Chapter 17
Antoine Augustin Cournot

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Introduction

Antoine Augustin Cournot[1] was born on the August 28th, 1801 at Gray, in Haute-Saône, in France. The family background was essentially rural, but an uncle of his was a public notary. He exercised a considerable influence in Cournot. He wrote in his *Souvenirs* concerning his birth: "Pour mon propre compte, je suis redevable de mon apparition dans ce monde à la révolution de 18 brumaire. Quelque temps après ce grand événement, mon père, parvenu à la quarantaine, crut les choses assez rassises et la liberté de conscience assez assurée pour songer à prendre charge de femme et d' enfants. Cependant, comme je suis né en 1801, six mois avant le Concordat, j' ai encore été, à la manière des temps primitifs, baptisé en chambre par un prêtre qui se cachait ou qui était censé se casher, car, dans la réalité, on ne craignait plus l' application des lois révolutionaires."[2]

He received his early schooling in his native town and his first special discipline in mathematics at the Lycée at Besançon in 1820. He lists in his *Souvenirs* the work of Laplace *Essai philosophique sur les probabilités* and Cordorcet's *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix* among the books which he read at this time and influenced him. In 1821, he entered the École Normale at Paris, where he continued his mathematical studies. He entered to the school with Auguste Walras, who was destined to become notable economist in his own right apart from being the father of Léon Walras.[3]

His stay at the École Normale was short for it was closed in 1822 by the government because of the alleged republican feelings of its students and Cournot had to transfer to the Sorbonne from which he graduated in Mathematics in 1823. His teachers included Laplace, Lagrange, and Poisson, who befriended him and helped him considerably in his later career. His stay at Sorbonne was very fruitful for him: "Je n'avais rien à lire, rien à composer, rien trouver, rien à projeter, je n'avais qu'à écouter et à réfléchir: Ce temps a été le plus heureux de ma vie."[4]

He became Professor of Mathematics at Lyons in 1834 on the recommendation of Poisson. One year earlier he was engaged in the translation of two works, one in mechanics, and one on astronomy.[5] He held the chair in Lyons for only one year, for in 1835 he was appointed, again on the recommendation of Poisson, Rector of the Academy of Grenoble. In 1836, he was provisionally appointed to the post of the *Inspecteur Général des Études*, an appointment which became permanent in 1838, the year of his marriage and the publication of his first book, the *Recherches*

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[1] The main source for Cournot's Biography is his *Souvenirs*, which were completed in 1859 but published in 1913 by Bottinelli. The main biographies of Cournot are contained in Moore (1905a, b, pp. 370, 521–543), Reichardt (1954), Moore (1991), Waffenschmidt (1991, pp. 57–69), Theocharis (1983).

[2] Quoted in Moore (1991, p. 19).

[3] Walras (1905) quoted in Theocharis (1983, p. 213).

[4] Moore (1991, p. 23).

[5] These were *Eléments de mécanique* de Kater et Lardner, which were *modified and completed* by Cournot (Paris 1842) and *Traité d'astronomie* de Herschel (Paris 1835). Both were translated from English.

sur les principes mathématiques de la théorie des richesses. He was made Knight of the Legion of Honour in 1838, and Officer in 1845. Parallel to the office of the Inspector-General, he held other educational offices during his career as an official, chiefly the membership of the "Commission des hautes études" and the presidency of the "Concours d' agrégation des mathématiques." He became Rector of the Academy at Dijon in 1854, but in 1862 retired from active teaching. The remaining years till his death on the March 30th, 1877 he spent in Paris engaged in philosophical meditation and writing.

The Works of Cournot[6]

Cournot was an economist, a philosopher and a mathematician. Among Cournot's mathematical writings mention can be made to his *Traité élémentaire de la théorie des fonctions et du calcul infinitésimal*, appeared in two volumes in 1841. This was followed in 1843 by the *Exposition de la théorie des chances et des probabilités* which is a systematic exposition of the calculus of probabilities and its application to statistics, and in 1847 by the *De l'origine et les limites de la correspondance entre l'algèbre et la géométrie.*

Cournot's philosophical works began to appear in 1851 when the *Essai sur les caractères de la critique philosophique* appeared.[7] This was followed by the *Traité de l'enchaînement des idées fondamentales dans les sciences te dans l'histoire* (1861) and the *Considérations sur la marche des idées et des évènements dans les temps modernes* (1872).[8] A last philosophical work entitled *Matérialisme, Vitalisme, Rationalisme: études sur l'emploi des données de la science en philosophie* appeared 2 years before his death in 1875.[9]

There are, however, his books in the field of Economics, which gave him fame and survive his name among future generations. Cournot started and finished his career as an author with an economic work. The *Recherches sur les principes mathématiques de la théorie des richesses* appeared in 1838[10] and the *Revue*

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[6] It is interesting to note and emphasize that Martin (1998) gave a "complete" bibliography, concerning not only Cournot's complete works, including the various French and foreign editions, as well as the different studies published about his works, but also a comprehensive review of all the references to Cournot in the world literature. Altogether there are 1,478 references of articles or books, listed and classified under 17 thematic headings. Cf. the reviews by Vatin (1999, pp. 310–312) and Larson (1999, pp. 377–378).

[7] It was published in two volumes. A second edition appeared in 1912 and a third edition in one volume in 1922.

[8] It appeared in two volumes. A new edition in 1934.

[9] A new edition appeared in 1923.

[10] The English translation bears the title *Researchers into the Mathematical Principles of the Theory of Wealth* translated by N.T. Bacon 1897, with an Essay and a *Bibliography of Mathematical Economics* by I. Fisher, 1927, New York, A. Kelley 1971, an edition to which we refer to. There is also a German translation entitled, *Untersuchungen über die mathematischen Grundlagen der Theorie des Reichtums* translated by W.G. Waffenschmidt, Jena: G. Fischer, 1924.

Sommaire des doctrines économiques in 1877,[11] the year of his death. A third book entitled *Principes de la théorie des richesses*, which is essentially a repetition of the *Recherches* without the mathematics, appeared in 1863.[12]

The Background of the "Recherches"

It has been a matter of considerable interest among all those who ever wrote about Cournot's economic work how he, an accomplished mathematician, was included not simply to turn to the study of economics but actually to appear for the first time before the wider public as an author of an economic treatise. It is difficult, as we believe, to answer this question and give an exact answer. We would like to make some assumptions.

First of all, the relationship between Cournot's economic works and French literature in Political Economy requires a preliminary questioning about Cournot's own relationships with economics. Let us recall that Cournot devoted his first book entitled *Recherches sur les principes mathématiques de la théorie des richesses* (1838) and his last book *Revue sommaire des doctrines économiques* (1877) to economics. In the intermediate period, he published ten books on other matters, such as mathematics and philosophy, the only exception being his book *Principes de la théorie des richesses* (1863).

Second, in spite of certain features, Cournot's contribution to economic theory does not belong to any French school of economic thought whatever. In fact, the so called French school of mathematical economics, to which Cournot is classically related, seems to be a mythical reconstruction. There were French economists[13] who used the mathematical method before Cournot (1838), but there were almost the French engineer economists from the eighteenth century up to now, who were especially fertile during the nineteenth century. However, Cournot was not an engineer. On the other hand, Walras worked hard in order to promote in France mathematical economics after 1860. Cournot was still alive, but he never accepted to be enrolled in the Walrasian campaign.

Third, Cournot's study of economics was a side interest. But having read Smith, Ricardo, and Say, as he himself admits,[14] he must have found their analyses vague

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[11] Reprinted by A. Kelley, New York 1968.

[12] Reprinted by Bizzarri, Rom 1969.

[13] There are C.-F.-J. d' Auxiron, *Principes de tout gouvernment* (1766), A.-N. Isnard, *Traité des Richesses*, London and Lausanne, 2 vols, 1781. L.F.G. de Cazaux, *Elémens d' économie privée et publique; Science de la valeur des choses et de la richesse des individus et des nations*, Paris – Toulouse, 1825. C. Courtois, *Mémoire sur différentes questions d' économie publique, relatives à l' établissement des voies de communication*, Paris, 1833. On Isnard's very rare book, see the excellent edition prepared by Van den Berg (2005, pp. 68–198). See Theocarakis' review in Theocarakis (2006). On Auxiron se Van den Berg and Dhesi (2004). On the French mathematical economists, see Theocharis (1961, pp. 66–69, 90–91). Theocharis (1988, pp. 265–273).

[14] Cournot (1938[1971], p. 4).

and confusing. He found that economic science was assuming "the dignity of a science of laws," and, as he was already influenced by A. Comte's ideas of a science of "social physics," the idea must have come to his mind that by developing the mathematical approach he could evolve a science of "economic physics." Cournot noted in the *Recherches* that of the previous attempts to apply mathematics to economics he had learned only the titles, except for one, Nicolas-François Canard's *Principes d' économie politique* (1801),[15] "a small work […] crowned by the Institut."[16] Although he asserted that its principles "are so radically at fault," as Cournot underlined, "and the application of them is so erroneous" he later wrote that Canard's *Principes* was his point of departure, albeit a discouraging one. Cournot said that Canard's work embodied a false point of view and that works such as it would not incline economists like Jean-Baptiste Say and David Ricardo to use algebra.[17] Cournot may have been familiar with A. Walras, which had mathematical leanings. Lastly,[18] it has been proved that Canard's *Principes* had a direct influence on Cournot.

The Use of the Mathematical Method

The aim of the *Recherches* is not to develop a theory of wealth, but to apply the mathematical method to those parts of the theory, which Cournot thinks are susceptible to such a treatment. "I have not set out to make complete and dogmatic treatise on Political Economy. I have put aside questions to which mathematical cannot apply, and those which seem to me entirely cleared up already," he writes.[19]

Cournot can be considered as a direct product of a French mathematical tradition. It is well known that he was preferred pupil of Poisson. He honestly confessed in his *Souvenirs* that he was not a first-rate mathematician, in spite of Poisson's hopes, but he possessed an excellent training in mathematics and a vivid sympathy for ideas and theorization.

Cournot never denied the existence of a link between mathematics and the "science of wealth" even when the refrains from mathematics, but he defends his position at two different levels. In the *Principes*, he reproduced the analysis of the philosophical foundations of economics. Economics develop the opposite point of view from the law and jurisprudence on the same topic. While the laws are concerned by individual cases, the economists study phenomena determined by large numbers. If not Cournot's economics belong to the family of the mathematical sciences, because it is rooted in the ideas of numbers and measurement, in the *Recherches*, he explains the use of mathematics in economics. He held that the solution to the general questions of the theory of wealth depend "not on elementary

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[15] For an evaluation of Canard's book in the history of economic thought, see Waffenschmidt (1958), Theocharis (1961, pp. 72–87), Tortajada (1990), Larson (1999, pp. 109–131).

[16] Cournot (1838[1971], p. 2).

[17] Cournot (1838[1971], p. 2.)

[18] Larson (1999, pp. 109–131).

[19] Cournot (1838[1971], p. 5).

algebra, but on that branch of analysis which comprises arbitrary functions, which are merely restricted to satisfying certain conditions."[20]

Thus, part of Cournot's dissatisfaction with the *Principes* must have been due to its use of a type of mathematics that he found inappropriate for economics.

Cournot's method is not aiming at finding directly numerical results; its aim is to ascertain what form of relation exists between two or more economic quantities and to apply there the theory of functions. Cournot underlined the fact that as only very simple conditions will be considered, "the first principles of the differential and integral calculus suffice for understanding this little treatise."[21] The determination of the relation may be vague but nonetheless the theory of functions will be applicable. Thus, the relation between quantity demanded and price may be presented by the function $D = F(p)$. It is sufficient to know some of its properties – in this case e.g., that it is decreasing and continuous – in order to find by means of analytical symbols "relations equally general which would have been difficult to discover without this help."[22]

This conception of the role of mathematics in economic theory struck, Cournot thought, at the roots of the argument of those authors, who although theorists of repute, mistakenly thought that "the use of symbols and formulas could only lead to numerical calculations, and as it was clearly perceived that the subject was not suited to such a numerical determination of values by means of theory alone, the conclusion was drawn that the mathematical apparatus, if not liable to lead to erroneous results, was at least idle and pedantic."[23]

Cournot was a mathematically sophisticated philosopher who, influenced by Fourier and his theory of heat, postulated that mathematical equations describing phenomenological entities were viable with any ontological commitments concerning the underlying phenomena.[24] Already in the preface to the *Recherches* he announced that in writing the book he had "put aside questions to which mathematical analysis cannot apply."[25] Further, the idea most authors had about the applicability of mathematical analysis to economics did not agree with Cournot's view:

> They imagined that the use of symbols and formulas
> could only lead to numerical calculations, and as it was
> clearly perceived that the subject was not suited to
> such a numerical determination of values by means of
> theory alone, the conclusion was drawn that the
> mathematical apparatus, if not liable to lead to
> erroneous results, was at least idle and pedantic.[26]

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[20] Cournot (1838[1971], p. 4).

[21] Cournot (1838[1971], p. 4).

[22] Cournot (1838[1971], p. 5).

[23] Cournot (1838[1971], p. 3).

[24] Vázquez (1997, p. 126).

[25] Cournot (1838[1971], p. 5).

[26] Cournot (1838[1971], p. 3).

Cournot's Forerunners and His Originality

It was Frisi's originality who first used the calculus in 1772 to determine when price would become a maximum or a minimum.[27] T. R. Malthus made in 1814 certain suggestions, in which he called attention to the potential usefulness of differential calculus for economics and related sciences.[28] Ten years later Perronet Thompson, who like Malthus had excelled as a student of mathematics at Cambridge, employed the calculus in economic analysis. The problem that Thompson posed was to maximize the pain of a government that purchases goods and services with paper money, the issue of which is attended by rising prices. Thompson's article entitled "On the Instrument of Exchange" was the first response to Malthus's suggested employment of the calculus.[29] In 1815, a continental writer, the German Graf Georg von Buquoy, who stressed the managerial side of economics, advised farmers to maximize their net revenue by holding production at a level at which the first derivative disappears and the second becomes negative.[30] Later on, when new economic problems emerged with the operation of railroads, similar ideas were advanced. In 1839 Charles Ellet, a noted American railroad builder, applied calculus to determine an optimum tariff that would maximize profits.[31]

Cournot's book does put things in a new place. It is astonishingly modern, and it contains, for those who take the trouble of reading it, many discoveries.[32]

The Content of the "Recherches"

Entering upon the book itself, we find that it naturally falls under three parts. These are (a) the pure theory of price[33], (b) the theory of rates of exchange and international trade[34], and (c) his theory of Social Income.[35]

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[27] On Frisi's notes to Verri's, *Meditazioni sulla Economia Politica*, Livorno 1772, see Theocharis (1961, pp. 27–34, 36–39), Luini (1996, pp. 127–147).

[28] Spiegel (1971, p. 507).

[29] Thompson (1824, pp. 171–205). On his contribution to mathematical economics, see Theocharis (1961, pp. 122–123).

[30] von Buquoy (1815[2005], p. 54). On this contribution see Theocharis (1961, pp. 112–113), Homberg (1971, pp. 61–62), Baloglou (1995, pp. 57–60), Bieri (1968, p. 138, n. 4) emphasized that v. Buquoy is Cournot's forerunner.

[31] Charles Ellet, *An Essay on the Laws of Trade in reference to the works of internal improvement in the United States* (1839). Cf. Theocharis (1993, pp. 21–40).

[32] Robbins (1998, p. 252).

[33] Cournot (1838[1971], pp. 7–116).

[34] Cournot (1838[1971], pp. 117–126).

[35] Cournot (1838[1971], pp. 127–171).

Value and Price

Chapter I[36] is devoted to defining wealth, the term Cournot uses in the sense of value in exchange. He carefully distinguishes this idea from "utility," with which he conceives the economist has no direct concern. What relations exist between wealth thus conceived and the welfare of the human race, Cournot regards as too difficult a problem to admit of present solution.

The second chapter entitled "Changes in Value, Absolute and Relative"[37] deals with the problem of value.

The very idea of value in exchange implies the necessity of comparison between two things; the idea of value is to fall into a logical contradiction. There can be absolute changes in one or both of the terms making up the ratio of value and these will affect the value of the ratio, but the idea of an absolute "change" in one of the terms of the ratio must be clearly distinguished from the idea of the ratio itself. "There are no absolute values" emphasized Cournot[38] "but there are movements of absolute rise and fall in values." Clinging to the physical analogy, Cournot cites the remarkable passage in Newton's *Principia*[39] in which an "absolute space" is supposed as a background for mechanical motion, distinct from the "relative space" made up of the system of moving points. He does despair of distinguished statistically absolute and relative changes, and observes that in case all commodities except one, such as gold or silver, preserve the same relative values; the probability to preserve the same relative value is greater that the one commodity has changed than that all the others have changed.[40]

The Law of Demand

The determination of price is the result of the play of the forces of supply and demand. Cournot believed that it was demand which played the essential part, "supply is the necessary counterpart of demand and consequently the accessory fact."[41] Cournot devotes a whole chapter, Chapter IV, entitled "Of the law of demand (De la loi du débit)"[42] to the discussion of demand, while his discussion of supply is hidden away as a discussion of costs in the chapter of Monopoly.

Cournot, though openly admitting that demand depends on utility, dispatched the embroiled classical discussions on the subject as ill-suited for the foundation of a scientific theory. Those ideas, he held, are by nature capable of neither enumeration

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[36] Cournot (1838[1971], pp. 7–17).

[37] Cournot (1838[1971], pp. 18–28).

[38] Cournot (1838[1971], p. 24).

[39] Cournot (1838[1971], p. 20).

[40] Cournot (1838[1971], pp. 25–26).

[41] Roy (1933, p. 17).

[42] Cournot (1838[1971], pp. 44–55).

nor measurement, and it is therefore plain that no algebraic law can encompass the behavior of prices.[43]

Cournot assumes that the demand for a commodity, in the sense of the quantity of it annually consumed, varies with – is a "function" of – its price. This relation may be generally written as[44]

$$D = F(p), \tag{17.1}$$

where D indicates the demand of a commodity during a given period, a year,[45] in a given market,[46] and p the average price of the same commodity during the year. In this case he considers price as the independent variable, but later in the treatment of oligopoly he gives the form of the function as $p = F(D)$, when the quantity becomes the independent variable.[47] The relation between price and demand is delineated by the new familiar "demand curve" which Cournot was the first to introduce.[48] The character of this relation depends on "the kind of utility of the article, on the nature of the services it can procure, on the habits and customs of the people, on the average wealth, and on the scale on which wealth is distributed."[49]

Cournot makes another assumption, that of the continuity of the demand function, from which it follows that there may be a linear approximation to it within short ranges. "The wider the market extends," says Cournot, "and the more the combination of needs, of fortunes, or even of caprices, are varied among consumers, the closer the function $F(p)$ will come to varying with p in a continuous, manner. However, little may be the variation of p, there will be some consumers so placed that the slight rise or fall of the article will affect their consumptions, and will lead them to deprive themselves in some way or to reduce their manufacturing output, or to substitute something else for the article that has grown dearer."[50]

The demand curve is not only downward sloping and continuous; as the price in the function $F(p)$ has been taken to mean the average price during a year, the curve $F(p)$ is "in itself an average off all the curves which would represent this function at different times of the year."[51] The demand curve has in general the form and in the following Fig. 17.1.

The total revenue $pF(p)$ is maximized, when

$$\frac{\mathrm{d}pF(p)}{\mathrm{d}p} = 0, \tag{17.2}$$

[43] Cournot (1838[1971], pp. 10, 47). Cf. Vázquez (1997, pp. 126–127).

[44] Cournot (1838[1971], pp. 47–48).

[45] Cournot (1838[1971], p. 51).

[46] Cournot (1838[1971], pp. 51–52), note *Cournot's book (1971).
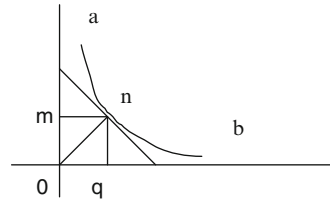
[47] Theocharis (1983, p. 138).

[48] Cournot (1838[1971], Fig. 1 of the Appendix).

[49] Cournot (1838[1971], p. 50).

[50] Cournot (1838[1971], p. 50).

[51] Cournot (1838[1971], p. 52).

Fig. 17.1 The demand curve

or by denoting by F' the differential coefficient of function F

$$F(p) + pF'(p) = 0. \tag{17.3}$$

The additional condition for maximization is[52]

$$2F'(p) + pF''(p) < 0. \tag{17.4}$$

The price oq which maximizes total revenue is found from Fig. 17.1 at such a point n on the curve anb, such as $on = nt$, where nt is the portion of the targent to the curve at the point n, which lies between n and the abscissa.[53]

In his discussion on the problem of maximization of the total revenue, Cournot further elaborate his concept of the elasticity of demand. For, he says, we would examine by statistical observation what happens to the total revenue $pD = pF(p)$, if there is a small change in price.

If the price becomes $p + \Delta p$, where Δp is a small fraction of p, the annual consumption would become $\Delta - \Delta D$. Then if[54]

$$\frac{\Delta D}{\Delta p} < \frac{D}{p}, \tag{17.5}$$

the increase in price will increase the total revenue $pF(p)$. The contrary would happen if

$$\frac{\Delta D}{\Delta p} > \frac{D}{p}, \tag{17.6}$$

when total revenue would decrease as a result of a rise in price and demand would be elastic.

It is, therefore, according to Cournot, of importance to know whether "the two values p and $p + \Delta p$ (assuming Δp to be a small fraction of p) fall above or below the value which makes the product under consideration a maximum."[55] Cournot suggests that "commercial statistics," as he says, should separate economically important commodities into two categories in accordance with their demand elasticity or, as he

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[52] Cournot (1838[1971], pp. 53–54).

[53] Theocharis (1983, pp. 140–141).

[54] Cournot (1838[1971], pp. 53–54).

[55] Cournot (1838[1971], p. 54).

puts it, "according as their current prices are above or below the value which makes a maximum of $pF(p)$."[56] As Professor R.D. Theocharis had emphasized, "long before Marshall himself, he fully elaborated the concept of the Marshallian elasticity of demand."[57]

Cournot points out that $pF(p)$ might have several maxima and pass through minimum values between, depending on the shape of the demand curve. He proves that whenever $F''(p)$ is negative or when the curve $D = F(p)$ "turns its concave side to the axis of the abiscissas, it is impossible that there should be a minimum, not more than a maximum. In the contrary case, the existence of several maxima or minima is not proved to be impossible."[58] On this question Cournot thinks that in actual practice, it is improbable that the function $pF(p)$ will present such a problem "inside of the limits between which the value of p can vary."[59] The question therefore is always whether within the limits of the oscillation of p, "the function $pF(p)$ is increasing or decreasing for increasing values of p."[60]

Given the law of demand, Cournot first supposes a complete monopoly of the commodity in question, and shows what price will yield the maximum profit.

We have already seen that Cournot had given as the condition of maximizing revenue, where there are no costs:

$$\frac{\mathrm{d}pF(p)}{\mathrm{d}p} = 0, \tag{17.7}$$

which leads to

$$F(p) + pF'(p) = 0. \tag{17.7a}$$

If there does exist a monopoly, as in the case of the proprietor of a mineral spring with exclusive salutary properties,[61] he will seek to maximize his revenue by applying (17.7a) above which gives as the maximizing price

$$p = \frac{F(p)}{-F'(p)}, \tag{17.8}$$

and the total revenue of the monopolist is

$$pF(p) = \frac{F(p)}{-F'(p)}^{2}. \tag{17.9}$$

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[56] Cournot (1838[1971], p. 54).

[57] Theocharis (1983, p. 142). It was worth to note that William Whewell (1794–1866) had used the concept of demand elasticity a little earlier than Cournot (Whewell 1829), but there is no indication that the latter was aware Whewell's contribution. Theocharis (1961, pp. 125–127), Rashid (1977).

[58] Cournot (1838[1971], p. 55).

[59] Cournot (1838[1971], p. 55).

[60] Cournot (1838[1971], p. 55).

[61] Cournot (1838[1971], p. 56).

Under monopoly, if there are costs, the net receipts to be maximized are[62]

$$pF(p) - \varphi(D), \tag{17.10}$$

and the maximizing condition is

$$\frac{\mathrm{d}pD}{\mathrm{d}p} - \frac{\mathrm{d}\varphi(D)}{\mathrm{d}p} = 0. \tag{17.11}$$

$$\frac{\mathrm{d}pD}{\mathrm{d}p} = \frac{\mathrm{d}\varphi(D)}{\mathrm{d}p}. \tag{17.11a}$$

The condition is the explicit formulation that the monopolist, the seller of a unique product, who is eager to maximize his net revenue, will charge a price at which marginal revenue equal marginal cost.

Cournot gives (17.11) in the form[63]

$$D + \frac{\mathrm{d}D}{\mathrm{d}p}\left(p - \frac{\mathrm{d}\varphi(D)}{\mathrm{d}D}\right) = 0. \tag{17.12}$$

Cournot denotes the change in cost in response to a change in quantity as $\varphi'(D) = \mathrm{d}\varphi(D)/\mathrm{d}D$. He uses the graphical representation to show how the monopoly price can be determined when there are costs.[64]

Cournot discusses further the effect of the monopoly price of a change in the various conditions of costs. He also discusses the effects of taxation on the price which is established under a monopoly.[65] These results depend on whether the tax is a fixed tax or direct levy proportional to the income of the seller (when there will be no effect on monopoly price or quantity) or whether the tax is a specific tax on the commodity (when there are repercussions as this means an additional cost to the producer).

The Theory of Oligopoly[66]

In passing from the study of perfect monopoly to that of perfect competition, Cournot considers also the intermediate case of a few, say, two, competitors. Cournot's treatment

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[62] Cournot 1838[1971] p. 57.

[63] Cournot (1838[1971], p. 57).

[64] Cournot (1838[1971], Fig. 5).

[65] Cournot (1838[1971], Ch. VI, pp. 67–78).

[66] The first use of the term "oligopolium" is in Thomas More's *Utopia* (1516), where he had argued that an increase in the number of sheep might not lend to a fall in their price because, though there was not "monopolium," as the sheep did not belong to a single person, there was an "oligopolium" as the sheep belonged to a few rich people who could afford to wait until they got the desired price. For the authors who contributed to the theory of oligopoly prior to Cournot cf. Theocharis (1983, pp. 151–155).

of this difficult problem is "brilliant and suggestive."[67] The central supposition is that each individual will act on the assumption that his rival's output is constant, and will survive only to so regulate his own output as to secure the largest profits.

Cournot entitles his Chapter VII of the *Recherches*, in which he discusses the theory of oligopoly, "Of the competition of producers." He now imagines two owners of two springs' of which the quantities are identical, and which, on account of their similar positions, supply the same market in competition.[68] As a result of this assumption there is only one price. He now, defines the price p as a function of the quantity demanded, so that

$$p = F(D). \tag{17.13}$$

The total quantity of sales D will be

$$D = D_1 + D_2, \tag{17.14}$$

whether D_1 the sales from the spring (1) and D_2 the sales from the spring (2).

If neither of the producers has any costs, the net revenue of the first will be his sales at the current price, pD_1 and the net revenue of the second will be pD_2.

The net revenue of the first will be

$$pD_1 = D_1 f(D_1 + D_2), \tag{17.15}$$

and that of the second will be

$$pD_2 F(D_1 + D_2). \tag{17.16}$$

Cournot makes two assumptions, which have great importance for his analysis. The first is that there is no collusion between the sellers. "*Each of them independently*"underlines Cournot,[69] will seek to make this income as large as possible". This is essential, "for if they should come to an agreement as to obtain for each the greatest possible income, the results would be entirely different, and would not differ, so far as consumers are concerned, from these obtained in treating of a monopoly.[70] The second assumption seems to be of the most crucial importance, because it assumes that either of the sellers seeks to maximize his revenue by assuming that his rival's quantity will remain unchanged. That leads to the fact that "proprietor" (1) can have no direct influence on the determination of D_2: all that he can do, when D_2 has been determined by proprietor (2), is to choose for the value which is best for him.[71] This assumption is followed by the next sentence where shows that Cournot did not exclude price adjustments: "This he will be able to accomplish by properly adjusting his price, except as proprietor (2), who, seeing himself forced to

[67] Fisher (1898, p. 126).

[68] Cournot (1838[1971], p. 79).

[69] Cournot (1838[1971], p. 79) (Italics by Cournot).

[70] Cournot (1838[1971], p. 80).

[71] Cournot (1838[1971], p. 80).

accept this price and this value of D, may adopt a new value for D_2, more favorable to his interest than the preceding one."[72]

The first seller's revenue will be a maximum, for constant D_2, when

$$\frac{d(D_1 p)}{dD_1} = p + D_1 f'(D) = 0, \qquad (17.17)$$

which may be written as

$$f(D_1 + D_2) + D_1 f'(D_1 + D_2) = 0. \qquad (17.17a)$$

The second seller's revenue will be a maximum, for constant D_1 when

$$\frac{d(D_2 p)}{dD_2} = p + D_2 f'(D) = 0, \qquad (17.18)$$

which may be written as

$$f(D_1 + D_2) + D_2 f'(D_1 + D_2) = 0. \qquad (17.18a)$$

Equations (17.17a) and (17.18a) form a system of equations,[73] the solution of which gives $D_1 = D_2$ as "ought to be the case, as the springs are supposed to be similar and similarly situated."[74]

The addition of (17.17a) and (17.18a) leads to

$$2f(D) + \left(D_1 + D_2\right)f'(D) = 2f(D) + D\frac{dp}{dD}$$

$$= 2p + D\frac{dp}{dD} = 0$$

Multiplying[75] this by dD/dp the result can become[76]

$$D + 2p\frac{dD}{dp} = 0.$$

Cournot uses also the graphical representation to solve this problem.[77]

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[72] Cournot (1838[1971], p. 80).

[73] Cournot (1838[1971], p. 81).

[74] Cournot (1838[1971], p. 82).

[75] Theocharis (1983, p. 221, n. 164).

[76] Cournot (1838[1971], p. 82).

[77] Cournot (1838[1971], Fig. 2). Cf. Theocharis (1983, pp. 158–159), Magnan de Bornier (2001, pp. 168–171).

Further Extensions of the Theory of Oligopoly

Passing on the case of "unlimited competition,"[78] Cournot shows that the price is, in this case, equal to the "marginal cost of production." Cournot himself does not use this term nor is any other verbal description of the magnitude involved. He confines himself to mathematical symbolism.[79]

If we plot the relation between the product of each individual and his resulting marginal cost, we have a system of individual supply curves. These may be combined into a single general supply curve, which Cournot uses.[80] He shows that the intersection of this general supply curve with the general demand curve determines price. It is significant "of the slow growth of economic science that these graphic pictures of supply and demand, now in almost universal use in textbook and classroom" as I. Fisher emphatically wrote,[81] "were ignored or forgotten by Cournot's contemporaries, and were only restored in 1870, when independently obtained by Fleeming Jenkin." It is worth to note that the German economist Karl Heinrich Rau (1792–1870) came independently[82] to the same result as Cournot, 3 years later (1841).[83]

In the same chapter Cournot enunciates two other principles which have become classic; the first one is in regard to the law of diminishing returns,[84] and the second is that a tax on a commodity subject to "unlimited competition" will raise the price by an amount less than the tax itself.[85]

The Oligopoly of Complementary Goods

Cournot next considers the "mutual relations of producers"[86] or the connections between complementary materials, such as copper and zinc, which enter jointly into the production of a composite, such a brass.[87]

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[78] Cournot (1838[1971], Ch. VII, pp. 90–98).

[79] Fisher (1898, p. 127).

[80] Cournot (1838[1971], Fig. 6).

[81] Fisher (1898, p. 127).

[82] Rau constructed the graphic representation of the law of demand and supply in 1841. Rau (1841a, p. 527), Rau (1841b, pp. 148–151). For the evidence that Rau came independently without Cournot's contribution to same result, see Baloglou (1995, pp. 160–167).

[83] Brandt (1968, pp. 90–91), Homberg (1971, pp. 97–100), Hennings (1979, pp. 1–14), Theocharis (1993, pp. 150–153), Baloglou (1995, Ch. 4), Vázquez (2002).

[84] Cournot (1838[1971], p. 91).

[85] Cournot (1838[1971], p. 93): *in all cases the rise in price will be less than the increase in cost* (Italics by Cournot).

[86] Cournot (1838[1971], Ch. IX, pp. 99–116).

[87] Cournot (1838[1971], p. 100).

Cournot assumes that there are two factors: (a) and (b), "which have no other use beyond that of being jointly consumed in the production of the composite commodity (ab)."[88] It is also assumed that there are no additional costs involved in the production of (ab), except for the reward of the two factors, which is paid to their owners. It is further assumed that the production of each factor costs nothing to its owner. Cournot assumes further that the two factors are used in the manufacture of the commodity in a fixed proportion $m_1 :: m_2$: "and $m_1 : m_2$ the proportion of copper to zinc in the brass," as Cournot says.[89]

This assumption leads to the equation

$$p = m_1 p_1 + m_2 p_2,$$

where p_1 is the factor of the price of the factor (a) and p_2 of the factor (b).

The quantity of the commodity demanded at price p is given by the demand function.

$$D = F(p) = F(m_1 p_1 + m_2 p_2).$$

If we suppose each of these to be handled by a monopolist, and "if we apply to the theory of the mutual relations of producers the same method of reasoning which served for analyzing the effects of competition,"[90] the condition of the maximization of the seller's revenue are

$$\frac{d(p_1 D_1)}{dp_1} = 0 \quad \text{and} \quad \frac{d(p_2 D_2)}{dp_2} = 0.$$

The development of these equation leads to the system.[91]

$$F(m_1 p_1 + m_2 p_2) + m_1 p_1 F(m_1 p_1 + m_2 p_2) = 0.$$

$$F(m_1 p_1 + m_2 p_2) + m_2 p_2 F(m_1 p_1 + m_2 p_2) = 0.$$

The solution of the above system gives as a result that the price of each will in equilibrium be such that the profits of the two sellers are equal

$$m_1 p_1 = m_2 p_2 = \frac{1}{2} p.$$

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[88] Cournot (1838[1971], p. 99).

[89] Cournot (1838[1971], p. 100).

[90] Cournot (1838[1971], p. 100–101).

[91] Cournot (1838[1971], p. 101).

The equilibrium price of the first will be equal to

$$p_1 = \frac{p}{2m_1},$$

and the equilibrium price of the second seller will be

$$p_2 = \frac{p}{2m_2}.$$

The addition of the equations of the above system gives

$$F(p) + \frac{1}{2} pF'(p) = 0,$$

which leads to

$$\frac{1}{2} p = -\frac{F(p)}{F'(p)}.$$

"The composite commodity," writes "Cournot,"[92] "will always be made more expensive, by reason of separation of interests than by reason of the fusion of monopolies. An association of monopolists, working for their own interest, in this instance will also work for the interest of the consumers, which is exactly the opposite of what happens with competing producers." That is, in the case of complementary commodities, it is better for the consumer to be at the mercy of one of monopolist than two. A levy of a tax on one of the two component commodities will raise the price of that commodity and of the composite commodity, but will lower the price of the other component.[93]

The Theory of Social Income

The solution of the problem of price determination was affected by Cournot under ceteris paribus conditions, which included the condition that incomes remain unchanged. But Cournot felt that this was only an approximation and that the ideal thing would be "to take the entire system into consideration."[94] This, he estimates

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[92] Cournot (1838[1971], p. 103).

[93] Cournot (1838[1971], pp. 112–116).

[94] Cournot (1838[1971], p. 127).

beyond the powers of mathematical analysis and he chooses to make another approximation and investigate how changes in prices of consumer's goods directly affect individual incomes and by implication the national income.[95]

Cournot defines social income[96] or national income[97] as "the sum total of individual incomes, of rents, of profits and of wages of every kind, in the whole extent of the national territory[98] and it includes 'the annual amount of the stipends by means of which individuals or the state sustain those classes of men which economic writers have characterized as unproductive, because the product of their labor is not anything or salable.'"[99]

Let us D denote the entire consumption of a "commodity for consumption" and p the price, "the product pD will express the sum to the extent of which this commodity co-operates in making up the social income."[100] If $p_0 D_0$ be the value of this product at one time, and $p_1 D_1$ at another, the difference between them, $p_0 D_0 - p_1 D_1$ expresses the diminution of social income. This diminution occurs in the incomes of the various persons contributing to the production of the commodity in question; and Cournot argues that the incomes of all other persons may be considered unchanged, for perturbations in the prices of other commodities are apt to occur as much in one direction as in the other.[101]

According to this reckoning, a dearth of a necessity of life may cause an increase of social income if the price rises faster than the quantity consumed falls. To overcome this difficulty, Cournot distinguishes between the "nominal" reduction of income $p_0 D_0 - p_1 D_1$ and a real reduction of income. He attempts to describe this real reduction of income without describing any "real income." The real reduction is found by taking into account the sacrifices that consumers of the commodity suffer in paying higher prices. Although it was already shown that the incomes of consumers, as a whole, may be considered as unchanged, still those who continue to buy after the price has risen have to pay the rise $p_1 - p_0$ on their purchase D, thus expending

$$(p_1 - p_0)D_1,$$

more income for precisely the same return. Hence they "are really in just the same situation as to fortune as if the commodity had not risen and their incomes had been diminished by $(p_1 - p_0)D_1$."[102] Adding this loss of income for consumers to the loss already shown for producers

[95] Theocharis (1983, p. 182).

[96] Cournot (1838[1971], p. 128).

[97] Cournot (1838[1971], p. 150).

[98] Cournot (1838[1971], p. 150).

[99] Cournot (1838[1971], p. 128).

[100] Cournot (1838[1971], p. 128).

[101] Cournot (1838[1971], pp. 129–132).

[102] Cournot (1838[1971], p. 134).

$$p_0 D_0 - p_1 D_1.$$

Cournot obtains

$$p_0 (D_0 - D_1),$$

as the total real loss.[103] He confesses, however, that, even with this amendment, he has not taken account of the loss to consumers who have ceased to buy the commodity because of the increased price, or of part of the loss to those who do buy, but buy less. He pleads in extenuation of this omission: "But this kind of damage cannot be estimated numerically [….]. Here comes in one of those relations of size which numbers can indicate, indeed, but cannot measure."[104] Edgeworth remarks at this point, that if Cournot had reached the conception of "consumers' rent," he would have seen that numbers can measures as well as indicate the damage in question.[105]

The Theory of International Trade

Cournot's contribution to the theory of international trade is elaborated in the last chapter of the *Recherches* entitled "Of the variations in the social income, resulting from the communication of markets."[106] The target of this chapter according to Cournot, is to prove "how commerce between two markets[….] causes the value of the social income to vary, as well in the importing as in the exporting market."[107] It is worth to note that, like in previous chapters, he again introduces losses and profits of the various involved agents.[108] Thus, Allais appears fully justified when he affirms: "Augustin Cournot should be credited with the merit of having introduced the concept of loss in economy (…) in 1838, i.e., 6 years before the first article of Dupuit, and of having approached the calculation of the first differential in simple cases."[109]

His analysis of the effects of international trade consists of three parts. In the first he develops a "highly ingenious,"[110] theory of foreign exchanges[111] the second deals with the effects of trade between markets, which were previously isolated, on

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[103] Cournot (1838[1971], p. 134).

[104] Cournot (1838[1971], p. 134).

[105] Edgeworth (1898, p. 628). Cf. Fisher (1898, p. 132).

[106] Cournot (1838[1971], Ch. XII, pp. 150–171).

[107] Cournot (1838[1971], p. 150).

[108] Alcouffe (2002, p. 10).

[109] Allais (1981, p. 168), quoted in Alcouffe (2002, p. 10).

[110] Edgeworth (1925, p. 446).

[111] Cournot (1838[1971], pp. 151–155).

prices.[112] Finally, the third part seeks to apply Cournot's ideas about social income and its variations to the theory of international trade.[113]

It is this chapter that had had the most negative critiques. The first critique was made by Karl Heinrich Hagen (1785–1856), a professor of Political Science and Economics at the University of Königsberg, in a booklet entitled *Die Nothwendigkeit der Handelsfreiheit für das Nationaleinkommen, mathematisch nachgewiesen*,[114] the professed aim which was to demonstrate, through the use of mathematical analysis, the necessity for free trade. In its concluding part Hagen[115] acknowledged that he had been led to his demonstration through the study of Cournot's *Recherches* and discussed the latter's treatment of the effects of international trade on social income. With the aid of a very crude analysis of the relation between price, quantity demanded, and costs, Hagen was led to an "importation" and an "exportation" formula, which according to him would express the national income effects of international trade.[116] Cournot himself had already employed an approach similar to that used by Hagen in order to analyze the effects of international trade or national income, through its effects on prices and quantities produced or consumed, that, is on gross revenue.[117] One criticism leveled by Hagen against Cournot's treatment concerned the latter's use of gross revenue for measuring national income, without taking costs into account.[118] He also criticized the fact, that Cournot had failed to take into account in examining national income effects the fact that, when a branch reduced its activity, the funds previously employed by it would flow to other activities, and the contrary would happen when a branch expanded.[119]

Cournot himself attempted later in his *Principes*[120] to answer Hagen's criticism. He argued that through his "principle of compensation of demands" he had taken into account "in the appreciation of average results, of the transfer of funds withdrawn from the demand of article A, to the demand of articles E, F...."[121] But to the observation of Hagen that the increase in the production of a branch can come about only at the expense of other branches, he concedes that "there may be circumstances when an industry will not be able thus to develop except at the expense of another."[122]

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[112] Cournot (1838[1971], pp. 155–160).

[113] Cournot (1838[1971], pp. 161–171). For an extensive analysis of Cournot's theory of international trade, see Theocharis (1983, pp. 185–194), Baloglou (1995, pp. 111–118).

[114] Hagen (1844). On Hagen's critique see Theocharis (1983, p. 196, 1990, p. 924), Baloglou (1995, pp. 128–129).

[115] Hagen (1844, pp. 30–32).

[116] Hagen (1844, pp. 11, 13). Cf. Theocharis (1993, pp. 170–172), Baloglou (1995, pp. 119–124).

[117] Cournot (1838[1971], pp. 150–171), Theocharis (1983, pp. 191–199).

[118] Hagen (1844, p. 31), Theocharis (1983, pp. 196, 231).

[119] Hagen (1844, p. 31).

[120] Cournot (1863[1981]).

[121] Cournot (1863, p. 212), quoted by Theocharis (1990, p. 924).

[122] Cournot (1863, p. 213).

The main criticism that could be leveled against Cournot's and Hagen's analysis[123] is that their discussion of the effects of international trade is carried in a partial equilibrium context.

Cournot in 1863

In 1863, 25 years after the publication of his *Recherches*, A. Cournot published his second economic work, the *Principes de la théorie des richesses.*[124] Deeply disappointed that his first work had not gained the recognition it deserved, he felt that what had gone wrong had been his use of the mathematical method. He declared in his work, that "I would like to see today whether I have erred basically in my ideas or only formally; and for this purpose I have again taken up my work of 1838 by correcting it, by developing it where the developments were missing, by completing it in those points which I had obtained from touching, and above all by absolutely stripping it of the algebraic apparatus which scares so much in these matters."[125]

Cournot considered that his *Principes* were his way of appealing against the sentence of non-appreciation imposed on his *Recherches*. "Since," he wrote,[126] "it has taken me 25 years to appeal against the first sentence, it goes without question that I do not intend, whatever may happen, to use another way of appeal. If I lose my case a second time, the only consolation left for me will be that the judgment, which condemns them, will be quashed 1 day in the interest of the law, that is, the truth."

It should be noted that, despite the above declaration of his intentions, 14 years later he published the nonmathematical *Revue Sommaire des Doctrines Économiques*,[127] which was his final attempt to reach the ever elusive goal of wider recognition.

Whereas, the *Recherches* did not provoke any reaction among the French circle of economists, the *Principes* were immediately commented by various authors, who did not share the same views on political economy: liberals such as Roger de Fontenay (1863), actuaries such as Chauveau (1864), and even the young Walras (1863).[128] Behind the *Principes,* their observations were mainly dedicated to the *Recherches*. Thanks to the *Principes*, Cournot's major economic work, the *Recherches* came to be recognized by several members of the French economic community a quarter of century after its publication. Unfortunately, however, this

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[123] Theocharis (1983, pp. 138–139).

[124] Cournot (1863[1981]).

[125] Cournot (1863, p. II).

[126] Cournot (1863, p. II).

[127] Cournot (1877[1968]).

[128] All these reviews have been reprinted in Cournot (1982).

late interest of the *Recherches* was overcompensated by a lack of sympathy, rapidly transformed into a total loss of interest in the *Principes*. In his last publication, Cournot summarized the situation in the following skeptical words:

> Mais voyez mon guignon. Si je gagnais un peu tard sans m'en être mêlé mon procès de 1838, je perdais mon procès de 1863 si l'on voulait bien faire rétrospectivement quelque cas de mon algèbre, me prose (j'ai honte à le dire) n' obtenait pas chez le libraire un meilleur succès.[129]

According to a long and still dominant tradition, the *Principes* would be only a pale translation in words of the mathematical content of the *Recherches* for strategic considerations of communications. More recently, a careful reading of the *Principes* leads to an opposite appreciation: Cournot would have changed his ideas on economics from the *Recherches* to the *Principes*, in their substance as well as in their methods.[130]

It is worth to note that the *Principes* had been received by Léon Walras, who as a student had become acquantainted with Cournot's *Recherches*. Walras underlined Cournot's contribution to introduce the mathematical method and emphasized them.

In the Preface of the *Principes*, Cournot underlined the continuity between the *Principes* and the *Recherches*. Several chapters of the *Principes* concerning the Law of Demand (Chapter VI), Monopoly and Competition (Chapter VII, Book 1), the Communication of the Markets (Chapter IV, book II), and the Social Revenue (Chapter V, Book II), are directly derived from the *Recherches*. On the other hand, Book II of the *Principes* entirely devoted to a criticism of economic optimism is quite new. The main argument in favor of a discontinuity is provided by the many digressions extracted from the philosophical *Traité de l' enchaînement des idées fondamentales dans la science et dans l' histoire* incorporated by Cournot in the *Principes*. As, for example, Cournot made a distinction between an absolute and a relative Maximum (or Minimum) and contest the possibility of an optimum, because of our limited knowledge of the economic order. Such views, which do not appear in the *Recherches*, reutilize previous developments on the same topic in the *Traité de l'enchaînement des idées*. Going through philosophy Cournot offered an opportunity for new insights into economics.[131]

Fact that Cournot "linked with an attempt to apply mathematics to Political Economy, a serious and honourable attempt, the first and only one of its kind which has been made, and about which it is impossible for us not to say a word, because it is of interest to a high degree for the future of Political Economy." Referring to the *Principes*, Walras expressed his disappointment at the abandonment of the mathematical method. He felt that if Cournot had chosen the course "of renewing his economic principles in order to apply again to the mathematical analysis" there might at last result, "if not a complete and definitive theory of change and of social

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[129] Cournot (1877, p. 111).

[130] Ménard (1978), Vatin (1998).

[131] Jaffé (1935) in Walker 1983 p. 18.

wealth, at least a new and precious chapter of pure political economy." Instead Cournot had rejected the original and fruitful mathematical element of his 1838 work, while he had retained and excessively developed its economic part.[132]

A second lengthy review of the *Principes* was published in 1864 by Roger de Fontenay (1809–1891), a graduate of the École Polytechnique and editor of F. Bastiat's works.

The reviewer started his essay by referring to Cournot's preface in the *Principes* where that author had explained why he had decided to present again the ideas of his original work of 1838 without the mathematical apparatus. Cournot had written in that preface that he had been wondering whether the failure of his *Recherches* had been due to basic errors in the ideas contained in that book or only in the form used. Fontenay expressed the opinion that the economic content in both of Cournot's books has not been quite up to the mark, as it was incomplete and sometimes wrong. Since he [sc. Cournot] has been able, as he says himself, to recast, correct, and even complete the first essay, said Fontenay, "by stripping it completely of the algebrical form, the ordinary economist appears to me to be entitled, up to a certain degree, to tell, him: Why have you amused yourself to talk to us in scary hieroglyphics, since you could present all this to us, and even better than this, in simple French prose and without algebra!"

According to de Fontenay, the algebraic process may either depart from precise and defines relations in order to arrive at numerical results and applications – this he calls the "triumphant" algebra; or, this process may involve the operation on vague formulas expressing relations, which are not reducible to numbers, in order to derive from them other theoretical forms and general laws – this he calls the "militant" algebra "of research, of progression and of theory." It was the second kind of algebra that Cournot chose to use in his Recherches by introducing functions of an indefinite nature and using the differential and integral calculus.

De Fontanay was a defender of the use of the mathematical method in economics. Algebraic analysis is a tool. It is "without doubt, the most powerful and the most extraordinary instrument of reasoning and investigation which the human genius has invented," and it was natural and absolutely justified for the able mathematicians to seek to apply their method to every science whose stage of development had reached a point where such application appeared feasible.

It is worth to note that de Fontanay recognized the disadvantages of the use of mathematics. First, there is the need of constantly seeking verification of the results obtained through this method. More important though is the limited scope and the uncertain nature of the results obtained. The application of the mathematical method requires right from the start the precise definition of all the initial data of the problem; it requires what we would today call the introduction of a model. This leads to the adoption of various devices, of subsidiary or simplifying assumptions etc. All these affect the result in such a way that in most cases the final conclusions reached are nothing else "but formulas which apply only to exceptional cases."

The most serious objection, according to de Fontanay, against the mathematical method is the fact that its very precision may be a serious handicap when it is applied

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[132] Theocharis (1993, pp. 234–235).

to sciences which have not yet been fully developed. In order to use mathematical analysis in such cases and in view of the inadequacy of the available data, one may either decide to make arbitrary assumptions, which would lead to uncertain and faulty results; or one may use only evident and incontestable, but inadequate, data, in which ease his base would be so thin as to lead to insignificant or null results.

It was the time when the economic science had not been fully developed at the time of writing the *Recherches* that, according to de Fontenay, Cournot's introduction of the mathematical method, despite its merits, could not be successful. "A political economy is not yet a nature and made up science in any of its part and as it was infinitely less in 1838," commented de Fontanay, "we must not be surprised that despite all his talent as dialectician and algebraist, M. Cournot has only arrived at results which are very mediocre from the point of view of economic interest, and which are sometimes more than questionable, as far as exactitude is concerned."

The reviewer emphasized and underlined the merits of the work, although he objects the acceptance of the Ricardian doctrine and Cournot's treatment on international trade. According therefore to de Fontenay "the important and capital thing is … the attempt made to give to political economy a mathematical foundation," which, as he said, explains why he devoted the major part of his review to the work of 1838 and not to the *Principes* of 1863. As far as the *Principes* concerns, he observed that it is the *Recherches* deprived of the conciseness and generality of its mathematics; and the second, which is interwoven with the first, is a presentation of ideas fundamental in the sciences and in history. The reviewer mentioned that Cournot introduced in *Principes* the concept of "economic equivalents," in the sense that such equivalents produce the same amounts of product. De Fontenay criticized Cournot's views regarding the effects of the introduction of machinery on the employment of workers. He also criticized Cournot's thought in the *Principes* against the system of economic freedom and its efficacy to obtain the optimum results from the point of view of human happiness. He even accused Cournot of not believing in the principle of economic liberty and in the ability of the economic system to self-adjust satisfactorily, as that author rejected both the existence of an organic economic harmony and the possibility of a mechanical adjustment of economic interests.[133]

Conclusion

A. A. Cournot's *Recherches sur les Principes mathématiques de la théorie des richesses*, which appeared in 1838, is the first consistent and systematic application of mathematical analysis, not simply to a single problem but to a number of topics – and this differentiates that book from earlier contributions to mathematical economics.[134]

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[133] This part is based on Theocharis' treatment. See Theocharis (1993, pp. 235–240).

[134] For a detailed analysis of the contributions of the authors to the mathematical economics prior to Cournot (1838), see Moret (1915, pp. 64–78), Weinberger (1930, pp. 36–42), Robertson (1949, pp. 523–536), Reichardt (1954, pp. 67–69), Bousquet (1958), Theocharis (1961). On the German authors prior Cournot see especially Homberg (1971), Baloglou (1995, pp. 29–53), Baloglou (2003, pp. 127–134), Vázquez (2006, pp. 533–541).

Cournot's book has the dual distinction of being the first economic treatise where, on the one hand, the calculus has been applied consistently and successfully throughout and, on the other hand, diagrams have been used extensively as an accepted form of exposition and analysis. The same can be said for Gossen's book.

Cournot is also the first author to put in clear mathematical terms the notion, that, ceteris paribus, the quantity demanded and the prices are functionally related; and to develop the concept of elasticity of demand long before Alfred Marshall. In discussing the conditions of supply, he introduces the idea of total and marginal cost and points out that under free competition the condition of equilibrium for the individual producer is the equalization of his price to his marginal cost.

Cournot determines, both analytically and graphically and under conditions of free competition, the static partial equilibrium of price, at the point where the total quantity demanded equals the total quantity supplied. He was the first to show that monopoly price would be fixed at the point where marginal cost equals marginal receipts and net revenue is a maximum. Cournot's approach to monopoly is very much alive today.

We have to underline that Cournot's contribution to the theory of oligopoly survives to the present day and it is truth, as R.D. Theocharis[135] has pointed out that "every author who has dealt with the problem of oligopoly price determination since the appearance of the *Recherches*, has not escaped the temptation to comment upon Cournot's solution, either critically or favourably."

References

Alcouffe A (2002) Cournot's international trade theory: between rejection and neglect. In: VI. Annual conference of the European Society for the history of economic thought (14–17 March 2002). Book of Abstracts, Rerthymnon, pp 10–11

Allais M (1981) La théorie générale des surplus. Paris Cahiers de l' I.S.M.É.A., n° 1 à 5, P.U.F., 1989, pp 18, 22, 161, 168, 191, 203–204, 229, 295, 297, 304, 325

Baloglou C (1995) Die Vertreter der mathematischen Nationalökonomie in Deutschland zwischen 1838 und 1871. Vorwort B. Priddat. Metropolis, Marburg

Baloglou C (2003) Researches in mathematical economics: the reception of Cournot's in Germany (1839–1871). Οργάνωση του Αθλητισμού 1(2):127–134

Bieri S (1968) Zur Stellung der frühen deutschen Finanzwissenschaft unter besonderer Berücksichtigung von Jakob, Sodenl, Lotz und Malchus, Zörich

Bousquet GH (1958) Histoire de l' économie mathématique jusqu' à Cournot. Metroeconomica X(III):121–135

Brandt K (1968) Die graphische Methode in der Nationalökonomie. In: Henn R (ed) Operations Research Verfahren, vol 5. Anton Hain, Meisenheim, pp 81–107

Cournot AA (1838) [1971] Researches into the mathematical principles of the theory of wealth (trans: Bacon N) (With an essay and a 'Bibliography of mathematical economics' by Fisher I). A. Kelley, New York

Cournot AA (1863) [1969] Principes de la théorie des richesses. Hachette, Paris [Bizzarri, Roma]

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[135] Theocharis (1993 p. 19).

Cournot AA (1877) Revue sommaire des doctrines économiques. Hachette, Paris [A. Kelley, New York]

Cournot AA (1982) Revue sommaire des doctrines économiques. Vrin, Paris

Edgeworth FY (1898) The pure theory of international values. Econ J 4:606–638

Edgeworth FY (1925) Papers relating to political economy, vol II. Macmillan, London

Fisher I (1898) Cournot and mathematical economics. Q J Econ 12:119–138

Hagen KH (1844) Die Nothwendigkeit der Handelsfreiheit für das Nationaleinkommen, mathematisch nachgewiesen. Grafe und Unzer, Königsberg

Hennings KH (1979) Karl Heinrich Rau and the graphic representation of supply and demand. Diskussionspapier. Fachbereich Wirtschaftswissenschaften Universität Hannover. Serie C, Volkswirtschaftslehre, No 35

Herschel JF (1835) Traité d'astronomie, par Sir John F.W. Herschel, traduit de l' anglais et augmenté d' un chapitre sur l' application de la théorie des chances à la série des orbites des cométes par A.- A. Cournot, Paulin, Paris

Homberg G (1971) Die Vertreter der mathematischen Nationalökonomie im deutschsprachigen Raum vor dem Erscheinen des Cournotschen Werkes (1838). Diss., Freiburg

Jaffé W (1935) Unpublished papers and letters of Léon Walras. In: Walker DA (ed) William Jaffe's Essays on Walras. Cambridge University Press, Cambridge, 1983, pp 17–35

Kater H, Lardner D (1842) Éléments de mécanique par le capitaine Kater et le Dr Lardner, traduit de l' anglais, modifiés et complétés par A.- A. Cournot, Paulin, Paris

Larson B (1999) Canard on direct exchange and taxation: a perspective on Cournot. Hist Polit Econ 31(1):377–378, Spring

Luini L (1996) Scienze naturali e scienze sociali. Le chiose matematiche di Frisi a Verri e Lloyd. In: Curzio AQ (ed) Alle Origini del pensiero economico in Italia. Il Mulino, Milano, pp 127–146

Magnan de Bornier J (2001) Magnan de Bornier on Cournot-Bertrand: a rejoinder to Clarence Morrison. Hist Polit Econ 33(1):167–174, Spring

Martin T (1998) Bibliographie Cournotienne, in collaboration with Jean – Philippe Massonie. Annales Littéraires de l' Université de Franche-Comte. Les Belles Lettres, Besancon

Ménard C (1978) La formation d' une rationalité économique: A. A. Cournot. Flammarion, Paris

Moore HL (1905a) The personality of Antoine Augustin Cournot. Q J Econ 19:370–399

Moore HL (1905b) Antoine Augustin Cournot. Rev Métaphys Morale XIIIᵉ année 521–543, mai

Moore HL (1991) Antoine Augustin Cournot (1801–1877). In: Schefold B, Moore HL, Alcouffe A, Fraysse J, Waffenschmidt W (eds) Vademecum zu einem Klassiker der mathematischen Wirtschaftstheorie. Wirtschaft und Finanzen, Düsseldorf, pp 19–33

Moret J (1915) L' emploi des mathématiques en économie politique. G. Briard & E. Brière, Paris

Rashid S (1977) William Whewell and early mathematical economics. Manchester Sch Econ Soc Stud 45:381–391

Rau KH (1841a) Grundsätze der Volkswirthschaftslehre, 4th edn. C. Winter, Heidelberg

Rau KH (1841b) (Communications); Economie politique. 1er et 2nd Extraits. Bulletin de l' Académie des Sciences et Belles – Lettres à Bruxelles VIII(pt I):145–152

Reichardt H (1954) A. A. Cournot. Sein Beitrag zur exakten Wirtschaftswissenschaften. J. C. B. Mohr, Tübingen

Robbins L (1998) A history of economic thought. In: Medema SG, Samuels W (eds) The LSE lectures (Foreword by Baumol W). Princeton University Press, Princeton

Robertson RM (1949) Mathematical economics before Cournot. J Polit Econ LVII:523–536

Roy R (1933) Cournot et l' école mathématique. Econometrica 1:13–22

Spiegel HW (1971) The growth of economic thought. Duke University Press, Durham

Theocarakis N (2006) Review: Richard Van den Berg, at the origins of mathematical economics: the economics of A. N. Isnard (1748–1803). Routledge, London 2005. Hist Econ Ideas 14(2): 147–151

Theocharis RD (1961) Early developments in mathematical economics (Foreword Lord Robbins). Macmillan, London

Theocharis RD (1983) Early developments in mathematical economics (Foreword Lord Robbins), 2nd edn. Porcupine Press, Philadelphia

Theocharis RD (1988) C. Courtois: an early contributor to cost-benefit analysis. Hist Polit Econ 20:265–273

Theocharis RD (1990) A note on the lag in the recognition of Cournot's contribution to economic analysis. Can J Econ 23(4):923–933

Theocharis RD (1993) The development of mathematical economics. The years of transition: from Cournot to Jevons. Macmillan, London

Thompson ThP (1824) The instrument of exchange. Westminst Rev 1:171–205

Tortajada R (1990) Produit net et latitude: Nicolas-François Canard. In: Faccarello G, Steiner P (eds) La Pensée Économique pendant la Révolution Française. Presses Universitaires, Grenoble, pp 151–172

Van den Berg R (2005) At the origins of mathematical economics: the economics of A. N. Isnard (1748–1803). Routledge, London [Routledge Studies in the History of Economics, No 76]

Van den Berg R, Dhesi D (2004) The equilibrium is never perfect: The dynamic analysis of C.- F.-J. d' Auxiron. Hist Polit Econ 36(1):1–29, Spring

Vatin F (1998) Économie politique et économie naturelle chez Antoine – Augustin Cournot. Presses Universitaires de France, Paris

Vatin F (1999) Book review: Thierry Martin, Bibliographie Cournotienne (1998). Eur J Hist Econ Thought 6(2):310–312

Vázquez A (1997) The awareness of Cournot's Recherches among early British economists. Res Hist Econ Thought Meth 15:115–137

Vázquez A (2002) Karl Heinrich Rau y el diagrama marshaliano. Rev Hist Econ XX(1):109–139

Vázquez A (2006) Slonimsky's view on Antoine – Augustin Cournot. Eur J Hist Econ Thought 13(4):533–541

von Buquoy G (1815) Die Theorie der Nationalwirthschaft nach einem neuen Plane und nach mehrern eigenen Ansichten dargestellt. Breitkopf & Hartel, Leipzig [(2005) Nachdruck mit einer Einleitung herausgegeben von Christos Baloglou und Bertram Schefold. G. Olms, Hildesheim]

Waffenschmidt WG (1958) Einleitung. In: Canard NF (ed) Grundsätze der Staatswirthschaft [German trans]. Kohlhammer, Stuttgart, pp 1–58

Waffenschmidt W (1991) Cournot und sein Werk. In: Schefold B, Moore HL, Alcouffe A, Fraysse J, Waffenschmidt W (eds) Vademecum zu einem Klassiker der mathematischen Wirtschaftstheorie. Wirtschaft und Finanzen, Düsseldorf, pp 57–69

Walras L (1905) Cournot et l' économie mathématique. Gazette de Lausanne 13 July, rept. in L. Walras, Mélanges d' économic politique et sociale, vol. VII of OEuvres économiques complétes d' Auguste et Léon Walras. Economica, Paris, 1987, pp 461–466

Weinberger O (1930) Mathematische Volkswirtschaftslehre. Teubner, Leipzig und Berlin

Whewell W (1829) Mathematical exposition of some doctrines of political economy 1829. Cambridge Philos Transact 3(1830):191–229