Ricardo, dubbed 'natural' or 'absolute' values. These are taken to express the idea of physical real cost-based values in an unadulterated way. Denoting the value of one unit of commodity i by p_i ($i = t, m, f$), we have

$$T_t p_t + M_t p_m + F_t p_f = T p_t$$

$$T_m p_t + M_m p_m + F_m p_f = M p_m$$

$$T_f p_t + M_f p_m + F_f p_f = F p_f$$

Only two of the three equations are independent of one another and allow one to determine relative prices. Fixing a standard of value whose price is ex definitione equal to unity, provides an additional equation without adding a further unknown. We may then solve the equations for the remaining dependent variables.

A numerical example taken from Sraffa's papers illustrates the important finding that the given socio-technical relations rigidly fix relative values:

$$2p_t + 15p_m + 20p_f = 17p_t, \quad p_t = 3p_m$$
$$5p_t + 7p_m + 4p_f = 28p_m, \quad p_m = \tfrac{2}{3}p_f$$
$$10p_t + 6p_m + 11p_f = 35p_f, \quad p_f = \tfrac{1}{2}p_t$$

Hence values emerge as the solution of a system of simultaneous equations. These values depend exclusively on necessities of production. They are the only ones that allow to restore the initial distribution of resources and thus guarantee the unhampered continuation of the process.

Once the problem is approached from a rigorous physical real cost point of view, the question of a 'common measure' of commodities loses much of the appeal it had in the

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[7] The notation above is not the one Sraffa employed in the late 1920s. At the time he used '+' instead of '$\oplus$' and '=' instead of '$\to$'. While he kept the plus sign throughout his work and also in his 1960 book, he replaced the equality sign by an arrow shortly after having resumed his constructive work in 1942 (see D3/12/23: 1).

contributions of the earlier authors. Or rather, any valuable thing – whether a single commodity or a basket of commodities – could serve as such a measure. One may also 'reduce' the value of one commodity to a certain amount of another commodity needed directly or indirectly in the production of the former. For example, one might reduce one unit of commodity *t* to an amount needed of commodity *m*. Hence one might say that each of the three commodities could serve as a common measure and that, for example, commodities *t* and *f* exchange for one another in the proportion 1:2 because commodity *t* 'contains' or 'embodies' twice as much of the third commodity *m* as commodity *f*. The scholastic idea of *tertium comparationes* is here almost driven ad absurdum, because there are infinitely many such common equivalents.

6. Production with a surplus: the classical heritage

We now turn to systems with a surplus - the real concern of the classical authors and Sraffa. The assumption of given real wages paid ante factum and thus belonging to the capital advanced at the beginning of the uniform cycle of production will however be retained throughout this and the next section. In the case of free competition, the surplus is taken to be distributed in terms of a uniform rate of profits on the 'capitals' advanced in the different industries. Before we turn to Sraffa's equations in Section 7, a few remarks on some earlier attempts to determine the general rate of profits are in place.

In the 1820 edition of his *Essay on the External Corn Trade* Torrens put forward the simplest conceptualization possible of the surplus approach to the theory of value and distribution: the *corn-ratio theory of profits*. He laid down as a 'general principle'

> that in whatever proportion the quantity of produce obtained from the soil exceeds the quantity employed in raising it, in that proportion the value of the manufactured goods will exceed the values of the food and material expended in preparing them. (Torrens, 2000, vol. II, p. 362).

Here the rate of profit in agriculture is determined as a ratio between two given quantities of corn: the surplus corn produced and the corn capital advanced in corn production (seed and corn wages). This rate of profit is then said to determine the price of manufactures which, in competitive conditions, yields the manufacturer the same rate of return on his capital advances as the rate obtained by the farmer.

Torrens expressed his indebtedness to David Ricardo's 'original and profound inquiry into the laws by which the rate of profit is determined' (2000, p. xix). This provides indirect evidence in support of Sraffa's corn-profit interpretation of Ricardo (Sraffa, 1951, pp. xxxi–xxxiii). According to Sraffa, 'The advantage of Ricardo's method of approach is that, at the cost of considerable simplification, it makes possible an understanding of how the rate of profit is determined without the need of a method for reducing to a common standard a heterogeneous collection of commodities' (1951, p. xxxii). It also provides a first confirmation of Ricardo's conviction that 'the great questions of Rent, Wages, and Profits must be explained by the proportions in which the produce is divided . . .'; and that the laws of distribution 'are not essentially connected with the doctrine of value' (*Works*,

vol. VIII, p. 194; emphasis added). The emphasis on *proportions* will become clear in the following.

It was, of course, clear to Ricardo and Torrens that, as Malthus had objected, the capital advanced in a single industry is never homogeneous with the industry's product. However, there may be homogeneity between product and capital in *aggregate* terms: the commodity composition of the surplus product may be the same (or at least roughly the same) as the commodity composition of social capital. In this case, the general rate of profits may again be conceived of (at least in a first approximation) in purely physical terms. In all three editions of Ricardo's *Principles*, we encounter a numerical example which satisfies this requirement. In the example of every 100 units produced of three commodities – hats, coats, and quarters of corn – workers are paid 25 (or 22) units of each of them and landlords are also assumed to receive 25 (or 22) units; accordingly, profits consist of 50 (56) units of each commodity (see *Works*, vol. I, p. 50). If capital consists only of the real wages bill, an assumption employed by Ricardo in much of his reasoning on profits and wages, the rate of profits can be determined independently of the problem of the valuation of the different commodities and amounts to $50/25 = 2$ (or $56/22 = 28/11$). Similarly, in his *Essay on the Production of Wealth*, published in 1821, Torrens put forward a circular flow example with two industries, one producing corn, the other suits of clothing, where both industries use both products in the same proportions (and actually in the same absolute amounts) as inputs (see Torrens, 2000, vol. Ill, pp. 372–373). With the social surplus and the social capital consisting of the same commodities in the same proportions, the general rate of profits can be determined without having recourse to the system of relative prices. Moreover, given the exceedingly simple conditions underlying the example, the exchange ratio of the two commodities corresponding to a uniform rate of profits is obvious: since both commodities exhibit the same physical real costs per unit of output, and thus the same amount of capital adavanced, a quarter of corn is necessarily exchanged for one suit of clothing.[8]

Torrens was critical of Ricardo's labour value-based theory of value and distribution and advocated instead a capital value-based theory. As regards the determination of the rate of profits, this theory applied in a straightforward manner to the example just discussed. However, physical homogeneity of product (and surplus) and capital cannot be expected to hold in any real economy other than in highly unlikely special circumstances. In his attempt to deal with more general cases he was thus of necessity confronted with the complexity of the relationship between income distribution and relative prices. In yet another attempt to contain this complexity and arrive at a clear-cut and simple determination of the general rate of profits, Torrens resorted to the special assumption we just encountered, namely,

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[8] In the debate about whether Ricardo or Torrens or any other classical author had put forward a 'corn model', this possibility has frequently, and surprisingly, been overlooked by some critics of Sraffa's interpretation. In order for a concept of the general rate of profits in purely physical terms to hold there is no need to discern in the classical authors the fiction of a single industry whose product is physically homogeneous with its capital. Corn models are, however, to be found in these authors and were quite widespread, as, for example, the case of Johann Heinrich von Thünen shows; see Kurz, 1999, pp. 154–157. Therefore, the concern with the corn model in the writings of some critics appears to be out of proportion with regard to the importance of that model in the classical authors: helpful as it may have been at an early stage in the conceptual development of the classical approach to the theory of profits, that approach can do very well without the corn model. See also De Vivo (2001).

that in all lines of production the same capital input proportions apply. Hence the capitals advanced in the different industries can easily be compared with one another. What is more, echoing the physical real cost approach in labour terms, the (circulating) *capitals* (means of production and means of subsistence) used up in the course of production can unambiguously be expressed as quantities *of accumulated labour*. In the preface to the *Essay* Torrens stressed:

> The principle that the *accumulated labour*, or, in other words, *the capital expended on production*, determines the exchangeable value of commodities, while it is derived from an extensive induction from particular cases, affords a *satisfactory solution* of some of the most important phenomena connected with the distribution of wealth. Without this correction or limitation of Mr. Ricardo's theory of value it is impossible to give a clear and unexceptionable demonstration of that gentleman's very original and valuable doctrine respecting the profits of stock. (Torrens, 2000, vol. Ill, p. vii; emphasis added)

It is clear that in the conditions specified Torrens's capital-value and Ricardo's labour-value theory amount to the same thing. It is also clear that Torrens's suggestion did not afford a generally 'satisfactory solution', as Torrens was inclined to believe. This became clear at the latest, if not earlier, in the context of the criticism of Marx's so-called 'transformation' of labour values in prices of production (see below, Section 7).

Not fully seeing their way through the complexities of the relation between relative prices and income distribution, given the system of production in use, applies in one way or another to all classical economists.[9] Ricardo, as is well known, struggled with the problem until his death: the manuscript fragments on 'Absolute Value and Exchangeable Value' (see *Works*, vol. IV) document in detail his attempts to come to grips with this problem and his failure to elaborate a fully correct theory. They also contribute to a better understanding of why Ricardo (and other classical economists) were so 'obsessed' with one version or another of the labour theory of value. This theory had allowed them to contain the complexities of the problem under consideration and determine, however provisionally and imperfectly, the general rate of profits. As long as no better theory was available, there was no compelling reason to abandon the labour value-based approach.

Sraffa was convinced that the move away from the 'loaf of bread' towards 'labour' had led into a dead-end and was partly responsible for the decline of the classical approach and its replacement by the marginalist one. This was unfortunate because the classical approach, he maintained, had been abandoned prematurely: what was deficient was the form in which it had been handed down, and not its substance.[10]

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[9] There is a notable exception: the critic of the Physiocratic doctrine, the French engeneer Achille-Nicolas Isnard; see Gilibert (1981) and Kurz and Salvadori (2000, pp. 159–161). However, Isnard's simultaneous equations approach in terms of quantities of commodities consumed productively and quantities produced can be said to have fallen flat on the ground and had hardly any impact on the development of the theory.

[10] Conversely, Sraffa was convinced that the alleged coherence of demand and supply analysis was more apparent than real and that the demand and supply schedules had no objective contents: nothing corresponded to them in the real world; see Sraffa (1925, 1926) and his notes written in summer 1927 (D3/12/3).

7. 'Second' equations: the with-surplus case

Systems with a surplus Sraffa began to analyse in terms of what he called his 'second' equations in November 1927. As early as June 1928 he was able to settle the problem of their solvability with his friend and colleague Frank Plumpton Ramsey (see D3/12/2: 28-9). Ramsey reformulated Sraffa's system of linear homogeneous equations by putting the system in its canonical form and then by setting the determinant of coefficients equal to zero in order to obtain a non-trivial solution (see Kurz and Salvadori, 2001, pp. 196–198). Here a few observations on some of the conceptual and doctrinal aspects involved must suffice.

For illustrative purposes we start again from system (Q), but now we assume that $T \geq \sum_i T_i$, $M \geq \sum_i M_i$, and $F \geq \sum_i F_i$, where at least with regard to one commodity the strict inequality sign holds. The case of a uniform rate of physical surplus across all commodities contemplated by Ricardo and Torrens,

$$\frac{T - \sum_i T_i}{\sum_i T_i} = \frac{M - \sum_i M_i}{\sum_i M_i} = \frac{F - \sum_i F_i}{\sum_i F_i} = r,$$

denotes a very special constellation: in it the general rate of profits, r, equals the uniform material rate of produce. Here we see the rate of profits in the quantities of commodities themselves, as having nothing to do with their values. In general, however, the rates of physical surplus will be different for different commodities. It cannot be even excluded that some of these rates are negative.

'Profits', Ricardo stressed, 'come out of the surplus produce' (*Works*, vol. II, pp. 130–131; similarly vol. I, p. 95). Unequal rates of commodity surplus do not, however, by themselves imply unequal rates of profit across industries. In conditions of free competition the concept of normal prices, or prices of production, implies that the social surplus is divided in such a way between the different employments of capital that a uniform rate of profits obtains. This is reflected by the following system of price equations:

$$
\begin{aligned}
(T_t p_t + M_t p_m + F_t p_f)(1 + r) &= T p_t \\
(T_m p_t + M_m p_m + F_m p_f)(1 + r) &= M p_m \\
(T_f p_t + M_f p_m + F_f p_f)(1 + r) &= F p_f
\end{aligned}
\tag{P}
$$

Flukes apart, these three equations are independent of one another and can be solved for relative prices and the rate of profits. Alternatively, fixing a standard of value provides a fourth equation and no extra unknown and allows one to solve the system for the dependent variables: the general rate of profits and prices.

The important point to note here is the following. With the real wage rate given and paid at the beginning of the periodical production cycle, the problem of the determination of the rate of profits consists in distributing the surplus product in proportion to the capital advanced in each industry. Obviously, as Sraffa expounded,

> such a proportion between two aggregates of heterogeneous goods (in other words, the rate of profits) cannot be determined before we know the prices of the goods. On the other hand, we cannot defer the allotment of the surplus till after the prices are known, for ... the prices cannot be determined before knowing the rate of profits. *The result is that the distribution of the surplus must be determined through the same*

mechanism and at the same time as are the prices of commodities. (Sraffa, 1960, p. 6; emphasis added)

This passage shows that the idea which underlies Marx's so-called 'transformation' of labour values into prices of production (see Marx, 1959, part II) cannot generally be sustained. Marx had proceeded in two steps, a procedure Ladislaus von Bortkiewicz (1906–1907, essay II, p. 38) dubbed 'successivist' as opposed to 'simultaneous'. In the first step, Marx had assumed that the general rate of profits is determined independently of, and prior to, the determination of prices as the ratio between the labour value of the social surplus and that of social capital, consisting of constant capital (means of production) and variable capital (wages). In the second step, he had then used this rate to calculate prices. Underlying his approach is the hypothesis that while the transformation of values into prices is relevant with regard to each single commodity, it is irrelevant with regard to commodity aggregates, such as the surplus product or the social capital, and the ratio of such aggregates. Yet this is a misconception.

The passage quoted also contains the key to Sraffa's critique of the long-period marginalist concept of capital. This concept crucially hinges on the possibility of defining the 'quantity of capital', whose relative scarcity and thus marginal productivity is taken to determine the rate of profits, independently of that rate. However, according to the logic of Sraffa's above argument the rate of profits and the quantity (i.e. value) of capital can only be determined simultaneously (see Kurz and Salvadori, 1995, Chapter 14).

We may now ask: is a two-step procedure which first ascertains the rate of profits independently of relative prices and only afterwards turns to the determination of the latter in principle impossible, or can a logically unassailable version of it be elaborated? In the next but one section we shall see that this is indeed possible, which confirms Ricardo's supposition that the laws of distribution 'are not essentially connected with the doctrine of value'. Sraffa proposed such a device in the context of an investigation of how the classical approach had to be modified in order to be able to cover the case in which wages, 'besides the ever-present element of subsistence, ... may include a share of the surplus product' (Sraffa, 1960, p. 9). However, we first have to prepare the ground by dealing briefly with the intricate analytical questions the participation of workers in the surplus raised and which Sraffa tackled in what he called his 'third' equations.

8. 'Third' equations: wages paid out of the product

The classical economists and Marx had already started to study the possibility of workers receiving more in the form of wages than mere sustenance (see, for example, Ricardo, *Works*, vol. I, p. 95). This raised immediately the following interrelated questions:

(i) Which wage concept was congenial to the new situation? Since the kind and quantities of commodities on which wages were spent could no longer be assumed to be given independently of and before relative prices were known, which wage concept should be put in place of the traditional one of real, or inventory, wages?

(ii) Since wages were no longer considered to be entirely paid out of the capital advanced at the beginning of the production period, could the assumption of ante factum payment

of wages be retained or should it give way to that of wages paid out of the product, that is, post factum?

(iii) Is it possible to ascertain the level of the rate of profits for given wages independently of relative prices?

(iv) How do the rate of profits and relative prices vary with hypothetical variations in the level of wages, given the system of production in use?

In this section, we focus attention on questions (i)–(iii); for a discussion of question (iv), see Kurz and Salvadori (1995, Chapter 4).

In his papers, Sraffa noted carefully the answers to these questions he encountered in the classical authors and Marx. He credited Ricardo with the proposal that a *share* concept of wages was appropriate in the new situation. More specifically, Ricardo had insisted that what could be taken as a given magnitude in the theory of value and distribution is 'the proportion of the annual labour of the country … devoted to the support of the labourers' (Ricardo, *Works*, vol. I, p. 49; see also pp. 274, 275 and 420). Ricardo's labour-based share concept was subsequently adopted by Marx in terms of a given 'rate of surplus value', S/V, that is, the ratio between the portion of the net (labour) value added that goes to capital owners, or surplus value, S, and the portion that goes to workers, or variable capital, V. While Sraffa also adopted a share concept, unlike Ricardo (and Marx) he defined wages as a proportion of the national income evaluated in terms of normal prices.

The next question was whether national income should be defined inclusive of the sum total of wages, or only inclusive of the surplus part of them, or exclusive of wages. It should come as no surprise that many of the classical (but also of the early neoclassical) authors vacillated a good deal when confronted with the alternatives. Adam Smith, for example, on the one hand insisted that the subsistence part of wages ought to be reckoned as a part and parcel of stock, i.e. capital, yet on the other hand anticipated modern national income accounting by treating total wages as revenue. Marx insisted against Smith and others that the two parts of wages – the subsistence and the surplus part – are fundamentally different in nature and ougth to be treated differently. Yet when it came to the determination of the general rate of profits and prices of production in volume III of *Capital* (Marx, 1959) he decided to continue to treat wages as a whole as variable *capital* advanced at the beginning of the period. In Ricardo we encounter essentially the same procedure. Hence, when dealing with the problem of value and distribution these authors retained an assumption, which, while perfectly sensible in the case of subsistence wages, loses much of its force in an environment in which workers receive a share of the surplus. While in the first period of his work Sraffa followed Ricardo in this regard, in late 1943, after some careful deliberation, he decided instead to treat wages henceforth as entirely paid out of the net product. As he was to stress in his book: 'We shall … refrain from tampering with the traditional wage concept and shall follow the usual practice of treating the whole of the wage as variable' (Sraffa, 1960, p. 10) – where the 'usual practice' referred to alludes, of course, to marginalist theory which considers wages as constituting a net income on a par with profits and rents. Sraffa left no doubt that this decision came at a cost, but confronted with a hard choice he opted in favour of what he deemed to be the less bad alternative.

As regards the third question, Sraffa showed that in basically all classical authors we encounter the view that for a given system of production there is an inverse relationship

between the *rate* of profits and (real or proportional) wages. However, this view gets every so often blurred because of analytical inconsistencies and contradictions in these authors. It was Ricardo who first firmly established what became known as his fundamental theorem concerning income distribution, namely, the constraint binding changes in the rate of profits and *proportional* wages: 'The greater the portion of the result of labour that is given to the labourer, the smaller must be the rate of profits, and vice versa' (*Works*, vol. VIII, p. 194). He was, thus, able to dispel the idea, generated by Adam Smith's notion of price as a sum of wages and profits (and rents) (Smith, 1976, WN, I.vi), that the wage and the rate of profits are determined independently of each other. Ricardo even contended that his above proposition was true irrespective of whether the technical conditions of production remained the same or changed. This was understandable, given the fact that in his respective observations he typically assumed social capital to consist only of wages (or to be entirely reducible to wages in a finite number of steps). Consequently, the rate of profits, r, was equal to the ratio of profits, P, to wages, W,

$$r = \frac{P}{W} \tag{R}$$

Obviously, Ricardo's theorem does not carry over to the case of production as a circular flow, since in this case capital can never be fully resolved in wages. However far the reduction is carried, besides the wage terms there will always be a 'commodity term' consisting of residual fractions of means of production.

According to Sraffa, it was Marx who carried the surplus approach a good deal beyond where Ricardo had left it. In particular, Marx is to be credited with establishing, however imperfectly, the inverse relationship between the rate of profits and the *share of wages* (in labour terms) within the circular flow framework of his schemes of reproduction. Since Sraffa's assessment of Marx's achievements is dealt with in some depth in another paper (see Gehrke and Kurz, 2005), a few remarks must suffice. At the beginning of the 1940s Sraffa read, or rather re-read, Marx's *Capital* and to his surprise learned what in the past he either does not seem to have seen or what had escaped his attention, namely, that Marx had been struggling partly with similar problems and had come up with similar solutions as he himself. While Marx had approved of the concept of proportional wages, he had chastised Ricardo for erroneously identifying the rate of profits with the rate of surplus value consequent upon his identification of social capital with the sum total of wages. Once this very restrictive assumption was abandoned, Marx had insisted, Ricardo's above theorem held no longer true any more: The rate of profits can fall (or rise) even if proportional wages remain constant.

Marx, Sraffa noted approvingly, had introduced two important concepts into political economy which paved the way to a better understanding of some of the properties of the modern economic system: first, constant capital, that is, that part of social capital that consists of the means of production; and, secondly, the 'organic composition of capital'. In his labour-value based reckoning the organic composition is given by C/L, that is the ratio of the labour embodied in constant capital, or 'dead labour', C, and the annual labour (of productive workers) performed in the economy during a year, or 'living labour', L (where $L = V + S$). The former concept expresses the fact that commodities are produced by means of commodities and can never be made to disappear completely. This involves that the rate

of profits is bounded from above: whereas in Ricardo vanishing wages would be reflected in a rate of profits that tends to infinity, in Marx there is a finite maximum rate of profits, Π. The maximum rate corresponds to zero wages and is equal to L/C; it is, thus, equal to the inverse of the organic composition of the system as a whole $\Pi = [(V + S)/C]$. The maximum rate of profits Marx assumed to be given and independent of the actual rate of profits and thus independent of relative prices.

We may now express Marx's labour value-based approach in the following way. The rate of profits is given by

$$r = \frac{S}{C + V} = \frac{S/L}{C/L + V/L} = \frac{1 - (V/L)}{1/\Pi + (V/L)} = \frac{\Pi(1 - \omega)}{1 + \Pi\omega} \tag{M}$$

The expression shows that the actual rate of profits depends on two magnitudes instead of on only one, as Ricardo had contended: it depends on proportional wages, ω (or the rate of surplus value, $(1 - \omega)\omega^{-1}$) and on the maximum rate of profits, Π. The above expression was meant to cover all cases, that is, both the case with a constant technical knowledge and the one with a change of it due to inventions. It describes the distributive core of the system under consideration without reference to the particular kinds of commodities produced and consumed and their respective prices. It was meant to contain the key to an investigation of the 'law of motion' of the modern economy – independent of commodity worlds passed through. Since Marx thought that expression (M) was valid even if instead of labour values prices of production would be used as measuring rods, it could be seen as providing a general confirmation of Ricardo's conviction that the laws of distribution could be established independently of the theory of prices.

The connection with the law of motion is close at hand, because it was essentially on the basis of Eq. (M) that Marx, in volume III of *Capital*, analysed the falling tendency of the rate of profits. Differentiating r partially with respect to Π, we get

$$\frac{\partial r}{\partial \Pi} = \frac{1 - \omega}{(1 + \Pi\omega)^2} > 0.$$

This expression shows why, according to Sraffa, Marx in his discussion of that tendency focused attention on the long-run development of the organic composition of capital, Π^{-1}. For, with a given (and even with a moderately falling) share of wages, the rate of profits is bound to fall as the organic composition rises (which is equivalent to a fall in the maximum rate of profits). Such a rise was in turn considered to be unavoidable if capital accumulates and there are no new technical inventions. The employment of previously invented methods of production which hitherto could not be employed, because it was not profitable to do so at the then going relative prices and income distribution, could not stop that tendency but only decelerate the speed at which it would occur.

This was, in a nutshell, Marx's major achievement as Sraffa saw it at the beginning of the 1940s. However, while Marx had succeeded in an important respect to improve upon Ricardo's contribution, he had not managed to elaborate a fully satisfactory analysis. Expression (M) cannot generally be sustained. This was clear to Sraffa long before he discovered Marx's achievements in the early 1940s. As early as toward the end of the first period of his constructive work, in the years 1929–1931, he had developed and investigated what he called his 'third' equations which were meant to apply to a with-surplus system and

given proportional wages. It was only then that Sraffa convinced himself that the concept of labour as a quantity could no longer be bypassed. Since wages, measured in some more or less abstract standard, were typically paid in relation to the number of hours or days or weeks worked, the quantity of labour employed in an industry entered his equations (see D3/12/7: 166 and 159 (1)). However, these magnitudes were only needed for the purpose of offering a basis on which to reckon wages. More precisely, the 'quantites of labour' inserted in the equations reflect not only the number of hours (or days or weeks) worked, but als given wage differentials between different kinds of labour. As Sraffa was to stress in his book: 'We suppose labour to be uniform in quality or, what amounts to the same thing, we assume any differences in quality to have been previously reduced to equivalent differences in quantity so that each unit of labour receives the same wage' (Sraffa, 1960, p. 10). Hence, there is no reason to presume that Sraffa's critical stance towards the concept of 'human energy', of which we have heard in the above, had changed by the time of his third equations. What was new was that the sharing out of the surplus product among different claimants now was an issue, which implied that the cost of production of a commodity could no longer be given exclusively in physical terms, as so many tons of steel and bushels of corn, but included also wages expressed in some more or less abstract standard.

With wages paid out of the product, this then led Sraffa to a system of equations which in our three-sectoral example read

$$(T_t p_t + M_t p_m + F_t p_f)(1 + r) + w L_t = T p_t$$
$$(T_m p_t + M_m p_m + F_m p_f)(1 + r) + w L_m = M p_m$$
$$(T_f p_t + M_f p_m + F_f p_f)(1 + r) + w L_f = F p_f$$

where the input quantities of the various commodities, T_i, M_i and F_i ($i = t$, m, f), now represent only the means of production used up, and w gives the share of wages in net income (defined inclusive of wages).[11] For a given w (or r) these equations allow one to determine r (or w) and prices. The above equations appear to confirm the impression we got already with regard to Eq. (P) that the rate of profits and prices of production can only be determined simultaneously and not successively. However, the next section shows that (i) with proportional wages and (ii) assuming a particular standpoint from which the economic system is observed by way of choice of a standard of value, the rate of profits can be determined independently of relative prices once the share of wages is given.[12]

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[11] To conceive of wages as a share, $0 \leq w \leq 1$, requires normalising total employment and the net social product as equal to unity.

[12] It goes without saying, that once the share of wages is given both the rate of profits and relative prices are fixed. Here we are only interested in the analytical possibility of ascertaining the former without having to go through the system of production equations in the above.

9. Looking at the actual economy through the lens of a construction

As early as February 1931 Sraffa, in the context of a discussion of the dependence of the prices of consumption and of capital goods on the rate of interest, had put forward the following idea:

> it may be said that the value of total capital in terms of total goods produced cannot vary {consequent upon a change in the rate of interest}, since the goods are composed exactly in the same proportions as the capitals which have produced them.

He added swiftly that this proposition is of course 'false, but may contain an element of truth' (D3/12/7: 157(3)). Some 12 years later, in a note composed in November 1943, he clarified that the proposition was based on the 'statistical compensation of large numbers' (D3/12/35: 28): whereas with regard to each and every single commodity the value of the commodity relative to the value of the capital goods employed in its production can be expected to change with a change in income distribution, these changes might perhaps be considered to largely compensate one another with regard to the corresponding aggregates. In this case, the value of social capital relative to that of social product would be the same irrespective of the level of the rate of profits (and the corresponding level of the share of wages). In the early 1940s, we encounter Sraffa's proposition again under the name 'My Hypothesis' or simply 'Hypothesis'.

Exploring the 'element of truth' mentioned, Sraffa eventually had to abandon the idea that any actual economic system in a self-replacing state could ever be expected to satisfy the hypothesis. There was no presumption that the value of the net product relative to that of social capital would remain constant irrespective of the level of the actual rate of profits. Marx's expression (M) was valid only for a particular level of this rate, that is, $r = 0$, the case in which the labour theory of value or, as Sraffa at the beginning of the 1940s preferred to say, a 'Value Theory of Labour' holds (see, for example, D3/12/44: 3 and D3/12/46: 24). Evaluated in terms of prices the ratio of the two aggregates – the net product and the means of production – would change with changes in the rate of profits. Since no actual system satisfied the Hypothesis and since prices depended on income distribution, the only remaining possibility was to construct an artificial system which mimicked the crucial property of the Hypothesis without, however, sacrificing any of the characteristic features of the actual system. Hence the construction under consideration had to be derived from the equations describing the actual system. This Sraffa accomplished in late January 1944 in terms of the devices of the 'Standard commodity' and 'Standard system' (see D3/12/36: 61 et seq.).

With wages paid post factum, workers' means of subsistence, or wage goods, no longer form a part of what Sraffa called the set of 'basic' products, that is, commodities needed directly or indirectly in the production of all commodities (Sraffa, 1960, p. 8). This set, as newly defined, formed the basis on which the Standard system could be built up by segregating from the actual system such fractions of the various basic industries 'as will together form a complete miniature system endowed with the property that the various commodities are represented among its aggregate means of production *in the same proportions* as they are among the products' (Sraffa, 1960, p. 19). In this system, the maximum rate of profits,

R, can be seen at a glance as the ratio of the net product to the means of production. Further, if the Standard net product is divided between wages and profits, where each share consist always, as the whole does, of Standard commodity, then the rate of profits in the Standard system appears as a ratio between quantities of commodities *irrespective of their prices*, and depends linearly on the share of wages, w:

$$r = R(1 - w) \qquad\qquad\qquad\qquad\qquad\text{(S)}$$

Since the actual system consists of the same basic equations as the Standard system, once the wage is given in terms of the Standard commodity, the rate of profits is determined for both systems: 'Particular proportions, such as the Standard ones, may give transparency to a system and render visible what was hidden, but they cannot alter its mathematical properties' (Sraffa, 1960, p. 23).

Setting aside different assumptions as to the payment of wages, Eq. (S) is the coherent version of Eq. (M). It confirms the view that the rate of profits depends on two magnitudes: a socio-technical datum, the maximum rate of profits, R (alias the inverse of the organic composition of capital, alias the inverse of the average period of production, for the system as a whole, each one appropriately defined), and on a distributional datum, the share of wages, w. It also provides a final confirmation of Ricardo's contention as to the relationship between the laws of distribution and the theory of value. For an analysis of the long-run, attention should focus on what happens to R and to w.

10. Joint production, fixed capital and natural resources

Up until now, the discussion was essentially limited to single-product systems of production. Given the empirical importance of joint production, fixed capital and scarce natural resources, the classical approach to the theory of production, distribution and value would be of little use and hence interest, if it were unable to address these phenomena. This was the view expressed by its critics, in particular the first generation of marginalist economists. More precisely, while the classical economists were credited with having put forward, in the theory of rent, a basically correct view of the role of land in production and distribution, they were chastised for having failed to see the possibility of developing 'a *unified general theory* to determine the prices of all productive services *in the same way*' (Walras, 1954, p. 416; emphases added).[13] This 'unified general theory', it was contended, involved generalising the principle of *scarcity*, which the classical economists had limited to natural resources, to all goods and factors of production alike.[14] In addition, it was contended that the classical (labour value-based) approach could not deal adequately with joint production

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[13] For a critical discussion of Walras's view, see Kurz and Salvadori (2002).

[14] William S. Jevons (1871, p. 198) saw his own theory as closely related to the received theory of rent: 'The general correctness of the views put forward … derives great probability from their close resemblance to the Theory of Rent, as it has been accepted by English writers for nearly a century'. It was indeed the generalisation of the principle of scarcity and of the concept of *intensive* diminishing returns from land and agriculture to all factors of production, including labour and capital, and all spheres of production which constitutes the core of the marginalist doctrine.

and fixed capital. As Jevons maintained, with joint production the number of processes of production operated will generally fall short of the number of products whose labour values (and prices) have to be ascertained. Hence there is a problem of underdeterminacy: 'it is impossible to divide up the labour and say that so much is expended on producing [commodity] *X*, and so much on [commodity] *Y*' (Jevons, 1911, p. 200). Essentially the same objection was put forward by Knut Wicksell with respect to durable capital goods which allow one to produce over a number of years. He argued that it is 'just as absurd to ask how much labour is invested in either one or the other annual use as it is to try to find out what part of pasture goes into wool and what part into mutton' (Wicksell, [1901] 1934, vol. I, p. 260).

The criticism levelled at the classical approach cannot be sustained. First, it has been shown that the principle of scarcity in the explanation of income shares in a long-period framework of the analysis cannot be generalised from land (rents) to labour (wages) and capital (profits), as Walras and other marginalist authors deemed it possible (see Kurz and Salvadori, 1995, Chapter 14, and 2002). The main difficulty consists in specifying the endowment of capital in a way that is both compatible with the long-period framework and meaningful. As was clear to authors such as Wicksell, in order to be consistent with the long-period framework the capital endowment of the economy could not be conceived as a set of given physical quantities of produced means of production, but had to be expressed as a *value* magnitude (in terms of some arbitrary numeraire). The reason for this is that if the capital endowment were to be given in kind only a short-period equilibrium, characterized by differential rates of return on the supply prices of the various capital goods, could be established. However, it is not clear what is the economic meaning of a capital endowment that is specified as a value magnitude: In which sense can such a sum of value be considered a 'factor of production' that is 'scarce' and determine – together with the other endowments, technical alternatives and preferences – the general rate of profits?

As to the objection that the classical approach cannot deal with joint production and fixed capital it suffices to point out the following. With joint production the proportions in which the products are produced need not coincide with those in which they are wanted. This raises the problem of the *disposal* of the overproduced quantities of certain products. We may distinguish between free and costly disposal. Interestingly, Adam Smith anticipated the former case when he wrote: 'The skins of the larger animals were the original materials of cloathing. Among nations of hunters and shepherds, therefore, whose food consists chiefly in the flesh of those animals, every man, by providing himself with food, provides himself with the materials of more cloathing than he can wear'. He concluded that the overproduced hides would be 'thrown away as things of no value' (Smith, WN, I.xi.c.4). Here we encounter, possibly for the first time in the history of economic thought, the 'Rule of Free Goods'. In this case, total cost of production would be imputed exclusively to the commodity that is not overproduced; and there would be no problem to ascertain its 'labour value'. In the more interesting and empirically important case of costly disposal, a production process would be combined with a disposal process, and Jevons and Wicksell's above premise, that the number of processes operated falls short of the number of products whose (labour) values (or prices) have to be ascertained, would no longer hold: two equations would simultaneously determine values. In this case the value of the overproduced product would be negative,

and the value of the other product would have to cover both total costs of production and disposal costs.

With a choice of technique, it is possible that the needs and wants of society are exactly met by a judicious combination of several joint production processes, where the number of processes operated equals the number of products produced. According to the classical approach to the problem of joint production the investigation of square systems of production has a genuine significance (see Sraffa, 1960, Chapters VII–IX). However, although there are important cases in which the cost-minimizing system of production has exactly as many processes as commodities, this is not generally true. Nonsquare systems can be analysed in terms of an appropriate transformation of the classical approach which makes it resemble the von Neumann approach to joint production, which is formulated in terms of inequalities. For details, see Kurz and Salvadori (1995, Chapter 8).[15]

The treatment of fixed capital as a joint product 'fits easily into the classical picture' and was in fact introduced by Robert Torrens and then adopted by Ricardo, Malthus and Marx (Sraffa, 1960, p. 94). Afterwards it seems to have fallen into oblivion. It re-emerged only with the works of von Neumann ([1937] 1945) and Sraffa (1960). The approach can also deal with the empirically important problems of capital utilization and the joint utilization of durable means of production or 'systems of machinery' (Marx); for a summary account of recent developments, see Kurz and Salvadori (1995, Chapters 7 and 9).

It has already been stated that classical economics is generally credited with having developed the principles of extensive and intensive diminishing returns with respect to the use of land in production. In the theory of differential rent land is commonly considered a renewable natural resource possessed of 'indestructible powers' (Ricardo, *Works*, vol. I, p. 69), that is, a nondepletable resource. However, the classical approach can also deal with renewable resources that are depletable, such as bird and fish populations or forests. A resource of this kind will actually be depleted if the levels of the periodic removal from its stock are unduly high and thus cannot be made good by the natural growth of the resource.

There have also been attempts to deal with the problem of exhaustible resources, that is, resources that are available in given stocks which can only be run down, but which do not increase (significantly) due to natural growth. These stocks are depleted each time parts of them are removed for productive or consumptive purposes. Typical examples of exhaustible resources are fossil fuels and oil. When stocks are run down, we are confronted with a dynamic problem where the price of the resource cannot generally be taken to remain constant: in order for the owner of a mine to obtain the same rate of profits as the one paid in conditions of free competition in any other business, the price of the resource must increase, where the rate of increase per period equals the ruling rate of profits. This is known as Hotelling's rule. While some of the classical economists showed an awareness of the fact that the prices of such resources would have to rise if no other developments (such as, for example, technical progress in extracting or processing the resource or the discovery of new mines) took place, which tended to thwart such a rise, they did not formulate this rule.

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[15] In this context, it deserves to be stressed that the von Neumann model (von Neumann [1937] 1945) shares crucial features of the classical approach to the theory of production, value and distribution (circular flow, physical real costs, asymmetric treatment of the distributive variables, profits based on social surplus); see Kurz and Salvadori (1995, Chapter 13).

They were rather of the opinion that exhaustible resources could be dealt with in terms of the theory of differential rent which was developed with respect to a nonexhaustible resource, land, within a framework of static prices. Here is not the place to enter into a discussion of how the classical authors and those working in their tradition attempted to cope with the intricate problem of exhaustible resources and the royalties obtained by the proprietors of stocks of them. It suffices to draw the reader's attention to a recent symposium devoted to this issue in the journal *Metroeconomica* (2001). In addition, it is perhaps worth quoting a passage taken from Piero Sraffa's hitherto unpublished manuscripts which points out some difficulties besetting dynamic economic theory. The passage reads:

> It is "a fatal mistake" of some economists that they believe that by introducing compli- cated dynamic assumptions, they get nearer to the true reality; in fact they get further removed for two reasons: a) that the system is much more statical than we believe, and its 'short periods' are very long, b) that the assumptions being too complicated it becomes impossible for the mind to grasp and dominate them - and thus it fails to realise the absurdity of the conclusions. (Sraffa Papers: D3/12/11 (33))

We may conclude by saying that the classical approach to the theory of production, dis- tribution and value has proved many of its earlier critics wrong who had contended that it was in principle unable to deal with important empirical problems such as joint production and fixed capital. More recently, the effort of those working in the classical tradition has fo- cused on problems such as exhaustible resources, costly disposal of discommodities, capital utilization, the treatment of services, and capital accumulation and economic growth. The future will show what can be accomplished in these respects using the classical approach.

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