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THE NORMATIVE BIAS OF THE WALRASIAN MODEL: WALRAS VERSUS GOSSSEN*

WILLIAM JAFFÉ

Though Walras's *Eléments d'économie politique pure* is couched in the language of pure theory and appears, on the surface, as a completely *wert-frei* synoptic view of the interdependent operations of an economic system under a hypothetical regime of perfectly free competition, nevertheless the implicit moral convictions that inform the model occasionally show through—nowhere better than in Walras's theorem of maximum social satisfaction. I propose, therefore, to concentrate my attention on that theorem: showing in the first part how, for want of proper attention to Walras's moral bias, the theorem came to be misunderstood by Walras's most eminent critics, and, in the second part, how this bias was brought out into the open in Walras's analysis of Gossen's theory of maximum subjective gain from trade.

I

A common complaint against Léon Walras is that he misconceived the maximization of social satisfaction. A good example of this complaint appears in William Baumol's *Welfare Economics and the Theory of the State*, where we are told:

One of the most unfortunate bits of circular reasoning in the history of the discussion of our problem is found throughout Léon Walras' otherwise invaluable works . . . [His] argument is the standard one, that given the price of any two commodities, the best any consumer can do for himself is to buy these commodities in such proportion that the ratio between their marginal utilities to him is equal to the ratio of their prices. Up to this point, no doubt, the argument is quite valid. But when one tries to argue further that in these circumstances consumers are as well off as human design and activity can make them, subject to the current state of technique and resources, that is an entirely different matter . . . The price system, and the particular market price of each commodity, is a product of human activity, and, indeed, may be partly the result of exactly that type of consumer decision just considered. While it may be true that consumers are doing as well as possible for

* I am indebted to my colleagues, John Buttrick and Rasesh B. Thakkar, for assistance in the preparation of this paper. An earlier version was presented in 1974 at a Seminar on Walras in Tokyo under the joint auspices of Keio University and the Japanese Section of the History of Economics Society, and also at the Economics Seminar of the University of British Columbia. On both occasions I benefited from comments and criticism, and since then from helpful suggestions of the referee. Research support from the Canada Council and a Killam Senior Research Scholarship (1975–1977) is gratefully acknowledged.

themselves given the level of various prices, it does not follow that they could not do still better for themselves, for example, by altering some of these prices from what they happen to be, or do better for others (in some sense in greater degree than they harm themselves) by e.g. paying higher prices or paying more than this role would require.

It is noteworthy that chary Wicksell . . . clearly saw the difficulties in the Walrasian argument . . .¹

I start with Wicksell's discussion, which Baumol considered "masterly," and which J. R. Hicks, more than thirty years earlier, had already declared definitive.² Though Wicksell held Walras's theory of exchange and production in the highest regard, he nevertheless took strong exception to Walras's theorem of maximum social gain under free competition, calling it "wrong or at least misleading" in one place,³ "undoubtedly wrong" in another,⁴ and "quite untenable" in still another.⁵ The criticism received its most ample development in the *Lectures on Political Economy*, where Wicksell showed himself, as Baumol put it, "chary," for he was careful to emphasize the specific constraint that Walras placed upon his theorem in its first enunciation in the *Eléments*. The theorem there reads:

*The exchange of two commodities for each other in a perfectly competitive market is an operation by which all holders of either one, or of both of the two commodities [obtain, changed in second edition to] can obtain the greatest possible satisfaction of their wants consistent with the condition that the two commodities are bought and sold at one and the same rate of exchange throughout the market.*⁶

Wicksell quite rightly understood Walras's conditioning clause to mean that Walras was speaking of a relative social maximum of utility subject to the constraint of uniformity of price; and Wicksell further perceived that implicitly the maximum is delimited by "existing proprietary rights."⁷ Wicksell, however, did not make clear all that Walras meant by uniformity of price, nor did he indicate that Walras intended the condition of uniformity of price to assure the

1. Baumol (1965), pp. 60–62.

2. Hicks (1934). Toward the close of his essay published on the occasion of the hundredth anniversary of the birth of Léon Walras (hereinafter referred to as L.W.), Sir John wrote: "Even he [L.W.] did not emancipate himself entirely from that sham utilitarianism which was the bane of his contemporaries, and which led them to suppose that the working of the free market 'maximized utility' for the community as a whole" (p. 348). Wicksell's inspiration was acknowledged in a footnote to this sentence, reading, "Cf. Wicksell's final exposure of this fallacy (*Lectures*, I, pp. 73 ff.)."

3. Wicksell (1893/1954), p. 19. Here, as elsewhere in these notes, the page references are to the English translations of Wicksell's writings.

4. Wicksell (1901/1934), Vol. I, p. 75.

5. Wicksell (1913/1958), p. 167.

6. Walras (1874/. . .), §99 of the 4th and subsequent eds. of the *Eléments d'économie politique pure*, Walras's italics.

7. Wicksell (1901/1934), Vol. I, pp. 72–83.

invariance of the existing distribution of wealth (or property, as he called it) under market trading. According to Walras, it is not by violating the principle of "justice in exchange" that the injustices of the existing distribution of property are corrected, but by applying another set of principles, those of "distributive justice."⁸

For a fuller explanation of what Walras meant, we must go to his article, "Théorie de la propriété," which was first published in the *Revue socialiste* in 1896.⁹ Only in this article did Walras specify the particular type of uniformity of price he had in mind as a constraint on the social maximum. It was none other than the uniformity that emerges spontaneously and automatically in a perfect market, i.e., a market so organized that there is no viscosity in the flow of buyers and sellers toward what each conceived to be his best bargain, nor any impediment to the flow. As Walras stated it, uniformity of price is a condition such that,

if, at any given moment, there were several different prices quoted [in the same market] for the same commodity, sellers would be free to move from a point where the price was lower to a point where the price was higher and buyers would be free to take the opposite course, so that, in consequence of these movements, price would find the same level [throughout the market].¹⁰

It should not be overlooked that in the "Théorie de la propriété" Walras designated the uniformity of price thus attained "as a condition of justice."

Another of Walras's requirements for "justice in exchange" was that the price to be held uniform be a real price, not a nominal or money price.¹¹ After all, in Walras's theories of exchange, production and capital formation, in which money plays no role as a store of value, the utility to be maximized, whether on the individual scale or the social scale, relates to real goods and not to a mere medium of exchange. That, it seems to me, is the reason why Walras included in his condition of uniformity of price the stipulation that when money

8. See footnotes 15, 27, and 28. Cf. Jaffé (1975), pp. 812-13, 816, and 819-21.

9. Walras (1896), pp. 212-13. The page references both here and in the following notes are to the republished version of the "Théorie de la propriété" in Walras (1896/1936).

10. *Ibid.*, p. 212; my translation. Cf. Cournot (1838/. . .), Chapter IV, §23, note 1: "On sait que les économistes entendent par *marché*, non pas un lieu déterminé où se consomment les achats et les ventes, mais tout un territoire dont les parties sont unies par des rapports de libre commerce, en sorte que les prix s'y nivellent avec facilité et promptitude."

11. See Walras (1896), p. 212: "*L'intervention de la monnaie* ne trouble pas non plus les conditions de justice de l'échange, si la monnaie ne change pas de valeur entre le moment où on la reçoit et le moment où on la donne. La fixité ou la régularité de variation de la valeur de la monnaie d'un point à un autre au même moment, et surtout d'un moment à un autre sur un même point, est donc essentielle à l'exercice du droit de propriété" (Walras's italics).

does intervene purely and simply as a medium of exchange, its purchasing power not only be the same at every point in the market at a given moment of time, but also remain the same in the course of receiving money and paying it out to effect a desired exchange of real goods. Walras's express purpose in specifying this requirement of his condition of uniform price was to eliminate any fortuitous redistributive backwash from exchange that might otherwise result when money intervenes in the process.

All this is in the *Eléments*, though not in so many words. We need only consider Walras's "equations of exchange" representing the budget constraint in his multi-equational model of general equilibrium.¹² These "equations of exchange" assert that at uniform prices expressed in terms of a given *numéraire*, every trader ends up with a vector of assets having the same *numéraire* value as his initial vector of assets. However much a trader may benefit subjectively by an exchange yielding him maximum satisfaction within the limits prescribed by competitively determined uniform prices, the *numéraire* value of his assets remains unchanged, so that he is no richer or poorer after the exchange than he was before. In Walras's theoretical model, the only way an individual can become richer is by capital formation through saving, and the only way he can become poorer is by consuming more than his income, but never richer or poorer by exchange at equilibrium prices in a perfect market operating under the rule of free (atomistic) competition. Thus, from the standpoint of the market as a whole, exchange by itself has no distributional effects, provided always that prices are determined by free competition and are uniform.

Obviously this does not preclude an individual from becoming richer or poorer from one period to the next in consequence of social changes in tastes, technology, resources, capital, etc. in the interval. In the *Eléments d'économie politique pure*, however, Walras assumed more or less tacitly that all such variables remain constant within the period in which the static equilibrium he defined is established. Even his theory of capital formation is confined to the determination of equilibrium at the moment decisions to invest are made on the basis of pre-existing data. This is as far as Walras could go in his heroic effort to squeeze capital formation into his static model. In his pure economics, therefore, he left out of account the economic consequences of investment once carried out.¹³ Only in a paper in applied

12. Walras (1874/...), §§118, 210, and 244 of the 4th and subsequent eds.

13. See Walras (1874/...), §274 of the 4th and subsequent eds. The telling phrase L.W. used in his theory of capital formation was "quantités à fabriquer de capitaux neufs" (*op. cit.*, §252 and *passim*, my italics).

economics, "La bourse, la spéculation et l'agiotage,"¹⁴ did he discuss the sequential effects of new investment and other changes in the data. These effects, he argued, could be held to a minimum, if the practice of stock market speculation, which he defended in principle as socially beneficial, were closed by law to all but well-informed specialists, with the public in general excluded from the stock market and restricted to spot trading in reputedly established securities of "blue-chip" quality at a retail counter.

Uniformity of competitively determined price represented for Walras not only an analytic ideal, but an ethical ideal as well, constituting an indispensable pillar of social justice.¹⁵ In the "Théorie de la propriété," Walras defined justice in exchange (or "commutative justice") in terms of two conditions: first, the complete freedom of every trader to pursue his own advantage in the market; and second, the complete elimination from the market of any chance for a trader to profit by exchange at the expense of his counterpart or anyone else.¹⁶ The first condition is satisfied by the assumed perfection of the market mechanism, and the second by the stipulated universality of the budget restraint from which no trader is exempt in the Walrasian general equilibrium schema. Walras's multi-equational system of general equilibrium thus appears profoundly moralistic, at least in terms of the individualistic, bourgeois moral outlook characteristic of nineteenth-century European culture.

It was on moral grounds rather than on grounds of economic efficiency that Walras momentarily dropped his pure theory mask in §§222 and 223 of the *Eléments* to interject a strong plea for the removal of all obstacles (such as restraints of trade and monopoly practices) that stand in the way of absolutely free competition and freedom of production. This was not because he was under any illusion that free competition in a perfect market stood supreme in generating the highest possible sum total of satisfactions for society, but because

14. Walras (1880/. . .).

15. See Walras (1860), pp. 44–45: "En tant qu'êtres libres et personnels, tous les hommes sont égaux. Les personnes s'opposent aux choses; mais toute personne, en tant que personne, en vaut une autre. Ce principe sert de base à une première forme de justice, la justice *commutative*, qui a pour attribut une *balance*."

"En tant qu'ils accomplissent librement leur destinée d'une manière plus ou moins heureuse ou plus ou moins méritoire, il se révèle chez les hommes des différences d'aptitudes, de talent, d'application, de persévérance, de succès qui les font inégaux; et cette inégalité est le fait sur qui se fonde la justice *distributive*, laquelle a pour symbole une *couronne*."

"*Egalité des conditions; inégalité des positions*, telle est alors la véritable formule sociale . . ."

16. See Walras (1896), p. 212: "Il s'agit à présent de passer du troc à l'échange économique en introduisant sur le marché plusieurs échangeurs concurrents, avec la condition qu'il n'y aura pour tous qu'un prix unique . . . et de savoir si les conditions ne favorisent ni les acheteurs ni les vendeurs au détriment les uns des autres."

it was a system superlatively designed to eliminate profit both from exchange and from production. He was far from condemning the pursuit of profit, which, in fact, he regarded as necessary for the working of his system, since profits and losses serve as indicators to direct the movement of entrepreneurs from branches of production yielding less utility to branches yielding more. He counted, however, on free competition in an ideally perfect market to perform automatically the moral function of stamping out profits¹⁷ that are something for nothing, unearned, the fruit of guile and antisocial conspiracy, and therefore unjust.

The constraint of zero profits (and zero losses) is implicit in Walras's second formulation of his theorem of maximum social utility, now amplified to read:

*Production in a market ruled by free competition is an operation by which services can be combined and converted into products of such a nature and in such quantities as will give the greatest possible satisfaction of wants within the limits of the double condition: [1] that each service and each product have only one price in the market, namely the price at which the quantity supplied equals the quantity demanded, and [2] that the selling price of the products be equal to the cost of the services employed in making them.*¹⁸

Here the first condition, requiring the uniform price to be the equilibrium price, was designed to obviate untoward distributional effects. If any transaction or partial transaction were allowed to take place at other than the equilibrium price, let us say in the course of a realistic *tâtonnement* process via firm contracts, i.e., without the use of Walras's imaginary "tickets" ("*bons*"), then one of the parties to the contract might find himself richer in terms of *numéraire* than he would have been had the price been the equilibrium price, and the other party poorer,¹⁹ in neither case as a result of saving or dissaving. The second condition, equating selling price to cost, is obviously synonymous with zero profit zero loss. In that case also, production is followed by no distributional effects either in favor of or against any entrepreneur.

I believe Walras had the same elimination of distributional effects in mind in his discussion of the theory of two-person, two-commodity barter, which he attributed to Jevons.²⁰ Walras envisaged the encounter of two prospective "Jevonsian" barterers as starting out with a proposal by one of them of a trial price and then turning into a

17. Walras (1874/. . .), §188 of the 4th and subsequent eds.

18. *Ibid.*, §221.

19. Jaffé (1967), pp. 2-3.

20. Walras (1896), pp. 207-09. L.W. interpreted Jevons's theory of exchange between two "trading bodies" as a theory of barter between two individuals. See Jevons (1871/. . .), pp. 95-101, corresponding to pp. 139-44 of the 1970 ed.

haggling contest, if, at the price first proposed, the quantities reciprocally offered and demanded of the two commodities did not match, but without any actual exchange of commodities intervening until the quantities demanded and offered became equal. "Alors le troc s'effectue," which means that only then does any effective exchange take place.

Walras was convinced of the theoretical possibility of the same sort of suppression of distributional effects in multiperson market trading, provided that the markets are appropriately organized. In §42 of the *Eléments*, he took the stock market as the closest approximation in the real world to his theoretically perfect market in order to illustrate how equilibrium price is established in reality. He described how the price of a given security is bid up or down in case of excess demand or excess offer until the excess is wiped out. In the second edition of the *Eléments* he added that on the appearance of excess demand, "Theoretically, trading should be suspended" ("Théoriquement l'échange doit être suspendu"); and on the appearance of excess offer, "Suspension of trading" ("Suspension de l'échange").²¹ If no trading is allowed at "false prices" in real markets, there too the possibility of distributional effects is eliminated.

When these clarifications of Walras's constraint on his theorem of maximum social utility are taken into account, Wicksell's critique of the theorem loses much of its force. The critique, which Hicks and Baumol echoed, assumed that Walras intended his theorem to state that, subject to uniformity of price, free competition yields a higher social maximum of satisfaction than any other system of price determination could achieve. I can find no evidence of this either in the *Eléments* or in any other of Walras's writings.

To be sure, at the threshold of his career as a theorist, Walras believed that the prices and quantities of goods produced under free competition are the best possible—until he was persuaded by Lambert Bey, a highly respected Saint-Simonist and engineer, that no economist had ever furnished proof of such a proposition. Wicksell, who cited Walras's account of the incident to set the tone of his critique, unfortunately misread the issue.²² He thought that Lambert Bey had made an "onslaught on the foundations of free trade theory" viewed as a system for procuring the best possible "exchange values." This, however, is not what Walras reported. According to Walras,

21. Walras (1874/. . .), §42 of the 4th and subsequent eds. See also p. 566 of the English translation, Collation Note [a] to Lesson 5.

22. Wicksell (1901/1934), Vol. I, pp. 73–74. For L.W.'s account of the Lambert Bey incident, see Walras (1898/1936), pp. 466–67.

Lambert Bey had merely argued that the case for free competition had not been made. Walras, though forced to agree, then and there conceived the youthful ambition to furnish the missing proof himself—only to encounter soon afterwards an insurmountable obstacle. His father, Auguste Walras, who had wrestled with the same sort of problem in his own way, pointed out to Léon that the satisfactions enjoyed by different persons are incommensurable and therefore cannot be aggregated.²³ Walras's mature formulation of his theorem of maximum social utility under free competition shows how well he learned his father's lesson.

Wicksell, failing to see this, got off on the wrong foot and misdirected his critique. At the outset, he properly chided Walras for not perceiving that in the case of multiple equilibria, the several equilibrium positions could not "simultaneously represent positions of maximum satisfaction,"²⁴ but made more of this oversight than it deserved. According to Walras's "remarque" at the end of §156 of his *Eléments*, "... multiple current equilibrium prices, which, as we have seen . . . , were perfectly possible in the case of exchange of two commodities, are, in general, not possible, in the case of exchange of several commodities one for another." As Walras's treatment of the two-commodity case appears to be little more than a preliminary exercise to prepare the way for his theory of the multicommodity case, which Walras imagined did not admit of "multiple equilibrium prices," his failure to probe the utility implications of multiple equilibria in the two-commodity case, though an analytical fault, is of trivial significance.

After this sally, Wicksell brought up his really heavy analytical artillery to complete the demolition of Walras's theorem; and he succeeded perfectly in demolishing a theorem that was not Walras's at all. The attack was misdirected because Walras had not assumed that the subjective advantages and disadvantages to different persons are commensurable or addable. Hence it was beside the point for Wicksell to argue, in opposition to Walras's theorem, that since uniform monopoly or administered prices could certainly procure for favored individuals more satisfaction than they could have obtained from competitive equilibrium prices, though everyone else might be worse off, a system of prices can always be found at which exchanges will produce a larger sum of utility for all individuals taken together than can be produced at competitive prices.²⁵ *Quis negavit?* Certainly

23. Jaffé (1972), p. 389.

24. Wicksell (1901/1934), Vol. I, p. 75.

25. *Ibid.*, pp. 79–80.

Walras never denied this. On the contrary, when we come to Walras's analytical discussion of Gossen, we shall see that Walras affirmed it; but for other than economic reasons, he refused to consider Gossen's maximum as beneficial to society.

Baumol was even more severe than Wicksell in his criticism. We have seen that he denounced Walras's theorem as "one of the most unfortunate bits of circular reasoning in the history of the discussion of our problem." Circular, yes; unfortunate, no. I do not think Walras intended it to be other than circular, for the simple reason that his propositions bearing on maximization of social satisfaction subject to specified constraints are, I should say, essentially definitional, like Pareto's concept of optimality for that matter. In my view, Walras's relative maximum of social satisfaction is nothing more and nothing less than an anticipation of Pareto optimality, with the same virtues and the same defects. The mathematical prelude to Walras's propositions simply demonstrate the internal consistency of the definition—and this demonstration has never been challenged, not even by Walras's severest critics.

The merits of any definition depend not only on its internal consistency, but also on its usefulness for solving a given problem or set of problems. Once we identify Walras's problem as, in essence, a conceptual problem, then his "stipulative definition"²⁶ of a relative maximum of social satisfaction falls into place, however unsuitable the definition may be for other purposes. Walras's aim, even in his "pure economics," was prescriptive or normative rather than positive or descriptive. His object was to formulate an economic system in conformity with an ideal of social justice²⁷ compatible with the inexorable exigencies of man's sentient nature and his environment.²⁸ We may quarrel with the para-economic purpose Walras had in mind and disagree with his natural law philosophy of social justice. That,

26. Hanson (1969, pp. 27–28) distinguished between "historical-authoritative" definitions that are factual reports of usage, and "stipulative-prescriptive" definitions that are arbitrary inventions specially designed for use in a proposed context.

27. L.W. expounded his ideal of social justice in his "Recherche de l'idéal social," a series of public lectures he delivered in Paris in 1867–1868 (Walras 1896/1936), pp. 25–202). These lectures attest to L.W.'s indebtedness to his father, Auguste Walras, for his social philosophy.

28. For L.W.'s views on the relation between social ethics and natural necessity, see Walras (1860), where we read, for example, "Ainsi, la théorie très ingénieuse de l'application de l'algèbre à la géométrie et les deux exemples cités ci-dessus, auxquelles on en pourrait ajouter mille autres, démontrent assez que c'est à la morale qu'il appartient de se subordonner aux sciences naturelles, sinon dans son principe, au moins dans ses applications, et que toute morale qui se permettrait de contredire la théorie du carré de l'hypoténuse, les lois de la réfraction, le fait de la circulation du sang, ou les résultats de la théorie de la valeur d'échange, serait une morale ridicule et caduque" (p. 32).

however, is another story, which has nothing to do with the formal properties of Walras's ideologically directed argument.

Wicksell's and Baumol's criticisms leave one with the impression that Walras fell into a "wrong or at least misleading" theory of maximum social satisfaction because he did not know any better. As far back as 1885, Wilhelm Launhardt held the same poor opinion of Walras's performance on that score. He called it a "serious error" for Walras to hold that, as Launhardt put it, "what is generally the best is most surely achieved by the natural operation of the force of free competition, by 'laissez faire, laissez passer.'" ²⁹ When this passage was translated from the German for Walras, he wrote immediately to Launhardt, on May 20, 1885, protesting in the following terms:

I stated that free competition procures maximum effective utility *within the limits of the condition of uniformity of price*, in other words a relative, not an absolute, maximum. It is perfectly clear that if one supposes commodities to be sold at a high price to the rich and at a low price to the poor, all that the rich will be compelled to forego are luxuries, while the poor will be able to afford necessities, and [consequently] there will be a considerable increase in effective utility . . . It remains, however, to be seen whether uniformity of price is required by justice. This is a question which falls outside the purview of pure economics, but which I shall be very careful to examine in that part of our science which deals with the distribution of wealth and in which considerations of social ethics must enter. All I can say for the present, and I think you will agree, is that the object is not an absolute maximum of economic welfare, but a maximum economic welfare *compatible with justice*.³⁰

Pareto, unlike Launhardt, saw perfectly clearly what Walras was driving at and did not like it. He wrote to Pantaleoni, on December 19, 1908, apropos of Walras's theoretical aims, ". . . I do not agree that pure economics *demonstrates* how events ought to occur . . . ; I find it inadmissible to study what *ought* to be rather than what is . . ." ³¹

29. Launhardt (1885/1963), p. 30; my translation. Paradoxical as it may seem in view of his own criticism of L.W.'s theorem of maximum social satisfaction, Wicksell rejected Launhardt's interpretation and correction of the theorem (Wicksell (1893/1954), p. 76, note 2).

30. Jaffé (1965), Vol. 2, pp. 49-51, Letter 652; my translation of the excerpts cited; Walras's emphasis.

31. De Rosa (1960), Vol. III, p. 121, Letter 590; my translation of passages from the following sentence: "Per riguardi personali non ho mai detto che dal Walras ho solo preso il concetto dell'equilibrio economico *in un caso particolare*; che non accetto in nessun modo il suo modo metafisico di trattare la scienza; che non posso trovare buono che adoperi il termine *rareté* ora in un senso ora in un altro, traendo in inganno il lettore; che non ammetto che ci sia, come dice lui, un metodo razionale superiore al metodo sperimentale; che non ammetto che l'economia pura *dimostri* come debbono seguire i fatti, mentre è l'inverso; che non accetto di studiare ciò che *deve* essere, ma che invece studio ciò che è; che è da bambini figurarsi che si dimostra colle formole dell'economia pura la convenienze per lo Stato di ricomprare le terre, di stabilire il bimetallismo, ecc. ecc."

II

More important than Walras's own declarations of purpose or the acute observations of others on the normative character of his "pure economics" is his analysis of Gossen's theory of maximum social gain from exchange. This analysis first appeared in §160 of the second edition of Walras's *Eléments*³² and was later expanded in his "Théorie de la propriété" already alluded to above.

In both places Walras directed his attention to Gossen's proposition that in two-person, two-commodity barter the greatest sum total of "happiness" ("*Genuss*," as Gossen called it) is achieved when the "value" ("*Werth*," in the sense of utility) of the last atom of each commodity exchanged is the same for one barterer as for the other.³³ This proposition is, like Walras's definition, essentially "stipulative," but in Gossen's case it is the definition of a maximum maximorum subject only to the real social budget constraint, that is, subject to the limitation in the quantities of goods available as compared with the quantities wanted for perfect bliss in the whole economic system. Walras translated Gossen's verbal statement into a mathematical proposition that can be derived and generalized as follows with the aid of a simpler notation than Walras employed.

Consider, as Gossen does, a universe of two commodities, (A) and (B), and two barterers, Messrs. 1 and 2. Let the initial endowments of commodities (A) and (B), which are designated by the first subscript, be

$$\bar{x}_{a1}, \bar{x}_{b1} \quad (\text{for Mr. 1})$$

$$\bar{x}_{a2}, \bar{x}_{b2} \quad (\text{for Mr. 2}).$$

Let the corresponding (unknown) amounts of (A) and (B) to be held on completion of Gossen's ideal barter be

$$x_{a1}, x_{b1} \quad (\text{for Mr. 1})$$

$$x_{a2}, x_{b2} \quad (\text{for Mr. 2}).$$

32. It was only in the interval between the first and second eds. of the *Eléments* that L.W. learned of the existence of Gossen's *Entwicklung*. See Walras (1874/. . .), p. 580 of the English translation, Collation Note [c] to Lesson 16.

33. See Gossen (1854/1927), p. 85. "*Es muss jeder der beiden Gegenstände nach dem Tausche unter A und B der Art sich vertheilt finden, dass das letzte Atom, welches jeder von einem jeden erhält, beiden gleich grossen Werth schafft.* Es ergibt dieses die einfache Betrachtung, dass bei jeder andern Vertheilung die Uebertragung des Atoms, welches bei dem einen geringern Werth schafft, auf den andern, bei diesem grössern Werth, also in Summa mehr Werth hervorbringen würde, und es folgt denn hieraus, dass, um dieses Grösste zu erreichen, in der Regel ungleiche Quantitäten gegen einander zu vertauschen sind."

Now, assuming with Gossen as with Walras that the utilities of the two commodities are independent, we may write the utility function of the j th individual ($j = 1, 2$) as

$$(1) \quad U_j(x_{aj}, x_{bj}) = \Phi_{aj}(x_{aj}) + \Phi_{bj}(x_{bj}).$$

Since these functions are twice differentiable, separable, and addable not only in the intercommodity sense but also (for Gossen, though not for Walras) in the interpersonal sense, we may write Gossen's social utility function of the two persons together as

$$(2) \quad U = U_1 + U_2 = \sum_{i=a}^b \Phi_{i1}(x_{i1}) + \sum_{i=a}^b \Phi_{i2}(x_{i2}).$$

Function (2) is a mathematical representation of the "happiness" that Gossen wanted to maximize. The maximization is, however, subject to the constraint that the respective total quantities of (A) and (B) remain constant. Without production (or destruction) of either commodity, Gossen's maximization of (2) is thus subject to the social budget constraint, which may be written as

$$(3) \quad \begin{aligned} x_{a1} + x_{a2} &= \bar{x}_{a1} + \bar{x}_{a2} \\ x_{b1} + x_{b2} &= \bar{x}_{b1} + \bar{x}_{b2}. \end{aligned}$$

To prove Gossen's theorem of maximum social "happiness," we need only form the Lagrangean function

$$(4) \quad \begin{aligned} L(x_{a1}, x_{a2}, x_{b1}, x_{b2}, \lambda_a, \lambda_b) &= \sum_{i=a}^b \Phi_{i1}(x_{i1}) + \sum_{i=a}^b \Phi_{i2}(x_{i2}) \\ &\quad + \lambda_a(x_{a1} + x_{a2} - \bar{x}_{a1} - \bar{x}_{a2}) \\ &\quad + \lambda_b(x_{b1} + x_{b2} - \bar{x}_{b1} - \bar{x}_{b2}), \end{aligned}$$

and set the first-order partial derivatives of the function with respect to the x 's and the indeterminate multipliers, λ_a and λ_b , equal to zero. Among the six resulting equations, we have

$$(5.1) \quad d\Phi_{a1}/dx_{a1} = d\Phi_{a2}/dx_{a2}$$

$$(5.2) \quad d\Phi_{b1}/dx_{b1} = d\Phi_{b2}/dx_{b2}.$$

It should be noticed that the solution given by equations (5.1) and (5.2) is much stronger than the usual (Walrasian) solution given below in (6), which merely asserts that the *ratio* of the marginal utilities of the two commodities is the same for the two individuals. Such as it is, Gossen's solution was restated by Walras in terms, but for the notation, of the above equations (5.1) and (5.2), in order to contrast

it with his own solution. In the fourth and definitive editions of the *Eléments*, Walras concluded significantly:

It is obvious that the maximum utility arrived at in this [Gossen's] way is not the relative maximum of free competition, nor is it compatible with the condition that all traders voluntarily exchange the two commodities at a common and uniform ratio. It is rather an absolute maximum which disregards the [two-fold] condition of uniformity of price and equality between effective offer and effective demand at that price, thus abolishing [private] property.³⁴

Walras's inference that Gossen's solution is incompatible with the right to private property was elucidated geometrically in the "Théorie de la propriété."³⁵ There, with the aid of four graphs based on Gossen's Figure 20,³⁶ Walras graphically juxtaposed his own (as well as Jevons's)³⁷ theorem of maximum satisfaction in two-person, two-commodity barter with that of Gossen in order to bring out the difference in their respective distributional effects. Walras's diagram is here reproduced as Figure I with the original axes transposed and the lettering changed to conform to the analytical notation used above. The four curves depict the independent marginal utility functions of commodity (A) and commodity (B) for Messrs. 1 and 2 between whom there is an exchange of the two commodities. Walras assumed that Mr. 1's endowment consists of the $\overline{0x_{a1}}$ units of (A) shown in graph i and none of (B) as the absence of an $\overline{0x_{b1}}$ segment in graph iii attests; while Mr. 2's original endowment consists of the $\overline{0x_{b2}}$ units of (B) shown in graph iv and none of (A), whence the absence of an $\overline{0x_{a2}}$ segment in graph ii.

In the Walras (and Jevons) schema, the barterers are supposed to haggle, without, however, effecting any exchange until they finally agree upon a mutually acceptable ratio of exchange or price of (B) in terms of (A), $p_{b,a}$, say 2. Then the whole exchange takes place in a single act in conformity with Jevons's "law of indifference." Thus at $p_{b,a} = 2$, Mr. 1 trades $\overline{x_{a1}x'_{a1}}$ of (A) for $\overline{0x'_{b1}}$ of (B), and Mr. 2 trades $\overline{x_{b2}x'_{b2}}$ of (B) for $\overline{0x'_{a2}}$ of (A). The price $p_{b,a}$ is the "market" equilibrium price determined by equality between the quantities offered and demanded, which are in turn determined by the utility maximization drive of each barterer, separately subject to his original en-

34. Walras (1874/. . .), §162 of the 4th and subsequent eds.; my italics. For changes in this statement from ed. 2 to ed. 4 of the *Eléments*, see English translation, p. 580, Collation Note [d] to Lesson 16. My translation here is slightly more literal than the version found in the *Elements of Pure Economics*. In ed. 4, L.W. inserted a footnote to this passage, reading: "Voyez *Etudes d'économie sociale. Théorie de la propriété*."

35. Walras (1896), pp. 210–11.

36. Gossen (1854/1927), p. 83.

37. See footnote 20.

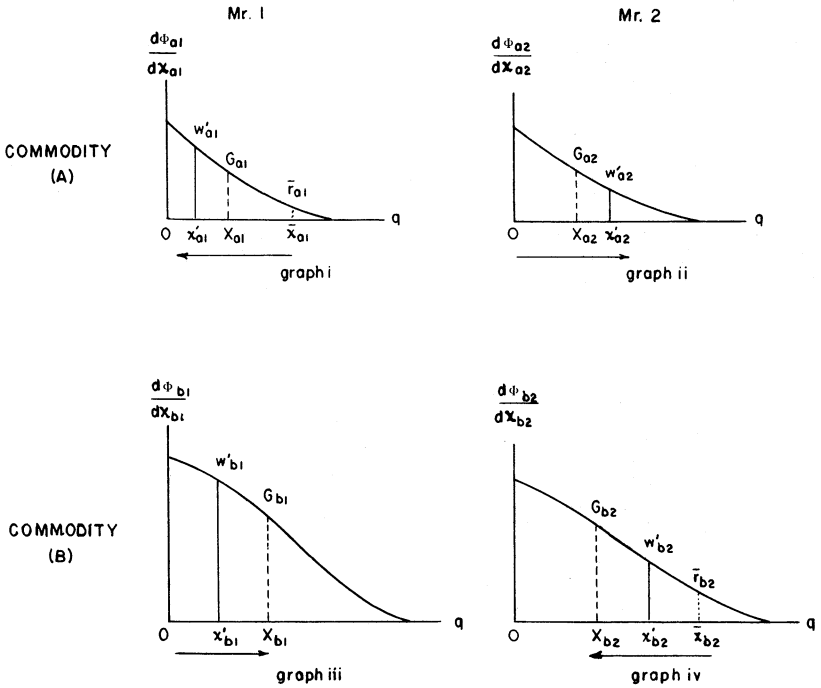


FIGURE I

downment. As is well-known, this simultaneous maximization result is defined analytically by

$$(6) \quad \frac{d\Phi_{b1}/dx_{b1}}{d\Phi_{a1}/dx_{a1}} = \frac{d\Phi_{b2}/dx_{b2}}{d\Phi_{a2}/dx_{a2}} = \frac{e_a}{e_b} = p_{b,a} = 2,$$

where e_a and e_b are, respectively, the quantities of (A) and (B) exchanging hands. These quantities are defined in Figure I as

$$e_a \equiv \bar{x}_{a1}x'_{a1} \equiv 0x'_{a2}$$

and

$$e_b \equiv \bar{x}_{b2}x'_{b2} \equiv 0x'_{b1}.$$

Graphically, then, the Walras-Jevons condition of constrained maximum, stated in equations (6), appears as

$$\frac{x'_{b1}w'_{b1}}{x'_{a1}w'_{a1}} = \frac{x'_{b2}w'_{b2}}{x'_{a2}w'_{a2}} = \frac{\bar{x}_{a1}x'_{a1}}{\bar{x}_{b2}x'_{b2}} = p_{b,a} = 2,$$

where

$$x'_{b1}w'_{b1} \neq x'_{b2}w'_{b2} \text{ and } x'_{a1}w'_{a1} \neq x'_{a2}w'_{a2},$$

except by chance.

In Gossen's schema, though the two individuals have the same initial endowments and the same marginal utility functions as in the Walras schema, they do not haggle nor do they really barter. They behave as though they threw their initial endowments into a common pool either voluntarily or under command in order that these endowments might be divided between them in such a manner as to maximize *the sum total of their utilities added together*. As we have seen analytically in equations (5.1) and (5.2), this entails a division of (A) and (B) between Mr. 1 and Mr. 2 such that, graphically now, $\overline{X_{a1}G_{a1}} = \overline{X_{a2}G_{a2}}$ and $\overline{X_{b2}G_{b2}} = \overline{X_{b1}G_{b1}}$, these marginal utilities being, for Gossen, interpersonally commensurable. Out of their respective initial endowments, Mr. 1 gives up $\overline{x_{a1}X_{a1}}$ ($= \overline{OX_{a2}}$) to Mr. 2; and Mr. 2 gives up $\overline{x_{b2}X_{b2}}$ ($= \overline{OX_{b1}}$) to Mr. 1. Since the final outcome is independent of the distribution of initial endowments between the two individuals, Walras saw in the Gossen division a total disregard of individual property rights.

It is evident that with both $\overline{OX_{a2}} < \overline{0x'_{a2}}$ and $\overline{OX_{b2}} < \overline{0x'_{b2}}$, Mr. 2 is worse off than he would have been had he traded freely at the assumed "market" equilibrium price of 2 of (A) for one of (B); and Mr. 1 with both $\overline{OX_{a1}} > \overline{0x'_{a1}}$ and $\overline{OX_{b1}} > \overline{0x'_{b1}}$ is better off. Moreover, in the particular example under consideration, Mr. 2's assets after the Gossen division are less than they would have been after a Walras-Jevons barter exchange, the final assets in the two situations being valued in *numéraire* at the same equilibrium "market" price $p_{b,a} = 2$:

$$\overline{OX_{a2}} + 2 \overline{OX_{b2}} < \overline{0x'_{a2}} + 2 \overline{0x'_{b2}};$$

and Mr. 1's assets are greater:

$$\overline{OX_{a1}} + 2 \overline{OX_{b1}} > \overline{0x'_{a1}} + 2 \overline{0x'_{b1}}.$$

This amounts to a transfer of assets from Mr. 2 to Mr. 1, which Walras denounced as incompatible not only with justice in exchange, but also with property rights, especially if the Gossen division is mandatory.

In conclusion, it is worth remembering that from his very first book on economics, *L'économie politique et la justice* published in 1860 to his last utterance on the subject at his jubilee in 1909,³⁸ Léon

38. Walras (1909).

Walras's dominant preoccupation was with the problem of social justice. By osmosis, as it were, this paramount preoccupation passed through the partition Walras himself erected to separate his normative economics ("économie sociale") from his pure economics.³⁹ Thus Walras's pure economics became imbued with a distinctive moral content and was given a normative direction. Walras's latent purpose in contriving his general equilibrium model was not to describe or analyze the workings of the economic system as it existed, nor was it primarily to portray the purely economic relations within a network of markets under the assumption of a theoretically perfect regime of free competition. It was, as Pareto perceived, rather to demonstrate the possibility of formulating axiomatically a rationally consistent economic system that would satisfy the demands of social justice without overstepping the bounds imposed by the natural exigencies of the real world. It is to the great credit of Walras as an economic analyst that in formulating this system, he invented a model that proved to be eminently serviceable to later economists who were more positivistic in their approach than Walras or who were inspired by different social goals. While it is undeniable, even when due account is taken of Walras's underlying purpose, that his model can still be shown to exhibit flaws and serious limitations that are open to criticism, it is not justifiable, in my opinion, to impugn as technical defects the inadequacies of the model for dealing with other problems than those it was originally designed to solve.

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39. Jaffé (1965), Vol. I, pp. 208-12, Letter 148.

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