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The Canonical Classical Model of Political Economy

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1. Adam Smith, David Ricardo, Thomas Robert Malthus, and John Stuart Mill shared in common essentially one dynamic model of equilibrium, growth, and distribution. When the limitation of land and natural resources is added to the model of Karl Marx, he also ends up with this same canonical classical model.

In its present version the model is stripped down to its minimal essentials. For brevity I employ modern mathematical tools, but only to characterize in modern terms the relations that were actually common to all these writers.

The reader should of course be warned that any simple codification of the classical economists' discursive writings must be an oversimplification: in some of their passages they qualify what they have written elsewhere; in some they provide negations and contradictions. Not a few of the stereotypes about the classical writers are, to paraphrase Voltaire, myths agreed-upon by later commentators—distortions that both improve and libel the originals. The relevant object of study for a modern scholar is the corpus of original texts and the commentaries on them, the latter not being genuinely of less interest than the former once we have succeeded in telling them apart.

To the fascinating question of whether classical political economy does, or can be made to, offer an *"alternative paradigm"*—in the sense of Thomas Kuhn [11, 1962]—to modern mainstream economics, the present investigation provides an instructive answer. So to speak, within every classical economist there is to be discerned a modern economist trying to be born. A Ricardo or Mill did not so much replace supply and demand by quite different mechanisms but rather sought to be able to say something significant and limiting about their properties, quite in the same way that we moderns endeavor to do. I describe and analyze here the basic classical model in its essential form.

2. Real output is divided interchangeably between *consumption* and *capital formation* (on a net or gross saving-investment basis). Ignoring details concerning the input-intensity differences between goods of different industries (much as can be done in a modern one-sector one–capital-good model), the classicists in effect assume that output is produced by a pro-

duction function involving land input and a *dose* of labor-*cum*-capital input. Competition among (1) landowners, (2) entrepreneurs who hire labor and needed raw materials to work with hired land, and (3) owners of labor and capital goods out to make the most favorable terms for themselves—all this leads to a determinate breakdown of competitive earnings and cost between (a) land rent and (b) the *combined return to the composite dose of labor-capital.* The breakdown of the combined return to the dose between its two components would be indeterminate as far as the demand side of the problem is concerned (at least this would be so if we stick literally to the fixed-proportions assumption usually alleged to be adhered to by the classical writers). The needed conditions come from the supply side.

3. The classical long-run theory postulates that the workers' wage rate is ultimately determined by (α) the real *subsistence* level needed to ensure reproduction and maintenance of the working population. Just as the classicists had a long-run horizontal supply curve for the subsistence wage, so they had a long-run horizontal supply curve for capital at (β) the *minimum-effective rate of accumulation,* that profit rate just low enough and just high enough to cause capital to be *maintained* with zero net algebraic saving. The long-run equilibrium number of total doses, with the implied long-run plateau of population and of capital stock, is just big enough so that the law of diminishing returns brings down the combined return of the dose to the *sum* of the needed wage-subsistence and needed minimum-profit rates. When accumulation has gone that far and population has grown in balanced proper degree, then in the absence of further technical change total land rent is *maximal.* Equilibrium prevails forever. (Mill went on to emphasize that technological innovation, continued in the long-run steady state, would imply *rising output forever;* we can show on Mill's behalf that, if the technical change is *land-augmenting* at a steady exponential rate, then labor and capital will grow forever at the same balanced exponential rate, just enough to match the growth of land measured in "efficiency units" and with the long-run wage rate and profit rate each just high enough above their respective bare minima to elicit the implied growth rates of the factors.)

4. The long-run equilibrium is stable in the sense that the system, if disturbed from it, will spontaneously return toward it. To grasp the short-run transient development of the system, suppose labor and capital goods begin in the balanced proportions needed for the technological dose, but with each at a level short of the long-run equilibrium level. Land rent then will begin below its long-run equilibrium; by the same token, the aggregate return to the composite dose will begin in excess of the long-run subsistence levels. The short-run breakdown of the dose's aggregate return among capital and labor will be determined by competitive auctioneering at that fractional breakdown just *needed to keep the two components of the dose growing at the same balanced rate* (which will be a uniquely determinate growth rate).

Thus, if population adjusts so rapidly to any surplus of real wage above subsistence that we can practically assume the truth of (what can be termed) Ricardo's "short-circuited" approximation, then the transient wage rate will be insignificantly different from the long-run subsistence level. The rate of profit is then determinate as a *residual* in the short run. And being thus determined *above* the long-run minimal profit rate, the system's saving propensities will determine the rate at which capital accumulates and population grows apace. Asymptotically, the growth of doses of capital as applied to a limited supply of land leads down the trail of diminishing returns to the rendezvous of long-run equilibrium.

Suppose we go beyond the short-circuited version and recognize that just as it takes an increment of profit rate to elicit positive saving and growth of capital, so too there must be an increment of real wage rate over the subsistence level to elicit the needed transient growth in population. Still, we shall find that there is a determinate short-term breakdown between the components of the dose's aggregate return that will be just enough to keep both labor and capital growing in the needed fixed-proportions way. The only difference in this more realistic scenario is what Smith envisioned so much more clearly than Ricardo—namely, that the real wage is higher in the transient state of progressive growth. Only in the final equilibrium when growth ceases is society in Smith's *dull* state of minimal real wages. Ricardo's predecessor and successor, Smith and Mill, are both more realistic than is Ricardo himself on wages adjustments. (By contrast, Ricardo is more realistic in 1817 [21] than Smith in 1776 [29] when it comes to recognizing that continuing new inventions will greatly delay the fall of the profit rate to its minimum and perhaps continue to do so permanently.)

Even Mill is not realistic enough in his modelling of innovation and the lagging supply of population in advanced economies. What observers like Kuznets have observed this past century is that the growth of technology has been enough *to keep the real wage growing at something like an exponential rate,* with the growth in population and saving not being fast enough to wipe out the rising trend in real wages. By contrast, the rate of profit has meandered more or less trendlessly depending on the qualitative structure of technical change, much as if population growth were more a bottleneck than were saving. It is curious that the Marxian variant of classicism, with its soft-pedalling of the limitations of land and natural resources, ought logically to have led to an even more optimistic scenario for the laws of motion of capitalistic profits than the Ricardo-Smith version.

Long-Run Equilibrium Diagrammed

5. *Figure* 1 shows the canonical classical equilibrium in the long run or steady state. The DD' relation, which looks like a modern demand relation, gives the competitive return to the composite dose of capital-*cum*-labor: the greater the number of doses competing for the same fixed supply of various qualities of land, the higher will be bid up land rents and the lower will be the dose price available to be divided between labor's wage rate and capital's profit-or-interest rate.

The DD' relation looks like a Clarkian neoclassical marginal-product curve for the variable composite doses applied to fixed land(s). But we shall be more in tune with the classicists' own mode of thinking if we delay giving DD' that admissible interpretation.[1] Perusal of the accompa-

[1] The numerical tables in the last part of Chapter II on rent of Ricardo [21, 1817] leave sketchy his notion that "successive portions of capital [doses 1, 2, 3, and 4] yielded 100, 90, 80, 70 [with total rent therefore being (100−70) + (90−70) + (80−70) + 0 = 60 and the total return to the 4 units of doses being {(100+90+80+70) − 60}/4 = 280/4 = 70]." It is clear that Ricardo believed that extra available doses would both work with lands of lower quality not previously worth cultivating and work old lands more intensively, thus altering both the *extensive* and *intensive* margins of cultivation.

Until the last half century, no one seems to have worked out rigorously the processes going on implicitly in the background, although Mountifort Longfield [12, 1834] came close to doing so as far as "reduced-form" descriptions are concerned. We shall outdo the classicists and underplay neoclassical versions of marginalism by first utilizing the following model, which is analyzed in Samuelson [24, 1959; 28, 1977].

(a) A strip of land declines continuously "eastward" in "fertility." (b) Every grade of land is cultivated

LONG-RUN EQUILIBRIUM

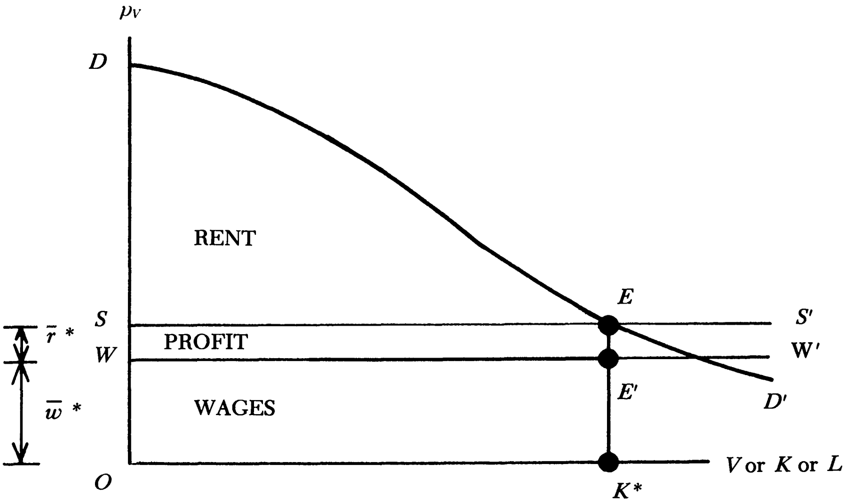

Figure 1. Long-run equilibrium is at *E*, where the *DD'* curve of return to the doses of *V*, made up of balanced proportions of *L* and *K* applied to a fixed profile of lands, just intersects the *SS'* supply curve that measures the wage subsistence and the minimum profit rate needed for steady reproduction of labor and capital. The equilibrium long-run wage rate is $\bar{w}^*$ and the long-run profit rate is $\bar{r}^*$, both exogenous parameters. Distribution of total product, *OK*ED*, is between the residual rent triangle *SED* and the dose's rectangle *OK*ES*. The breakdown between wages and profit is given by the rectangle's breakdown into *OK*E'W* and *WE'ES*. (Possibly $\bar{r}^*$ could be zero. *DD'* can be given the ultra-classical interpretation as the *average product* of the dose on the *external* margin of a continuum of lands of different grades; but, also, it could be the dose's common *marginal product* at the varying *intensive* margins of all lands used.)

by composite doses of exactly the same internal *relative* labor-capital proportions. (c) Fixed proportions of dose to land prevail at each grade's longitude, with (d) return of product per doses applied declining continuously as we move eastward. Then, when total available doses are few, all land to the east is not worth cultivating; as doses increase in availability, they all are applied ever further eastward (with no change in density of doses on prime westward land, but of course with increased *differential rent* earned on those prime acres). For each total of doses available, there is an external frontier of zero-rent land: the height of *DD'* at any given *V* is the *average product* (not the yet-to-be invented "marginal product") of output per unit of *V*, namely *Q/V* there. Footnote 7 below will show when *DD'* also can be interpreted as a true marginal product of the variable composite dose, *V*.

nying footnote shows how we might also interpret *DD'* as the curve of *average product of the varied dose* out at the moving external frontier of zero-rent grade of land.

6. The supply of labor is given by the horizontal line *WW'*, representing the subsistence cost of reproduction of labor in steady-state numbers. Followers of Smith, Ricardo, and Mill were prepared to recognize non-physiological components in the subsistence level of wages required by workers before their patterns

of marriage, procreation, migration, and labor-force participation were just adequate to keep total labor employed constant. Marx shifted the determinants of WW' away from Malthus's emphasis on biological elements of marriage, procreation, and mortality toward his own emphasis on the reserve army of the unemployed, labor-saving inventions, and inmigration to the industrial regions from the over-populated rural areas.

So long as we choose conventional units for labor, capital goods (or "leets"), and composite doses that agree in numerical magnitude, the height of WW' represents the *real wage rate* per unit of labor, $\bar{w}^*$, that must prevail when labor power's cost of reproduction is just achieved and its long-run total is in stationary equilibrium.

7. Superimposed on WW', to achieve the long-run supply response SS' for the composite dose, is the long-run profit per unit of capital goods needed if the profit rate per annum is to be at the effective rate of accumulation, $\bar{r}^*$, just enough to choke off further net saving but not so low as to cause dissaving and eating up of the previously existing stock of capital goods.

8. The distance WS, or $E'E$, represents both the rental rate of capital goods *and* the ("own") profit rate per unit time (such as .10 when 10 percent is the rate of profit to be earned on assets) when our numeraire for output is capital-goods per unit time. When differences in factor intensities between the consumption- and the capital-goods industries are ignorable, as in *Figure* 1, there is no difference between using consumption goods or capital goods as numeraire provided both kinds of goods are actually being produced—as will be the case for any stationary or growing system.

This long-run profit rate might, in some theories, be zero (after, of course, all allowances for depreciation and replacement of principal have separately been allowed for; after any needed actuarial premia for probable accidents and losses had been properly allowed for; and after any wages of managing capital assets had been provided for). If $\bar{r}^*$ is zero, SS' would coincide with WW' and E with the intersection of DD' and WW': the vertical distance between them measures the long-run *perpetual net rental* (if any) to be earned by owners of maintained capital-goods (leets) in the steady state when they are just motivated to cease net saving or dissaving.

9. The *residual of land rent* is measured on the diagram by the curvilinear triangle SED. It is what is left of total product, OK^*ED, after the composite doses are paid their needed long-run aggregate of OK^*ES. Whereas J. B. Clark and such neoclassicals as Philip H. Wicksteed, Knut Wicksell, Léon Walras, and Paul Douglas would split up the non-rent aggregate between labor and capital by a marginal-product calculation in which variability of the labor-to-leets components is brought to the optimal degree of substitution, the present classical paradigm denies such smooth substitutability *within* the composite dose and at best tolerates it between land and the composite of the fixed-components dose.

From the horizontal long-run supply curves of the components, WW' and SS', and from them alone comes the classical system's determinate *long-run* distribution theory of the non-land factor shares.[2]

[2] If $V = \text{Min}[L, K]$ were replaced by a neoclassical-first-degree-homogeneous smooth function, $v[L, K]$ with well-defined partial derivative, $\partial v[L, K]/\partial L$, the same $\bar{w}^*$ and $\bar{r}^*$ levels would prevail in the long run: but now the 2-dimensional *Figure* 1 would be inadequate to depict the determination of the resulting L^* and K^* levels: marginal productivity conditions, involving DD' and $\partial v[L, K]/\partial L$ would be the necessary and sufficient conditions to determine the extra unknowns of the rephrased problem, as in §19 below.

10. As noted in the Physiocratic version of the classical system of Samuelson [23, 1959], under long-run equilibrium all goods can be decomposed into their (marked-up) socially-necessary *land* contents: a shift in tastes and final demand from one good to another, toward more cloth and less corn, would have no effect on long-run prices. But such a shift toward less–land-intensive and more–labor-intensive goods would lower rent's share in ultimate national income; it would also raise the plateaux of population and of capital in any model where they combine in doses of the same proportions in all industries (an implausible special case). The point is obvious that any classicist who thinks he can separate "value" from "distribution" commits a logical blunder. He also blunders if he thinks that he can "get rid of land and rent as a complication for pricing" by concentrating on the external margin of no-rent land: where that external margin falls is an *endogenous* variable that shifts with tastes and demand changes so as to vitiate a hoped-for labor theory of value or a wage-*cum*-profit-rate theory of value.

11. For given technological knowledge, there is defined a unique steady-state ("factor-price") frontier relating (α) the *profit or interest rate* to (β) the *real wage rate* (expressed in terms of market basket of subsistence goods or in terms of any specified good) and (γ) the *rate of land rent* earnable by a composite unit of *all* grades of land weighted by their actual importance in the system. With the profit rate and the real wage given at their long-run supply levels, the rent rate is maximal at the long-run rendezvous of the system. (For fixed profit rate, the trade-off between real wage and rent rates can be shown to be convex, no matter how many the sectors or capital goods.)

Any reader uninterested in the rigorous analysis of this classical model may skip the next section's mathematical exposition and concentrate on the subsequent section's graphical analysis of classical growth and development.

Mathematical Version of the Canonical System

12. To define the system's behavior both in long-run equilibrium and also in transient movements toward equilibrium, here are the equations implied by this version of the classical system.

Real output, Q, is divided into real consumption, C, and net capital formation, dK/dt. It is produced at time t out of land and a composite dose of labor and capital goods ("leets"), $V_t = \text{Min}[L_t, K_t]$, where the units in the dose and in L_t and K_t are related so that one dose involves one unit of labor and one unit of capital goods. With land (possibly of various grades) fixed, we can omit the symbol for it (T, standing for a scalar, a vector, or even possibly a function of a parameter denoting a continuum of grades) in the economy's production function.

The basic production function becomes

$$Q = 1(C) + 1(dK/dt) = f(V). \quad (1)$$
$$V = \text{Min}[L, K], \quad (2)$$

where $f(V)$ is a *concave* function with $f'(V) \geq 0, f''(V) \leq 0$. [Warning: only the expositional need to compress the model into a single sector and the desire to exaggerate the differences between classical and neoclassical writers can justify so simple and strong an axiom of fixed proportions. When we relax this by quoting passages in classical writings, we need to augment the system with extra equations that help determine the extra unknowns.]

Total land rent, R, is given residually by

$$R = f(V) - Vf'(V). \quad (3)$$

The non-rent real return to the total dose, p_V, expressed in output units as numeraire, is equal to the sum of the wage component and the profit component: it is given by

$$f'(V) = p_V = 1w + 1r, \qquad (4)$$

where w is the real wage in output units, r is both the own rate of interest and the real rental rate of capital goods expressed in output units (*i.e.*, interchangeably in capital-good or consumption units), $f'(V)$ is the increment in product resulting from an extra dose of V, applied to fixed land(s); its reciprocal is the competitive marginal cost of output in terms of extra needed V requirements.[3]

13. As *Figure* 1's WW' and SS' horizontal lines indicated, w and r have to be at the well-defined $\bar{w}^*$ and $\bar{r}^*$ levels in long-run equilibrium. Thus, *Figure* 1's E is defined by the long-run equilibrium equations:

$$f'(V^*) = \bar{w}^* + \bar{r}^* = p_V^* \qquad (4^*)$$
$$R^* = f(V^*) - V^*f'(V^*) \qquad (3^*)$$
$$L^* = K^* = V^* \qquad (2^*)$$
$$Q^* = C^* + 0 = f(V^*). \qquad (1^*)$$

Any increase in the subsistence wage, $\bar{w}^*$, or in $\bar{r}^*$, must lower V^*, R^*, $K,^*$ and L^*, Q^*, and C^*. The absolute shares ($\bar{r}^*K^*$, $\bar{w}^*L^*$) can move in either direction relative to R^*: if $\bar{r}^*/\bar{w}^*$ rises, the profit/wage share rises, but how a change in $\bar{w}^*+\bar{r}^*$ affects the $(\bar{w}^* + \bar{r}^*)V^*/R^*$ ratio must depend on how changes in V affect the elasticity of the $f(V)$ curve (more precisely, on what we today call the elasticity of substitution of the $f(V)$ production function).

[3] As will be seen later, when one of the L or K inputs exceeds its needed proportions—*i.e.*, when $L/K > 1$ or $K/L > 1$—the price of the redundant's input in (4) is zero, corresponding to a free good. So (4) must be augmented under ruthless perfect competition by

$$w = 0, \quad L > K \qquad (4a)$$
$$r = 0, \quad K > L \qquad (4b)$$
$$w + r = f'(\mathrm{Min}[L, K]), \quad L \gtrless K. \qquad (4c)$$

For all their talk about the importance of the problem of distribution between land rent, labor wages, and profits, the classicists succeeded in saying little definite (and correct!) on levels of and changes in relative factor shares.

14. The dynamic laws of growth of population, $(dL/dt)/L$, and of the accumulating stock of capital, $(dK/dt)/K$, must be specified for the canonical model. When the real wage rate, w, is above the subsistence real wage rate, $\bar{w}^*$, the population grows—and grows at a greater rate the greater is the excess in wage rates:

$$\epsilon(dL/dt)/L = \lambda[w - \bar{w}^*]; \qquad (5)$$
$$\lambda[0] = 0, \quad \lambda'[\,] > 0.$$

Here ϵ is a non-negative parameter determining the slowness of the growth response of labor supply to surpluses over subsistence wages: if $\epsilon = 0$, the adjustment is instantaneous of short-run w to long-run $\bar{w}^*$ level of subsistence as L grows at whatever pace is needed to achieve $\bar{w}^*$; this is what I call the Ricardian "short-circuited" version of dynamics. If ϵ is a large positive parameter, evidently the more realistic case historically, the population grows only slowly during a high-wage era. By definition, in the long run, $(dL/dt)/L = 0$ when $w = \bar{w}^*$.

15. For Smith, Ricardo, and Mill, saving and investing never fail to be equated at full-employment conditions; only Malthus expressed doubts, envisaging in 1820 [15] the possibility of oversaving and violation of what we know loosely as "Say's Law." The rate of saving-investment is positive when r exceeds $\bar{r}^*$, the effective rate at which net accumulation ceases. Crudely, we write:

$$(dK/dt)/K = \sigma[r - \bar{r}^*]; \qquad (6)$$
$$\sigma[0] = 0, \ \sigma'[\,] > 0,$$

where $\sigma[r - \bar{r}^*]/r$ is the fraction that saving will bear to total profit incomes.

16. Our dynamic canonical classical system is almost complete.[4] If it *always* started with initial L_0/K_0 in the balanced configuration of unity and remained always in a balanced configuration, it would in fact generate determinate motions of all our variables: $L(t)$, $K(t)$, $V(t)$, $w(t)$, $r(t)$, $p_V(t)$, $C(t)$, $Q(t)$.

One such complete version is the "short-circuited" case already referred to. In it, we make the unrealistic polar Ricardian assumption that population adjusts virtually instantly, so that w falls or rises immediately to the $\bar{w}^*$ subsistence wage rate. Now (5) is replaced by:

$$w = \bar{w}^*, \; L(t) \equiv K(t) \equiv V(t) \qquad (5')$$

Our new system [(1)–(4), (5′), (6)] can now be reduced to

$$(dK/dt)/K = \sigma[f'(K) - \bar{w}^* - \bar{r}^*], \quad K(t_0) = K_0 \qquad (7.1)$$
$$Q = f(K) \qquad (7.2)$$
$$C = f(K) - dK/dt \qquad (7.3)$$
$$L = V = K \qquad (7.4)$$
$$w = \bar{w}^* \qquad (7.5)$$
$$R = f(K) - Kf'(K). \qquad (7.6)$$

The equilibrium at the K^* root of $f'(K) = \bar{w}^* + \bar{r}^*$ is globally stable: for any initial positive K_0,

$$\lim_{t\to\infty} [K(t), L(t), r(t), R(t), \ldots] = [K^*, L^*, \bar{r}^*, R^*, \ldots], \qquad (7.7)$$

where the starred long-run equilibria are precisely those of (1*)–(4*). The global stability follows from the fact that $\sigma[x]$ always has the sign of x and $-dK/dt$ therefore always the sign of $K - K^*$.

17. Not even Ricardo adhered to the short-circuited version in which the population is instantly variable so that the wage rate could be regarded as adjusting to the long-run $\bar{w}^*$ rate instantaneously. Ricardo realized that labor as well as capital would have to share in the transient surplus of the dose's return: how much of the maximum "wage fund" that could go to wages rather than to profits was never worked out in proper supply-and-demand detail by the classical writers but was left implicit by Ricardo and his contemporaries. Our supply relations (5) and (6) explicitly bridge the logical gap. (See *Figures* 3(a) and 3(b) for the diagrammatic details.) The full classical system of (1)–(6), if started out with initially balanced (K_0, L_0) sufficiently near to (K^*, L^*), will forever after grow with K/L in the needed balance and with neither factor redundantly free. Subject to such balanced conditions, the canonical system of (1)–(6) can be reduced to the determinate dynamical system:

$$(dK/dt)/K = \sigma[f'(K) - w - \bar{r}^*], \quad K_0 = L_0 = V_0 \qquad (8.1)$$
$$(dK/dt)/K = (dL/dt)/L = \sigma[f'(K) - w - \bar{r}^*] = \epsilon^{-1}\lambda[w - \bar{w}^*]. \qquad (8.2)$$

Between (8.1) and (8.2) we can eliminate w as an unknown, solving for it uniquely in terms of K:

$$w = \omega(K; \epsilon) \qquad (8.2')$$
$$\partial w(K;\epsilon)/\partial K < 0,$$
$$\omega(K^*;\epsilon) \equiv \bar{w}^*.$$

The less is ϵ, the faster w approaches final equilibrium, the sign of $\partial\omega/\partial\epsilon$ being that of $-(K - K^*)$.

Our determinate system becomes:

$$(dK/dt)/K = \sigma[f'(K) - \omega(K;\epsilon) - \bar{r}^*] \qquad (9.1)$$
$$0 \equiv \sigma[f'(K^*) - \omega(K^*;\epsilon) - \bar{r}^*]$$
$$\lim_{t\to\infty} K(t) = K^*$$

for all K_0 near enough to K^* for (8.2) to have the solution of (8.2′). The smaller is ϵ, the closer the solution of the full-fledged canonical system (1)–(6) to the short-circuited version.

[4] With the (4a) and (4b) relations of footnote 3, the dynamic system would be complete (as will be discussed later in footnote 6).

18. There remains only the task of showing that the canonical system is determinate and globally stable from any *initial conditions* of positive K and L, balanced or unbalanced.[5]

Suppose we start the system off with excess supply of one of the factors—say with more of capital goods (leets) than can be manned by labor. With $K_0 > L_0$, the short-term rentals of redundant capital goods would fall to zero under ruthless competition. At a current profit rate of zero (really negative if we recognized depreciation), there would be no profit income to save, and presumably there would be every incentive for all holders of capital assets to want to dissave at as rapid a rate as possible. Meantime labor's wage is getting all of the gross return to the composite dose, and population growth will be rapid. Therefore, very quickly, K/L will diminish toward balanced proportions without redundancy— the case already analyzed in (8) or (9).

Similarly, if we begin with redundant L/K, labor will be a free good with a zero competitive price or wage. Under laissez faire, people will die like flies; even if poor-law relief slows down the process of genocide, after an interval L/K will have dropped to balanced proportions suitable for the earlier analysis. (If one more realistically replaces perfectly fixed proportions by some variability of techniques, the r/w factor-price ratio will not gyrate so violently to zero or infinity and the more neoclassical model of §19 will better approximate reality.)

In every case, ours is a deterministic system for $(L, K, dL/dt, dK/dt)$ and the other variables.[6]

Digression on Neoclassical Elaboration of the Classical Model

19. Ricardo and Marx were not so naive observers as to believe literally in fixed proportions between capital goods and labor. Their knowledgeable commentaries on current events presuppose recognition that, at certain price and profit rates, substitutions will be made that would not be competitively viable at other price and profit rates. So it is a caricature to insist on fixed-proportion doses, $V = \text{Min}[L,K]$.

On the other hand it would be ahistorical to read into the classicists a full-fledged post-Clarkian model of neoclassical type. Nonetheless, if we wish to flesh out the torsos of their logically incomplete models, we must supply the equations missing for their additional unknowns. And, once we commit ourselves to (α) free-entry and widely-shared knowledge, (β) constant-returns-to-scale technology, and (γ) smooth variability of the (L_t, K_t) components of the V_t dose, ruthless competition will enforce the neoclassical marginal productivity relations in the canonical model whether or not the classicist is yet aware of those relations and is able to apprehend them. (Before Isaac N.'s birth, apples and the moon fell toward the earth in accordance with inverse-square-of-distance gravitational laws!)

To evaluate the question of how different the classical paradigm was from to-

[5] Strictly speaking, if initial L_0 is astronomically large, starvation and insurrection might kill off the system in one fell swoop. To do justice to this realism, we would have to make $\lambda[w - \bar{w}^*]$ minus infinity at $w = 0$ and perhaps make $f(V)$ turn down for overly large V. Along with the classical writers, I forbear from modelling scenarios of *extinction* from overpopulation. Darwin would no doubt deem this a fault.

[6] The dynamic system is most generally defined by

$$dL/dt = \epsilon^{-1}\lambda[g_1(L, K) - \bar{w}^*]$$
$$dK/dt = \sigma[g_2(L, K) - \bar{r}^*]$$

$$g_1(L, K) + g_2(L, K) = f'\ (\text{Min}[L, K]),\ K \gtreqless L$$

$$g_1(L, K) \equiv 0,\ L > K;\ g_2(L, K) \equiv 0,\ K > L.$$

For any initial (L_0, K_0), this system will approach §13's (L^*, K^*) asymptotically.

day's mainstream economics, it is worth sketching briefly the consequences of replacing $f(\text{Min}[L,K])$ by smooth constant-returns-to-scale technology. To relate the discussion more easily to classical "wage-fund" notions, I work with discrete-time variables, $K_{t+1} - K_t$ instead of dK/dt, and so forth. Writing $(T_1, T_2, \ldots) = \mathbf{T}$ for prescribed amounts of different grades of land, we have:

$$Q_{t+1} = C_{t+1} + K_{t+1} - K_t = F(L_t, K_t; T_1, T_2, \ldots), \quad (10.1)$$

where $F(\ )$ is a first-degree-homogeneous, concave function.[7] For this section, F's partial derivatives are assumed to be well-defined everywhere, in contrast to $\text{Min}[L_t, K_t]$'s assumed non-substitutability. The present section does not rule out that $F(\ )$ might have the separability property of $F(v[L,K]; T_1, T_2, \ldots) \equiv f(v[L,K])$, where $v[L,K]$ is now a smoothly substitutable first-degree-homogeneous and concave function. In this last case, $\partial F/\partial L$ and $\partial F/\partial K$ would be equivalent to $f'(v[L,K])\,\partial v[L,K]/\partial L$ and $f'(v[L,K])\partial v[L,K]/\partial K$, with $wL + rK = f'(v[L,K])\,V$ still.

We complete our system with the relations:

$$r_t = \partial F(L_t, K_t, T_1, \ldots)/\partial K_t \quad (10.2)$$

$$w_t = \frac{\partial F(L_t, K_t, T_1 \ldots)/\partial K_t}{1 + r_t} \quad (10.3)$$

$$R_{t+1} = F(L_t, K_t, T_1, \ldots) - r_t K_t - w_t L_t(1 + r_t). \quad (10.4)$$

Note that the workers who are paid *at the beginning* of the period receive only their *discounted* marginal productivity. Note that the total of profit includes (α) interest on wages advanced to the workers (in consumable output) plus (β) the interest earned on the capital-goods used in production K_t. Land gets as residual rent under competitive bidding that part of end-of-period product not preempted by competitive bidding for laborers and capital goods; if rent is payable in advance, competitive arbitrage will ensure that it too will be discounted by the $1/(1 + r_t)$ factor.

Long-run equilibrium comes when all dated variables are starred constants determined by (10.1)–(10.4), when (10.2)–(10.4) have had inserted inside them $\bar{w}^*$ and $\bar{r}^*$ exactly as in §13.

To generate the dynamic growth path of the classical system, we complete it by the supply conditions of saving and of population growth:

$$\frac{K_{t+1} - K_t}{K_t} = \sigma[r_t - \bar{r}^*], \quad (10.5)$$

$$\sigma[0] = 0 < \sigma'[\ ]$$

[7] As Ricardo and J. H. von Thünen understood, larger totals of L and K involve more intense cultivation of previously cultivated grades of good land—say of T_1, at the same time that T_2 newly comes into cultivation. In the background, the function $F(\ )$ has been defined implicitly *as if* by a maximization process. Thus

$$F(L, K; T_1, T_2, \ldots) = \underset{L_i, K}{\text{Max}}\{F_1(L_1, K_1, T_1) + F_2(L_2, K_2, T_2) + \ldots\}$$

subject to

$$L_1 + L_2 + \ldots = L,$$
$$K_1 + K_2 + \ldots = K.$$

If $F_i(\ )$ are concave in (L_i, K_i) and $\partial F_i(\ )/\partial L_i$ and $\partial F_i(\ )/\partial K_i$ are well-defined, or if, for the composite dose $V_i = \text{Min}[L_i, K_i]$, $\partial F_i(V_i, T_i)/\partial V_i$ is well-defined for $F_i(\ )$ concave in V_i, then for all lands actually in use, there will be common marginal productivities of the transferable factors, equal as the case may be to $\partial F(L,K; T_1, T_2, \ldots)/\partial L$ or $\partial F(L,K; T_1, T_2, \ldots)/\partial K$ or to $\partial F(V; T_1, T_2, \ldots)/\partial V$. Therefore, in *Figure* 1 and later figures, the DD' curve of $f'(V)$ represents not merely the *average* product at the *external margin* of continuous-grade lands (as in my footnote 1's ultra-classical interpretation); DD' alternatively represents the true *marginal* product (at the *internal* margin on every land used of whatever grade) of the variable dose there applied. Note that residual rent, $R = F(V; T_1, T_2, \ldots) - V\partial F(V; T_1, T_2, \ldots)/\partial V$, can *also* be given the post-classical interpretation as being a *marginal-product imputation to lands:* it is logically $R = [T_1 \partial F(V; T_1, T_2, \ldots)/\partial T_1] + [T_2 \partial F(V; T_1, T_2, \ldots)/\partial T_2] + \ldots$ when $F(V; T_1, T_2, \ldots)$ is smooth and obeys constant-returns-to-scale—whether or not Adam Smith had ever known anything of the work of his contemporary, Leonhard Euler!

$$\frac{L_{t+1} - L_t}{L_t} = \lambda[\mathrm{w} - \bar{w}^*], \qquad (10.6)$$

$$\lambda[0] = 0 < \lambda'[\ ]$$

$$\lim_{t\to\infty} [L_t, K_t, r_t, w_t, \ldots] = [L^*, K^*, \bar{r}^*, \bar{w}^*, \ldots]. \qquad (10.7)$$

The stability property of (10.7) holds under wide conditions.[8]

It would not be hard to include in (10.5) explicit handling of the wage-fund component in the total asset base upon which capitalists earn profits. For that matter, the capitalized value of land could, in the fashion of A. R. J. Turgot [34, 1770] and Franco Modigliani [18, 1966], be included in the asset base of life-cycle saving. But to handle these items and the public debt rigorously would be to mete out more than justice to the classical writers.

Actually, the classical economists did less than justice to their own model. To suppose that the real wage of any period must merely be the ratio of however many workers present themselves for jobs, divided into that part of C_t which capitalists have decided not to consume but instead have dedicated to the wage fund is not so much a falsehood as a triviality. There could be a period so short in which that version of the wage fund might even be formally correct. (But even this is dubious: the potatoes coming into Manchester need not go by that night into some worker's belly; they might be stored for another day or be destined for the stomach of one of Jane Austen's genteel rentiers.) For a more sympathetic appraisal of wage-fund, see George Stigler [32, 1976].

In the long-run steady state, the fraction of C^* that is adapted to wage-earner's consumption will have been *endogenously* determined. In the transient growth phase of the classical system where each month or year is not very different from its predecessor or successor, the competitive system will anticipate and forestall unpleasant surprises: so the "wage fund" will have been adapted to the viable real wage and total of employed population rather than itself constituting a *deus ex machina* to predetermine wages. John Stuart Mill had reason to dither when various of his wage-fund expositions came under attack—which is not to disagree with the attack in Frank W. Taussig [33, 1896] on the vulgar view of Henry George that

[8] For $\sigma'[\ ]$ and $\lambda'[\ ]$ sufficiently small, the difference equations of the above neoclassical model will be at least locally stable. For any number of factors, $(T, L, K, \ldots) = (x_0, x_1, x_2, \ldots)$, the following version will be stable:

$$(dx_i/dt)/x_i = s_i[p_i - \bar{p}_i^*], \quad (i = 1, \ldots, n)$$

$$p_i = \partial F(1, x_1, \ldots, x_n)/\partial x_i = \partial f(x_1, \ldots, x_n)/\partial x_i$$

$$s_i[0] = 0 \leq z s_i[z] x_i, \qquad x_i > 0$$

$$F(x_0, x_1, \ldots, x_n) = x_0 f(x_1/x_0, \ldots, x_n/x_0)$$

$f(x_1, \ldots, x_n)$ a strictly-concave function.

We may set $x_0 = 1$; and denote by $(x_1^*, \ldots, x_n^*)$ the unique roots of

$$\bar{p}_i^* = \partial f(x_1, \ldots, x_n)/\partial x_i, \quad (i = 1, \ldots, n).$$

Then, for all positive (x_i^0),

$$\lim_{t\to\infty} x_i(t) = x_i^*, \qquad (i = 1, \ldots, n).$$

To prove this theorem on global stability, consider the following maximization process of *total rent*,

$$R(x_1, \ldots, x_n) = F(\bar{x}_0, x_1, \ldots, x_n) - \sum_1^n \bar{p}_j^* x_j$$

$$\dot{R} = \sum_1^n [F_j(\bar{x}_0, x_1, \ldots, x_n) - \bar{p}_j^*] \dot{x}_j = \sum_1^n [F_j(\ ) - \bar{p}_j^*] x_j$$

$$s_j[F_j(\ ) - \bar{p}_j^*] > 0 \text{ when } x_j \neq x_j^*$$

$$\therefore \lim_{t\to\infty} R(t) \equiv \operatorname{Max} R = R^*$$

$$\lim_{t\to\infty} x_j(t) = x_j^*, \ (j = 1, \ldots, n).$$

Note that $\dot{R} = dR/dt$, etc., and that the sign-preserving property of $z_j s_j[z_i] x_j$ has been exploited in the above proof of global stability. I owe thanks to Hiroaki Nagatani of the MIT graduate school for suggesting that $d\{R^* - R(t)\}/dt$ be used as a Lyapunov function for this stability proof. When dx_i/dt is replaced in this footnote's first equation by $x_i(t+1) - x_i(t)$, it can be shown that the resulting difference equations will assuredly be locally stable provided all $s_i'[0]$ are small enough positive numbers.

SHORT-CIRCUITED CASE

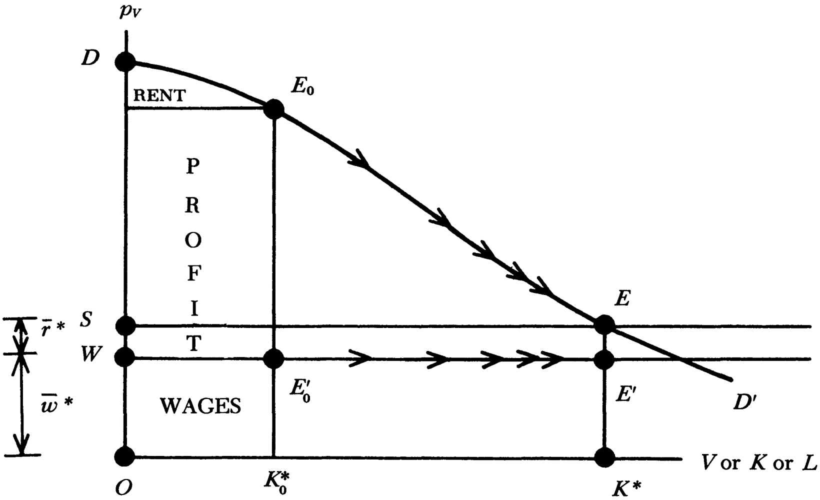

Figure 2. When population supply adjusts virtually instantly, $w(t)$ is always at $\bar{w}^*$ along WW'. With capital (and labor) initially scarce at K_0, rent begins low as shown by the small triangle at E_0: *all* the gain to the composite dose goes to profit as a short-term residual, as shown by E_0' being down on WW'. But the excess of $r(t)$ over $\bar{r}^*$ generates accumulation, as shown by the growth arrows at E_0 and E_0'. Gradually, as $E_0 \rightarrow E^*$ and $r(t) \rightarrow \bar{r}^*$, growth shuts itself off as shown by the shortening of the arrows near the long-run equilibrium E.

production of outputs by inputs is instantaneous and automatically synchronized, a view that seemed to have been surprisingly condoned by J. B. Clark [7, 1899] and Frank Knight in his many writings of the 1930's, and a view properly questioned in Eugen von Böhm-Bawerk [2, 1906; 3, 1907] and echoed a generation later by Fritz Machlup [13, 1935] and Friedrich von Hayek [9, 1936].

* * *

After this neoclassical digression, the reader may return to the canonical classical system. Whether or not he has sampled the mathematical expositions of §12–19, he should be able to follow the next section's graphical depiction of the canonical classical model's path of dynamic development.

Diagrammatics of Classical Growth Theory

20. *Figures* 2, 3(a) and 3(b) provide a self-contained derivation of how the classical system is self-propelled into development by capital accumulation and parallel population growth whenever it initially starts from scarcity of capital and labor relative to their long-run equilibrium rates when they barely earn their costs of reproduction.

For pedagogical simplicity, it is well to begin with the "short-circuited" version of virtually instantaneous population adjustment and the real wage practically always at the subsistence level $\bar{w}^*$. *Figure* 2 portrays this archetypical case, essentially embodying the equations of (7) from §16. The legend should be self-explana-

CANONICAL GROWTH

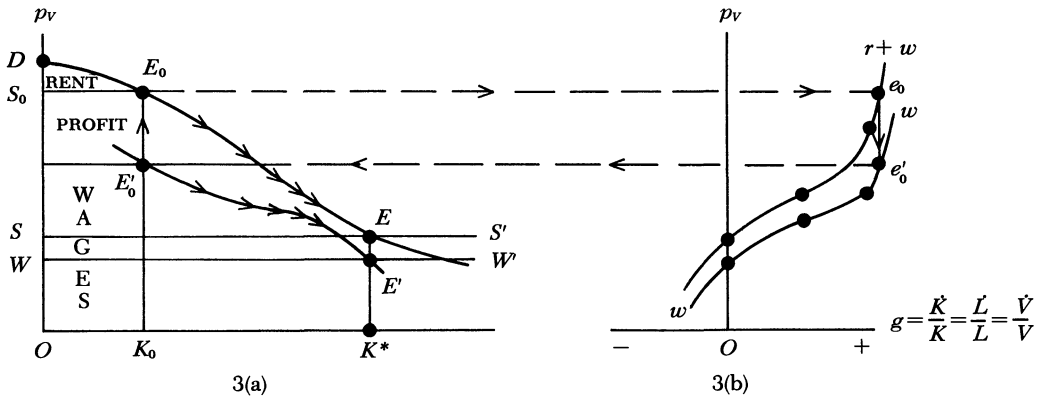

Figure 3. Labor and capital grow in concert from initial E_0. But now competition determines a fractional breakdown of the dose's surplus return between *both* components of the dose, $w(t) - \bar{w}^*$ as well as $r(t) - \bar{r}^*$, at those fractions that just succeed in evoking the same balanced growth in the supplies of the respective factors $L(t)$ and $K(t)$. The E'_0E' locus in 3(a) gives the breakdown of the fruits of transient progress between capitalists and laborers: the faster the relative supply responsiveness of population, the nearer will E'_0E' be to WW' and the greater profit's transient share; the steeper is E'_0E' northwest of E', the greater the transient share of labor in the above-subsistence surplus. (Short-run distribution of total product, of OK_0E_0D, is shown by the rent triangle S_0E_0D; the dose's remaining rectangle $OK_0E_0S_0$ is divided between profit share and wages share by E'_0 on the E'_0E' locus.)

3(b) shows exactly how the E'_0E' locus is determined. The locus *ww* shows the real wage needed to elicit each algebraic growth rate of labor in balance with the composite dose, $g = \dot{K}/K = \dot{L}/L = \dot{V}/V$ (where $\dot{L} = dL/dt$, etc.). The $r + w$ locus shows the p_V composite returns needed for the combined dose to grow, its needed wage rate plus needed profit rate, to induce balanced growth g—with $w(t)$ read from the lower curve and $r(t)$ from the interval between the curves. One begins at the computational cobweb E_0 in 3(a) then moves horizontally to e_0 in 3(b) and down to e'_0. Going back to 3(a) gives appropriate height of E'_0 in 3(a) and appropriate short-run distribution of non-rent income between profit and wages, thus filling in at each new time any logical gaps in the "wage-fund" palaver of the classicists.

tory, as the system moves the short-run equilibrium point E_0 down the path of diminishing returns on fixed land: as E_0 approaches E^*, with $w(t)$ always on the WW' level of $\bar{w}^*$ and $r(t)$ therefore falling toward $\bar{r}^*$, the motive for saving shuts itself off and the system eases itself asymptotically into long-run equilibrium with minimal profit and wage rate and maximal land rent.[9]

[9] If scarcity of land is ignored, the law of diminishing returns is negated and DD' becomes a horizontal line above WW'. This yields the perpetual exponential growth of Marx's *Tableau of Extended Reproduction*, with the growth arrows never shortening and with finite E undefinable.

21. *Figures* 3(a) and 3(b) are interrelated diagrams that handle the more general case. Eschewing the naiveties of the short-circuited pole, they portray the short-term equilibrium in which the transient shortfall of capital and labor leads to both wage and profit being above their ultimate subsistence levels in relative degrees determined by the short-run elasticities of these factors' growth responses: the auction markets for goods, lands, labor, and capital determine short-term equilibrium factor and goods prices that provide allocation between profits and wages of the composite dose's transient surplus return.

To supplement the legends of *Figures* 3(a) and 3(b), the reader will want to understand what determines the dynamic path $E_0'E'$ in 3(a), the path that summarizes how much goes to above-subsistence wages and how much to what Schumpeter would have called *transient* profits (which, he thought, would soon cease to exist in the absence of technical change and entrepreneurial innovation because $\bar{r}^* = 0$ for Schumpeter).

To test his understanding, the reader should be able to realize that Smith's cheerful rise in real wages would be enhanced if the *ww* curve were made more vertical in 3(b) and the $r + w$ curve were made virtually parallel to it.[10] By contrast, the short-circuited case will be understood as that in which *ww* is virtually horizontal while $r + w$ is not. In every case, the dashed-line cobweb $E_0e_0e_0'E_0'$ determines the position of the points on the $E_0'E'$ path of wage-profit allocation and the decelerating growth rate of the classical system as land scarcity invokes the law of diminishing returns and the relapse into long-run equilibrium.

Ricardo's readers should not have been shocked by his third edition discovery that invention of machinery could depress the real wage and lower the population and the total of product in the short and long run. Already in his earlier editions, and quite independently of the *durability* of capital goods, there was present to a truly sophisticated eye the possibility that DD' could twist *upward and inward*, the *only* limitation on the *long*-run viability of an invention being that it *raise* the *SED* rent triangle!

Final Qualifications and Extensions

22. The classicists earned for our subject Carlyle's title of the dismal science precisely because their expositions erred in overplaying the law of diminishing returns and underplaying the counterforces of technical change. They lived during the industrial revolution, but scarcely looked out from their libraries to notice the remaking of the world.

Thus, as innovation plucks the DD' string outward, it would in all likelihood lift real wages and profit rates above their minima. Before they and the string can dampen down, a new invention plucks again the string. Therefore, a Brownian dance or Schumpeterian fluctuation of real wages and profits at average levels *above* the minima would be the proper and realistic generalization of the notion of gloomy equilibrium. Indeed, if one steps up the rate of innovation enough, an upward trend in the level of E_0' and real wage may be called for as the putative laws of motion of developing capitalism, which could have made economists in

[10] *Figures* 3(a) and 3(b) in effect solve the simultaneous equations (8.1) and (8.2) for each inherited level of K and its accompanying balanced L. In the pre-balanced stage where one of L or K might be redundant, the diagrams must be reinterpreted. Thus, suppose K/L initially unbalanced above unity. Then only the *ww* curve in 3(b) is relevant: we run from E_0 over to it and short-run E_0' coincides with E_0; V_t grows with L_t and, being redundant, K_t's level is irrelevant. Once L_t rises to K_t, 3(a) and 3(b) apply as shown. To handle the case of initially redundant L_0, the reader should vertically subtract *ww* from $w + r$, labelling the result as *rr*; then, erasing *ww*, he should proceed as in the previous several sentences but with the factors being interchanged in the logically obvious way.

Carlyle's eyes the complacent scientists and the apologists for the system.

Just as one example, suppose land-augmenting technical change takes place according to Malthus's *arithmetic* progression. Then if he could analyze correctly his version of the canonical system, he would find that population comes to grow in an *arithmetic* and not in a *geometric* progression [14, 1798]. An amusing irony? Perhaps, but not a joke on Malthus: for, asymptotically, the real wage would then indeed approach his subsistence wage, $w(t) \rightarrow \bar{w}^*$. But, whatever the warrant for geometric progressions in biological reproduction, Malthus never had any plausible reasons behind his gratuitous effusions about arithmetic progressions. If the wrangler had remembered from his Cambridge education three rather than two kinds of progressions, Malthus's impact would have been weakened, but his analysis would have been less special.

23. The present model narrows the classical focus to a single sector. Thereby, one succeeds in freeing their distribution theory from the dreaded complications of value theory. Thereby, one fabricates the Ur-Ricardo model, which determines the system's profit rate from the corn sector alone: having only one sector, it is the corn sector that Ladislaus von Bortkiewicz [5, 1907], Piero Sraffa [30, 1951], Nicholas Kaldor [10, 1956], and others liked to think about; and, contrary to enemies of neoclassicism, there is nothing in the model of a corn-sector-that-determines-its-own-profit-rate which is alien to neoclassicism.

But of course many of the classical problems—as for example the actual share of wages to profits or rent—were recognized by them to depend on many-sector demands. Reducing the Corn-Law tariff on imported food shifted the mix of English production to less land-intensive goods and lowered rent's share. Ricardo knew that in 1815 and 1821: no external margin can logically save his pseudo-labor-theory-of-value from "the complications of value theory and resource scarcities." In my bicentennial appreciation of Smith [28, Samuelson, 1977], I sketched a many-commodity version of the present system, and in my classroom lectures on Ricardo, I show how a 2-primary-factor time-phased system *must* depart from the simplicities of a labor-only technology. It would be easy here to deal with many capital goods of differing durabilities.[11] But it is ludicrous to think that problems that haunt a post-neoclassical writer today—the 1966 Hahn problem of foresight to determine the warrantable allocation among microsectors and durable goods, reswitching, *etc.* [8, 1966]—were themselves absent from the century of 1750–1850 or were better handled by some lost paradigm of the capitalist writers. Under a powdered wig you find the usual head, like yours and mine, sometimes inflated and sometimes sage, but quite innocent of magic charms and skeleton keys to banish complexity.

24. Much of what has been called history of economic thought deals with questions like, "What did Ricardo mean when he said . . . ?" And "Was Smith right and Malthus wrong in alleging . . . ?" On this occasion it has not been my purpose to find and quote the pages in which Smith or Marx or Mill did or did not define an exogenous reproduction wage or profit rate, $\bar{w}^*$ and $\bar{r}^*$. Like the Bible, the canon of classical political economy contains passages that seem to assert and to deny the same thing. If, in some mood or for some problem, an ancient writer denies some axiom of what has here been called the canonical classical system, that does not

[11] Thus, we might replace K in (2) by the vector $\mathbf{K} = (K_1, K_2, \ldots)$ and dK/dt in (1) by $\Sigma c_j(dK_j/dt)$, recognizing that depreciation of each K_i occurs at the rate $\delta_i K_i$.

dispose of the problem. It raises the question of what he then intended to provide for the now-missing equation of the new system.

The canonical model is not so realistic in its features or pretty in its logic that any classicist, if he really understood it in all its interrelations and implications, would want to go into a very hot oven to defend it. As you read the letters of debate and agreement between Malthus and Ricardo, the treatises of Smith and Mill, you realize that theirs was not an age where one set out in Whitehead-Russell or even Spinoza purity the structures of their models. Their quarrels lasted because often they were quarrels over misunderstandings and definitions. (I was intrigued a few years ago when Professor Dorfman came from Harvard to my MIT seminar to report on Malthus's theoretical system: it turned out—Say's Law aside—to be isomorphic with my earlier report to the seminar on Ricardo's system, even though Dorfman and I had never compared notes! Yet Ricardo and Malthus thought they had different and irreconcilable views on microeconomics, and most commentators have judged Ricardo the victor in the debate.)

On reflection, I think that the present version of the classical system agrees in behavioral essentials with that understood by John Ramsey M'Culloch, William Nassau Senior, Samuel Bailey, Karl August Dietzel, Francis Y. Edgeworth, Edwin Cannan, Frank W. Taussig, Jacob Viner, and Piero Sraffa. I have checked my relations and behavior equations against those of Nicholas Kaldor [10, 1956], Luigi L. Pasinetti [20, 1960], Mark Blaug [1, 1978], Hans Brems [6, 1960], and Samuelson [22, 1957; 23, 1959; 24, 1959; 25, 1971; 26, 1974; 27, 1977] and believe they all tell essentially the same classical story. Left to the Appendix is a cursory sampling of the semantic quarrels of the classical writers.

Literary Appendix on Doctrinal Disputes Among Classicists

I ought to address myself, even if briefly, to the following queries. "Have you not minimized the basic differences between the classical writers in formulating for them a common canonical model? After all, didn't Ricardo set out to write his *Principles* in considerable degree because he thought Smith in error on important matters?"

The considered answer I would give is this: "Yes, Ricardo differed with Smith; and thought those differences important. But upon detailed examination, we find that their differences do not mainly involve differences in their behavior equations, short-run or long-run, but rather involve their semantic preferences about what names could be given to the same agreed-upon effects. To moderns, it is for the most part a quarrel about nothing substantive, being essentially an irrelevant argument carried out by Ricardo, often with somewhat unaesthetic logic."

I shall illustrate with no less than Ricardo's Chapter 1, Section 1 [21, 1817]: Here Ricardo wishes to relate changes in any good's "value" to changes in its embodied labor content alone; and here he chides Smith for replacing embodied labor content by how many hours of labor a good can command (or, in some Smithian moods, by what need not be quite the same thing, by the amount of corn or means-of-subsistence goods basket that the good in question may trade for or command).

A 3-good version of the canonical model will show Smith and Ricardo in absolute agreement on *all* substantive facts. Corn, ballet, and gold are each producible by land, labor, and possibly out of themselves as needed raw materials or durable goods in a time-phased way. Here are test cases.

Case 1. Land is redundant and rent

zero. The profit rate is zero ($\bar{r}^* = 0$). In this initial rude state, each good has a market price in proportion to its labor contents; each good commands precisely those same labor contents. A drop in labor requirements for a good like gold cheapens gold relative to corn and ballet and relative to a day's labor. Gold has dropped in "value," both writers agree. (Note: "money prices" expressed in gold rise for corn, ballet, and a day's labor.)

Case 2. Replace the invention in gold production by a similar one in corn production only. Now only corn has dropped in "value" for Ricardo. Gold and ballet have remained unchanged. The same holds for Smith's labor-command version of value. But now of course Smith's corn-command measure of value must diverge from Smith's labor-command measure: in terms of the former, Smith would say that gold, ballet, and a day's labor have risen in "value" and corn by definition has not changed.

If this were the end of the matter, despite Einstein's shrugging of shoulders, Ricardo's terminology and Smith's here-identical first-version terminology would seem slightly preferable to Smith's second-version terminology. I suspect Smith would agree for this case.

Case 3. But Smith—and Ricardo in Chapter 1, Section 1—would not expect the matter to end with this new short-run equilibrium. With the real wage now above the (previous) subsistence level, population would grow and in *Figures* 2–3 we would move along E_0E'. With land superabundant and the population adjustment parameter ϵ in (5) very fast (as suggested by Ricardo's words "in no long time" or "probably at the end of a very few years"), the corn invention would raise the profit rate above zero (actually to 100 percent per period if the labor requirements for corn halved) keeping the corn wage down near $\bar{w}^*$ and making Smith's two versions now agree with each other. But now they differ from Ricardo's version.

And which is semantically more appealing? I believe the jury will say, if *case* 3 is at all the common one in history, then Smith's terminology is more appealing: For Smith, the rise in the prices of gold and ballet in terms of both corn and a day's work (these last are in the same exchange rate as before) represents an increase in their "value." Moreover, Smith's quantitative degree of rise in their "value" does *exactly* match their rise in relative price. By contrast, Ricardo says that gold and ballet are completely unchanged in "value," while corn has halved in "value"; relative to corn's "value," they have exactly doubled in "value"—whereas actually, both men agree that their prices could have increased respectively by 10 percent and 999 percent or by 99 percent and 1 percent or by *any* quantitative degrees, depending on what a 100 percent profit rate does in marking up their competitive prices!

Ricardo's debating gaffe is to chide Smith for departing from concepts appropriate only for a $r = 0 = R$ world, and then in Section 1 himself adopting such departures as the ammunition for his criticisms of Smith.

Case 4. To amplify the point, let's suppose land is scarce and rent no longer zero. For simplicity, posit $\bar{r}^* = 0$ and concentrate on comparing long-run equilibria. Suppose all wages are spent on subsistence corn, which is produced by land and labor. Suppose all rent is spent on a luxury good (say gold), which is produced by labor alone. *Figure* 1, with WW' and SS' coinciding, determines the corn employment level at E (and coinciding E'). Knowing the ratio of the *SED* rent triangle to the wage rectangle formed by E, we know the ratio of labor employed in gold to that employed in corn. So our 2-good canonical long-run equilibrium is determinate in all details.

Now let a labor-augmenting invention in the corn industry make one laborer be the equivalent to two laborers. After a transient rise in profit and wage rates, $r(t)$ and $w(t)$ settle back to 0 and $\bar{w}^*$ by that determinate change in corn labor that reflects the shifted E intersection. In the end, the prices of corn, a day's labor, and gold are in *exactly the same ratios* as they started out. Both of Smith's verbiages well describe the facts: neither corn nor gold have changed in command over a unit of labor; gold is unchanged in its command over corn; only rent has risen in its command over everything else, corn, gold, and a unit of labor.

Ricardo, by contrast, is in a pickle. Gold's embodied labor is unchanged; but corn's embodied labor, measured by *total corn labor/total corn,* rises if rent's share of corn cost rises and falls if that rent share falls: either case is possible depending on a 1932 elasticity-of-substitution undreamed of by Ricardo or Mill. Who would find it useful to follow Ricardo in saying that corn's "value" has changed when *all* (P_c, P_g, W) ratios are unchanged?

There is one way out for Ricardo—a disastrous one. Suppose Ricardo measures corn's embodied labor content, not by *average* labor content, L_c/Q_c, but by "*marginal* labor content"—measured by labor per corn output on *external* margin land, or on the *internal* land margin's $1/(\partial Q_c/\partial L_c)$. To coin a phrase, this neoclassical version of Ricardo (call it Clark-Ricardo) is a disaster for his debate with Smith *because, using it, Ricardo finds himself here in exact agreement with* the labor-command *doctrine of Smith* that *Section* 1 is attacking![12]

* * *

Other beefs with Smith by Ricardo reduce to similar semantic snarls.[13] We are left with the essential unity of the classical model, the progressions and retrogressions being primarily in the modes of explanation.

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[12] As Viner [35, 1930, pp. 79–80] pointed out in his famous review of Cannan, a marginal-labor theory of value is isomorphic with a marginal-land theory of value or with a marginal-fertilizer theory of value: when *n* goods are each producible out of transferable-indifferent labor and transferable-indifferent fertilizer, it is as trivial to say that any competitive price ratio, P_i/P_j, is equal to relative marginal-fertilizer–requirement ratios as to marginal-labor–requirements. It would be anticlimactic if the labor theory of value, from John Locke through Marx, reduced down to this (envelope) triviality.

[13] One such is Ricardo's accusation that Smith makes land rent price determining. After a careful audit, we should agree with the dissenting verdict of Stigler [31, 1952, p. 205]: ". . . the tenor of Smith's theory of rent, which was not given a coherent statement, was that aggregate rents are a residual, but that the rent of any one use of land is a cost determined by alternative uses of land. Ricardo ignored the multiplicity of uses of land." Another Ricardian *non sequitur,* Sraffa [30, 1951, p. xxxvii], is his invalid inference that Smith's equating of price to the *sum* of wages + rent + profit implies that Smith believed that such a price was necessarily higher than in the rude state (when productivity was so low as to make rent zero and the earnable-profit zero). Again, it is the critic, Ricardo, who seems to have nodded.

Professor Stigler points out to me that I have been rather charitable to Smith in attributing to him knowledge of diminishing returns; and less than just to Ricardo in not crediting his Chapter 1 with having succeeded in showing that changes only in wage rates will not affect relative values. I ought to mention that the similarity of Ricardo and Malthus ends when we deal with the macroeconomics of Say's Law. Professor Blaug also doubts that the differences between Ricardo and Smith are usefully dismissed as being merely semantic; in any case, upon review, I find no thought experiments proposed by Ricardo to which he has given a different substantive answer than would Smith's system; outright slips by Smith seem if anything less than those in Ricardo and are of secondary importance in both cases.

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