The Rise and Fall of Walrasian General Equilibrium Theory: The Keynes Effect

D. Wade Hands
Department of Economics
University of Puget Sound
Tacoma, WA USA
hands@pugetsound.edu
Version 1.3
September 2009
15,246 Words

This paper was initially presented at The First International Symposium on the History of Economic Thought "The Integration of Micro and Macroeconomics from a Historical Perspective," University of São Paulo, São Paulo, Brazil, August 3-5, 2009.

Abstract: Two popular claims about mid-to-late twentieth century economics are that Walrasian ideas had a significant impact on the Keynesian macroeconomics that became dominant during the 1950s and 1060s, and that Arrow-Debreu Walrasian general equilibrium theory passed its zenith in microeconomics at some point during the 1980s. This paper does not challenge either of these standard interpretations of the history of modern economics. What the paper argues is that there are at least two very important relationships between Keynesian economics and Walrasian general equilibrium theory that have not generally been recognized within the literature. The first is that influence ran not only from Walrasian theory to Keynesian, but also from Keynesian theory to Walrasian. It was during the neoclassical synthesis that Walrasian economics emerged as the dominant form of microeconomics and I argue that its compatibility with Keynesian theory influenced certain aspects of its theoretical content and also contributed to its success. The second claim is that not only did Keynesian economics contribute to the rise Walrasian general equilibrium theory, it has also contributed to its decline during the last few decades. The features of Walrasian theory that are often suggested as its main failures – stability analysis and the Sonnenschein-Mantel-Debreu theorems on aggregate excess demand functions – can be traced directly to the features of the Walrasian model that connected it so neatly to Keynesian macroeconomics during the 1950s.

> *We may digress to point out that the general point of view and habit of mind reflected in the Hicks-Slutzky analysis has wide ramification in recent literature and has led to utter confusion in the whole body of economic thought. We refer, of course, to the huge corpus of discussion beginning with Keynes's General Theory and following the lead of that work. (Knight, 1944, p. 300)*

0. Introduction

Pronouncements of the death of Walrasian microeconomics have become commonplace in recent years. For an increasing number of economists, the research program that was once the discipline's showpiece of rigor and technical sophistication "has finally run out of gas" (Rizvi, 1998, p. 274) and should be moved from the front lines of economic research to the back burner of retrospective reflection (Bowles and Gintis, 2000). In many cases the target for the narrative of demise is narrowly focused – the Arrow-Debreu version of Walrasian general equilibrium theory – and in such cases the story is usually that it succumbed to a host of internal technical difficulties: particularly those associated with stability analysis and the Sonnenschein-Mantel-Debreu (SMD) theorems on excess demand functions (Kirman 1989, 2006, Rizvi 1998, 2003). In other cases the target is much broader – neoclassical economics or rational choice theory in general – and here the downfall is generally associated with the theory's poor empirical track record and recent developments within research programs such as behavioral economics, experimental economics, and the economics of complexity (Colander 2000, 2006; Colander, Holt and Rosser 2004a, 2004b, Davis 2006, 2008). Commensurate with, although relatively independent of, these narratives about the fall of Walrasian microeconomics, a body of historical literature has developed during the last few decades which gives us a deeper understanding of the various forces that contributed to the Walrasian rise to dominance and how the resulting theory came to take the particular form that it did. A few of the many books covering aspects of this recent historical literature include Amadae (2003), Giocoli (2003), Ingrao and Israel (1990), Mirowski (2002), and Weintraub (1985, 1991, 2002), but the relevant research is quite extensive and this is only the tip of the iceberg.[1]

[1] In addition to books of course, the history of general equilibrium theory, and mid-twentieth century microeconomics more generally, has received extensive discussion in history of economic thought journals,

This paper will also discuss the rise, and to a lesser extent the fall, of Walrasian general equilibrium theory, but it will focus on an aspect of the story that has received very little attention: Keynesian economics. Of course there already exists an extensive literature on the relationship between the Walrasian and Keynesian research programs. For example, the histories of macroeconomics offered by the Cambridge-centered critics of IS-LM Keynesianism that Alan Coddington (1983) labeled the "Fundamentalist Keynesians" (Robinson 1975, Pasinetti 2007), clearly emphasize the relationship between Walrasian and Keynesian economics. They argue, as do post-Keynesians of a variety of stripes, that Walrasian ideas – initiated by John R. Hicks's original IS-LM paper (Hicks 1937) – influenced, and ultimately corrupted, the central message of John Maynard Keynes's General Theory (1936).[2] The economists Coddington labeled "Reconstituted Reductionists" (Clower 1965, Leijonhufvud 1968) have a different take, but they too have drawn attention to, and criticized, the Walrasian influence on textbook Keynesianism. But identifying the Walrasian imprint on standard Keynesian theory is not exclusive to those who would call themselves Keynesians. Milton Friedman's Marshallianism was associated in part with his identification and criticism of Walrasian theoretical influences within Keynesian macroeconomics (see DeVroey 2009, Hoover 1988, or Mayer 2009 for example). Finally, even Hicks himself, when explaining the origins of the IS-LM model, stressed the influence of Walrasian ideas on the "Keynesian" theoretical framework he set in motion: "the idea of the IS-LM diagram came to me as a result of the work I had been doing on three-way exchange, conceived in a Walrasian manner" (Hicks, 1980-81, p. 142).

So there clearly is an extensive literature on the relationship between Walrasian and Keynesian economics, but none of it really focuses on the issues I will examine here. All of these authors, and most others who have examined the relationship between Walrasian and Keynesian theory, have directed the explanatory arrow from Walras to Keynes. The two main questions have traditionally been: How did Walrasian ideas influence, condition, or possibly determine, what came to be the standard textbook Keynesian theory? and, Was that Walrasian influence a good thing or a bad thing (with respect to either the scientific adequacy of the resulting theory or its fidelity to Keynes's own thinking)? My focus will be quite different. First and most importantly, I will run the

---

Handbooks and Companions, and to some extent mainstream journals in economic theory. A number of History of Political annual conferences have also focused on aspects of the story.

[2] As Coddington put it: "what Hicks was supposed to have done was to have taken the pristine work of Keynes's General Theory and, via a kind of Walrasian sleight of hand, transformed the profound and intellectually subversive message into something innocuous, insipid and even lifeless" (1983, xi).

explanatory arrow in the opposite direction: from Keynes to Walras. I want to explain not how Walrasian ideas played a role in shaping what became standard textbook Keynesian macroeconomics, but rather how Keynesian ideas played a role in shaping what came to be the standard textbook Walrasian microeconomics. Secondly, my interest will be more explanatory than evaluative; I will focus on identifying influences and explaining the profession's theoretical preferences, not on evaluating whether those influences and preferences were scientifically a good thing or a bad thing.

The paper is organized in the following way. The first section lays out some definitions and presuppositions relevant to the overall discussion. Given that the argument cuts across such a wide swath of time, individuals, and ideas, it is useful to be clear right up front how important terms will be used and to point out some of the things that will be taken as givens in the paper. The second section is the heart of the argument and the paper's main contribution. This section argues that Keynesian ideas played a role in the Walrasian rise to dominance and also contributed to the form and content of the particular "Walrasian" theory that ultimately emerged. The neoclassical synthesis was not, as it is often presented, just a case of Walrasian and Keynesian ideas coming together in a way that influenced the character of the latter; it was in fact a two-way street with "influence" flowing both ways. Mid-twentieth century versions of both "Keynesian macro" and "Walrasian micro" were joint products of the neoclassical synthesis. The third section discusses the connection between the neoclassical synthesis and the fall of Walrasian economics. The final section contains a brief summary and a review of the main themes of the paper.

1. A Few Presuppositions and Stage-setting for What Follows

I will talk about "Walrasian economics" and "Keynesian economics" as if they were research programs that can clearly be distinguished from other theoretical frameworks in economics and are sufficiently stable to be identified, and re-identified, across various points in time. Although I do assume that both research programs contain certain hard core propositions/conceptualizations, this does not mean that I have captured the "essential nature" of these programs, or that such an essence even exists. The hard core propositions are simply empirically identifiable features of a particular sort – reliable identifiers of family resemblance – and their stability is always subject to particular time constraints. They are not essential in any deep philosophical sense, they do not last forever, and like all empirical observations, they require a trained eye, but they are sufficiently persistent and identifiable for me to talk sensibly about Walrasian and Keynesian economics. For example, Hicks's Value

and Capital (1946) is very different from Kenneth Arrow and Frank Hahn's General Competitive Analysis (1971), and there is no reason to believe that either captures the true essence of the Walrasian system, yet they do both contain common, identifiable, propositions/concepts that are present in "Walrasian" economic models and not present in models grounded in other economic research programs; similar things can be said for various renditions of Keynesian economics. The mathematical character of the Walrasian program may constrain intra-programmatic variation to a greater degree than it is constrained within Keynesian economics, but if that is the case it is only a difference of degree, not of kind. Roy Weintraub's six hard core propositions (Weintraub, 1985, p. 109) do a reasonably good job identifying the core of the Walrasian research program, and the core of the Keynesian program would include propositions such as: the short run aggregate level of output and employment is determined by aggregate expenditure, the interest rate is determined by the supply of and demand for liquidity, the marginal propensity to consume is positive but less than one, etc. Notice that accepting such hard core propositions – reliable identifiers of family resemblance – leaves a lot of room for variation and debate. It may be possible to identify some hard core propositions associated with Christianity or Marxism – but history has demonstrated that this still leaves a lot of room for variation and internecine strife – so too for Walrasian and Keynesian economics.

It is also useful to identify two presumptions about the history of twentieth century economics that will be assumed throughout the discussion. Both seem relatively uncontroversial, but it is useful to state them explicitly since they are taken as given in all of what follows. The first is that mainstream economics was dominated by the neoclassical synthesis from sometime during the mid-1950s until roughly the mid-1970s.[3] The neoclassical synthesis was a product of contributions by a number of different economic theorists – key texts in the following discussion include Hicks (1937, 1946), Lange (1944), and Samuelson (1947) – and although there were clearly differences among the various contributors, one of the main impacts of the synthesis was that the discipline came to be seen as an amalgam of two separate, but related, consistent, and completely non-antagonistic parts: macroeconomics and microeconomics.[4] As Paul Samuelson put it in the sixth edition of his famous Economics textbook: "the economist is now justified in saying that the broad cleavage between microeconomics and macroeconomics

[3] This was primarily an Anglo-American phenomenon, but given the historical context of the immediate post World War II period, it came to characterize mainstream economics more generally.

[4] Note the "neoclassical synthesis" here and throughout refers to the original neoclassical synthesis and not the "new neoclassical synthesis" of dynamic stochastic general equilibrium (DSGE) models (Clarida, Gali, and Gertler 1999, Goodfriend and King 1997).

has now been closed" (Samuelson, 1964, p. 361). The macroeconomics of the synthesis was Keynesian and the microeconomics (at least the "high theory") was Walrasian. By the 1960s the "synthesis" manifested itself in both the undergraduate and graduate economics curriculum of essentially every university in the United States – an institutional condition that remains in effect even today (though there may be some recent signs of change) – and the two paradigms together formed the theoretical and conceptual backdrop for effectively all "serious" research in economics.[5] As Brian Snowdon and Howard Vane explain:

> The synthesis of the ideas of the classical economists with those of Keynes dominated mainstream economics at least until the early 1970s. The standard textbook approach to macroeconomics from the period following the Second World War until the early 1970s relied heavily on the interpretation of the General Theory provided by Hicks (1937) and modified by the contributions of Modigliani (1944), Patinkin (1956), and Tobin (1958). Samuelson's best selling textbook popularized the synthesis … making them accessible to a wide readership and successive generations of students. It was Samuelson who introduced the label 'neoclassical synthesis' into the literature in the third edition of Economics in 1955. The synthesis of classical and Keynesian ideas became the standard approach to macroeconomic analysis, both in textbooks and in professional discussion … (Snowdon and Vane, 2005, p. 23)

The second historical presupposition of the paper is the pluralism and diversity that existed in microeconomics during the interwar period. As heterodox economists have long argued, the interwar period was a bubbling cauldron of diverse economic ideas where versions of the

[5] I will follow tradition and use the term "neoclassical synthesis," but in fact the term "synthesis" does not really capture the relationship very well. A synthesis suggests two things coming together to form a third that is unique and different from each of the things that entered into it: like the synthesis of water from hydrogen and oxygen. But the neoclassical synthesis was not like this. Microeconomics and macroeconomics remained identifiable and distinct fields; they did not disappear as separate entities upon the formation of the neoclassical synthesis. The main point of this paper is that although Walrasian economics had a certain hard core that was identifiable over time, it also evolved and changed in response to, and because of, its contact with Keynesian economics. This seems much more like co-evolution than synthesis. Each program remains distinct – it retains its own genetic material and some aspects of its earlier behavior – but also changes in various ways because it has formed a partnership with another research program. I will argue that what Walrasian economics was in 1960s was in part because of its relationship with Keynesian economics – and the interaction of the partnership with the environment in which these two sets of ideas competed – and yet it always maintained a separate identifiable existence. Without wandering too far a field, perhaps the term "marriage," or other word signifying a romantic partnership, is a better way of thinking about the relationship. Given the giddy optimism of the early years, the willingness to overlook differences and have faith they could be worked out over time, and the ultimate irreconcilable differences that emerged in the 1970s, perhaps Neoclassical-Keynesian marriage is a better expression.

heterodox big three (Institutionalist, Marxist, and Austrian economics) and an array of other theoretical frameworks all vied for position within the economics profession – a diversity that ended with the stabilization of the neoclassical synthesis (Backhouse 2003, Morgan and Rutherford 1998). But while such broad inter-programmatic diversity undoubtedly existed, that is not the diversity that will be emphasized in the following discussion. The diversity emphasized in this paper is a more intra-programmatic diversity – the diversity among various economists who were broadly marginalist or neoclassical (although not all would label themselves as such) and shared a commitment to certain modeling strategies, mathematical tools, and types of evidence – yet who promoted and defended very different economic theories.

Focusing on demand theory, a partial list of these various approaches would include: defenders of the Marshallian tradition in either cardinal utility (Robertson 1952) or compensated demand (Friedman 1953) form; those who, Cournot- or Cassel-like, started from demand functions rather than individual choice [these took different forms including, among others, statistical (Moore 1914, Schultz 1928) and mathematical (Evans 1930) versions]; Slutsky (1915); Bernardelli (1952); Knight (1944); Hicks and Allen (1934), Allen's non-integrable interpretation of Hicks and Allen (Allen 1936); Nicholas Georgescu-Roegen's psychological threshold (Georgescu-Roegen 1936) and directed choice (Georgescu-Roegen 1950) models; Harold Hotelling's entrepreneurial demand function model (Hotelling 1932), Ragnar Frisch's conditional preferences approach (Frisch 1926), Oskar Morganstern's reconstituted demand theory (Morgenstern 1948), Paul Samuelson's radical behaviorism in his first "revealed preference" paper (Samuelson 1938), and W. E. Armstrong's just-perceptible-differences theory (Armstrong 1939). By the late 1950s this diversity of ways of explaining consumer choice and demand had been replaced with a "Walrasian" theory originating in the work of Leon Walras (1954) and Vilfredo Pareto (1971), but getting its final (calculus-based) form in Hicks and Allen (1934) and Slutsky (1915). Early influential book-length statements include Hicks's Value and Capital (1946), Samuelson's Foundations (1946), and Henry Schultz' Theory and Measurement of Demand (1938); these n-good multivariate calculus-based versions of the theory formed the basis for the standard graduate microeconomics textbooks of the 1960s and 1970s (lower-level textbooks offered the same theory, but presented it in one and two-dimensional diagrams). The argument will be that Keynesian economics had something to do with Walrasian demand theory emerging as the (rather than a) theory of demand as well as why certain aspects were emphasized and particular theoretical formulations emerged as they did.

The last two remarks I would like to make in this section are more comments on, than presuppositions for, what is to follow. The first is

that in all of my discussion about how Walrasian economics was, or particular aspects of Walrasian economics were, "consistent with" or "conditioned by" Keynesian economics, I will always mean to the relevant theorists: the community of those engaged in the research in question. These remarks – in fact, my entire argument – in no way implies an endorsement of the view that "Walrasian economics" and "Keynesian economics" are fundamentally consistent or could co-exist in a theoretical partnership indefinitely. In fact I generally agree with those who argue that the neoclassical synthesis exhibited a certain "theoretical schizophrenia" (Snowdon and Vane, 2005, p. 21).[6] The "fit" that formed the backbone of the neoclassical synthesis was at a best a temporary equilibrium. It existed because of the particular way the two research programs co-evolved, the historical situation (politically, economically, and epistemologically), the persuasive power of certain individuals, concerted effort, luck, and many other historically contingent factors.

Second, I think it is useful right up front to be clear about what I am not arguing. My argument is not that Keynesian economics – or anything else – was the only reason that the Walrasian version of neoclassicism emerged triumphant or that Walrasian economics took the particular form that it did during its heyday. The reason research programs rise to dominance and the transformations they go though during their evolution is always a very complex story. The story for research program A will generally be quite different than the story for research program B, and the story for program A from t_0 to t_1 will be different than program A from t_1 to t_2. History is like that and the history of modern economic thought is, well, history. In the first paragraph I cited a number of authors/texts who have recently made contributions to our understanding of the ascent and character of Walrasian general equilibrium theory. The argument here is not an alternative to the issues discussed in those and other narratives;[7] it simply provides an additional, unrecognized, factor that needs to be considered.

2. Why and Which Walrasian Economics?

[6] "The Keynesian propositions of market failure and involuntary unemployment expounded within macroeconomics did not rest easily alongside of the Walrasian theory of general competitive equilibrium, where the actions of rational optimizing individuals ensure that all markets, including the labor market, are cleared by flexible prices. In the Walrasian model, which dominated microeconomics, lapses from full employment cannot occur. Although Paul Samuelson and other attempted to reconcile these two strands of economics, producing a 'neoclassical synthesis,' Keynesian macroeconomics and orthodox neoclassical microeconomics integrated about as well as oil and water. During the 'Golden Age' this problem could be ignored. By 1973, with accelerating inflation, it could not." (Snowdon and Vane, 2005, p. 21)

[7] Including the other factors I have examined in previous research (e.g. in Hands 1994, 2006, 2007, 2008a, 2009; Hands and Mirowski 1998; or Mirowski and Hands 1998).

This section will discuss four ways (2A-2D) in which the apparent compatibility between Walrasian and Keynesian theory helped the former win out over its immediate competitors and how the theoretical structure of Walrasian theory was pulled in various directions that enhanced the fit.

2A. The Centrality of Market Demand

It should be uncontroversial that "demand" (as opposed to supply, production, or cost) is central to Keynesian economics. There are many different interpretations of the General Theory, but common to all is the central idea that aggregate demand (aggregate expenditure, aggregate spending, …) is the major determinant of output and employment.

Of course demand theory is also an important part of Walrasian economics. The core idea is that demand functions are the result of consumers solving a particular constrained optimization problem: choosing the most preferred bundle (the utility maximizing bundle) from the set of affordable bundles. The consumer's preference ordering is the key primitive in the analysis; preferences are assumed to be well-ordered (complete, transitive, etc.) and thus can be represented by an ordinal utility function $U(x)$.

Writing out the standard consumer choice problem we have:

$$\begin{aligned} &\underset{x}{\text{Max}}\, U^h(x) \\ &\text{Subject to: } M^h = \sum_{i=1}^{n} p_i x_i, \end{aligned} \qquad \text{(CCP)}$$

where $p_i > 0$ is the price of good i and $M^h > 0$ is consumer h's money income. Given the standard assumptions on preferences and the linearity of the budget constraint, the utility function will have sufficient mathematical structure to guarantee the existence of a well-behaved solution.

The solutions to the consumer choice problem are the n individual demand functions. The demand for good i by individual h is given by:

$$x_i^h = d_i^h(p, M^h) \text{ for } i = 1,2,\ldots,n, \qquad \text{(ID)}$$

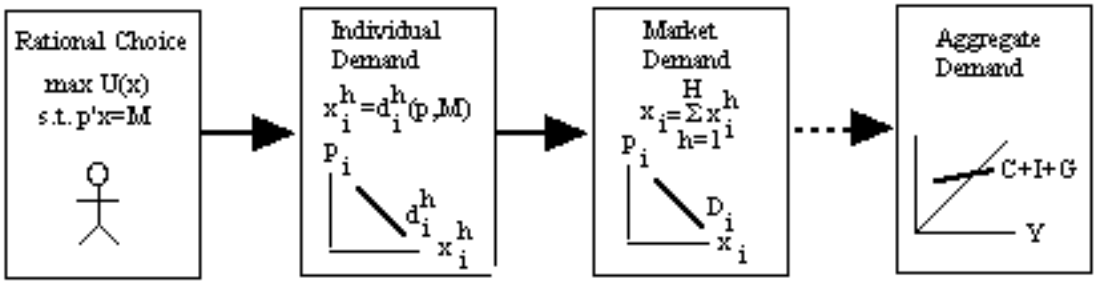
where $p = (p_1, p_2, \ldots, p_n)$. Market demand functions are obtained by adding up the individual consumer demand functions, so assuming there are H individuals, the demand for good i is given by:

$$x_i = D_i(p, M^1, M^2, \ldots, M^H) = \sum_{h=1}^{H} d_i^h(p, M^h).^8 \qquad \text{(MD)}$$

As noted above, this Walrasian demand theory – now simply the theory of demand – comes down to contemporary textbooks from Pareto, through Slutsky (1915) and Hicks and Allen (1934), and the influential presentations in Hicks (1946), Samuelson (1947), and Schultz (1938).

In relating this microeconomic theory of demand to aggregate demand in Keynesian macroeconomics, it is useful to note that there are really three separate parts to the micro side: rational choice (the behavior of individual economic agents), individual demand (an individual's demand for a particular good), and market demand (the total market demand for the good). The market demand functions should then relate in some consistent way to macroeconomic aggregate demand. Of course this last step has proven to be one of the many controversial issues in the microfoundations literature (Weintraub 1979). Fortunately we do not need to address this macroeconomic issue in order to make the point I want to make here. I will focus on the three separate parts of the micro side.

Consider the following picture of the relationship between macroeconomic aggregate demand (right side) and the three different parts of microeconomic demand theory (left side). Almost all



microeconomic theories of demand have some version of all three of these aspects, but most also emphasize one of these aspects more than others. For example, going back to the partial list of various pre-synthesis demand theories given in section one, some of these focused primarily on the psychological specifics of human decision making (e.g. Armstrong,

[8] Only under very restrictive conditions can the market demand function be written as a function of the n prices and total income $M = \sum_{h=1}^{H} M^h$. This is one version of the notorious aggregation problem in demand theory.

Bernardelli, Georgescu-Roegen, and to a lesser extent Allen and Frisch). Although such theories frequently came up with something like a market demand function (though it might be thick or discontinuous), their main focus was on individual choice (i.e. the left-hand side of the above picture). In some ways this individual-choice-theory-first tradition has recently been revived by experimental and behavioral economists (although it is seldom recognized as a revival since the experimental and expected utility aspects of the recent literature tend to blur its relationship to 1930s demand theory[9]). On the other hand, other theorists tended to focus primarily on market demand functions and had only a very thin, and in some cases non-existent, theory of individual behavior (e.g. Cournot, Cassel, Evans, Moore, Schultz 1928 but not 1938, and others). Those theorists tended to focus more on the right-hand side of the picture (and some, Cassel and Moore in particular, did not have, or believe it was necessary to have, a theory of individual behavior at all).

Given this differentiation between choice-centered and market demand-centered theories of demand, it is important to recognize that the version of Arrow-Debreu general equilibrium theory that became dominant during the 1960s was much more of a right-side theory than a left-side theory. Of course the neoclassical synthesis era Walrasian models assumed rational economic agents with well-ordered preferences acting under constraint, but explaining individual behavior was never the main task of the theory. Arrow-Debreu theory was primarily a right-side (i.e. market-focused) view where all the theoretical heavy-lifting was done by restrictions on market excess demand functions. In fact one could conduct all of the analysis of existence, stability, uniqueness, and comparative statics of Arrow-Debreu theory using a model specified entirely in terms of market excess demand functions. As Kenneth Arrow and Leonid Hurwiz explained in there influential work on stability theory:

> This work is characterized, in the main, by being based on models whose assumptions are formulated in terms of certain propensities of the individual economic units, although in the last analysis it is the nature of the aggregate excess demand functions that determine the properties of equilibria. (Arrow and Hurwicz, 1958, p. 522)

For example, if the market excess demand for each good i is given by $z_i(p)$ and the model assumes a sufficient amount of continuity and interiority, the only two assumptions needed on the $z_i(p)$s to do "general

[9] See Hands (2008b).

equilibrium analysis" are zero degree homogeneity (H) and Walras's Law (W):

$$z_i(p) = z_i(\lambda p) \text{ for all } \lambda > 0 \text{ and for all } i = 1,2,\ldots,n, \qquad \text{(H)}$$

$$p^T z(p) = \sum_{i=1}^{n} p_i z_i(p) = 0. \qquad \text{(W)}$$

Granted, the reason why one might think excess demand functions have these two properties comes from assumptions on the behavior of the underlying agents, but once one has well-behaved market excess demand functions satisfying (H) and (W) it is possible to kick away the rational choice ladder and focus entirely on market excess demand. In fact this is the main message of so-called Sonnenschein-Mantel-Debreu (SMD) theorems on excess demand functions (Debreu 1974, Mantel (1974, 1977), Sonnenschein (1972, 1973).[10] Basically these results say that any continuous function that satisfies (H) and (W) can be an excess demand function for a Walrasian economy. Another way of saying this is that Arrow-Debreu general equilibrium theory has very weak microfoundations; the Walrasian theory of individual consumer behavior doesn't put much structure on market excess demand functions and what structure it does impose is exhausted by (H) and (W). Thus Arrow-Debreu theory has almost nothing to say about the behavior of individual economic agents.

One way to read the argument in this section is to reduce it to simply praising Hicks (1937) for having a good eye for finding the best microeconomic theory to hook up with Keynesian macroeconomics. If it is the late 1930s and one is looking for a microeconomic theory to connect up with Keynesian economics, then choosing the Walrasian program with its right-hand side focus on market demand and its lack of emphasis on the behavior of individual economic agents (for Keynes a notoriously unreliable source of policy insights), does seem to be a very wise move. But one can say more than this. The Walrasian program in the hands of Pareto (1971) and later Schultz (1938) was more focused on individual choice behavior than the Arrow-Debreu theory that came later; in fact Pareto had no market demand functions at all in the Manual.[11] These transitional general equilibrium theorists never sought the serious psychological underpinnings that concerned some of the competitors to Walrasian theory, but their approach was certainly "more" left-hand side than the Arrow-Debreu framework that characterized Walrasian general equilibrium theory at its peak. This would suggest not only that Hicks

[10] See Shafer and Sonnenschein (1982) for a survey and Rizvi (1998, 2003, 2006) for more historical discussion.

[11] See van Daal and Walker (1990) on the difference between Walras and Pareto on this matter.

did in fact have a good eye, but also that Walrasian theory ultimately came to take the particular (right-hand side) form it did in part because of the context of the neoclassical synthesis and its relationship with Keynesian economics.

2B. Tâtonnement Stability and Related Issues

Walras's main focus in the Elements was the formal characterization of competitive equilibrium: specifying the basic equations of the general equilibrium model and proving the existence of a solution (which for him meant demonstrating that the number of equations was equal to the number of unknowns). Walras did though, throughout the various editions of the Elements, also attempt to show how the theoretical solution would actually be reached by the competitive market process. As Walras himself explained in the final 4th definitive edition: "Now let us see in what way this problem of the exchange of several commodities for one another to which we have just given a scientific solution is also the problem which is empirically solved in the market by the mechanism of competition" (Walras, 1954, p. 169).[12] His approach to this "empirical" question was to specify an adjustment mechanism where prices changed "by a process of groping ['par tâtonnement']" under the rule that if "the demand for any one commodity is greater than the offer, the price of that commodity in terms of the numéraire will rise; if the offer is greater than the demand, the price will fall" (Walras, 1954, p. 170).

Although to the contemporary (post-synthesis) reader Walras's words may suggest the system-of-ordinary-differential-equations version of the tâtonnement popularized by Samuelson (1941, 1942, 1944, 1947) that went on to become the standard characterization during the 1950s:

$$\frac{\partial p_i}{\partial t} = H_i\left[z_i[p_1(t), p_2(t), \ldots, p_n(t)]\right] \quad \text{for all } i = 1, 2, \ldots, n \qquad \text{(T)}$$

[12] My discussion of the tâtonnement of Walras (as opposed to the Walrasian tâtonnement) will focus primarily on his analysis of the pure exchange case. A detailed discussion of what Walras said about tâtonnement processes in general is not necessary for the task at hand. There were at least three (nested) models in the Elements – pure exchange, production, and capital formation – and the book went through five editions (counting the 4th definitive) and Walras offered different characterizations of the tâtonnement in different editions as well as for different models within various editions. In particular, the assumption of "no disequilibrium trading" or no trading at "false prices" was handled differently in various editions and models. The variation among editions is greatest in his analysis of production and capital formation, where his introduction of "tickets" ("*bons*") in the 4th edition provided a version of the "no trading at false prices" restriction for these models. Although there is some variation in his analysis of the pure exchange case, the core characterization offered in the 2nd edition remained basically intact in the later editions and that is the version of Walras's tâtonnement discussed here. Those interested in the details of how Walras's view of the tâtonnement changed across various editions and models can consult the various detailed discussions in the secondary literature (i.e. Donzelli 2006, 2007; Jaffé 1967, 1981, Morishima 1977).

(where $p_i(t)$ is the price of the ith good at time t, $z_i[\bullet]$ is the excess demand function for the ith good, and $H_i' > 0$), Walras did not employ this version of the adjustment process. Walras's own explanation involved a fairly elaborate "sequential" process of clearing one market at a time based on changing only the price of the good in that market. From any initial disequilibrium position the price of good 1 is adjusted on the basis of the rule that if excess demand is positive the price would be raised and if it is negative it would be lowered until the excess demand for good 1 is equal to zero. Then the same procedure is applied to the market for good 2, then good 3, and on and on in sequence. Obviously in the standard case where the excess demand for each good depends on the prices of all goods, there is no reason that the first iteration will be sufficient to reach equilibrium, so the process would need to be repeated again and again. But under the assumptions of Walras's original model there is no reason to believe this sequence of iterations will ever converge to the general equilibrium.[13]

Walras's sequential tâtonnement was clearly different than (T): the standard way the tâtonnement came to be written in the post-Samuelson era. As Walras's translator William Jaffé explained: "The current reformulations of the theory, though they proudly bear the Walras patronymic, display only a distant family resemblance to their ancestral prototype, for the infusion of new technical refinements has all by obliterated any recognizable similarity between the descendant theories and their progenitor" (Jaffé, 1967, p. 1). To see why this difference is important for the issue of the relationship between Walrasian and Keynesian economics, it is useful to rewrite the later version of the tâtonnement (T) in its common "speed of adjustment" form:

$$\frac{\partial p_i}{\partial t} = k_i z_i[p_1(t), p_2(t), \ldots, p_n(t)] \quad \text{for all } i = 1, 2, \ldots, n, \qquad \text{(T')}$$

where $k_i > 0$ is the speed of adjustment for the ith market (Arrow and Hurwicz, 1958, p. 525). As will be discussed in more detail below, this form makes it clear that some markets can be "slower" or "stickier" in the process of price adjustment than others, allowing for Keynesian-type behavior in certain markets while staying broadly within the Walrasian framework. Of course one can question whether this characterization of disequilibrium accurately captures what Keynes had in mind, but that is not the issue. The point to note here is that (T') – or (T) since it is just a more general version of (T') – accommodates Keynesian ideas much

[13] Uzawa (1960) noted that Walras's iterative process was a version of the Gauss-Seidel algorithm and proved that it converges under the assumption that all goods are gross substitutes.

better than Walras's original sequential process. According to Walras's version, each market will be in equilibrium at a certain point (and generally multiple times during the iterative process), a framework that makes it much more difficult to accommodate the idea that some particular markets are consistently slower or stickier in their adjustment than others.

In addition to and perhaps even more important than the fact that Walras's original sequential formulation of the tâtonnement was difficult to combine with Keynesian theory, is that between Walras's Elements and Samuelson (1941), general equilibrium theory systematically moved away from any discussion of the competitive price adjustment mechanism. As Jaffé explains (Jaffé 1967, 1981), Walras recognized that the "realistic" or "empirical" dynamics[14] that he was attempting to model would involve trading at "false prices" which in turn would involve "income" or "endowment" effects that could potentially change the equilibrium price vector. This is a problem even in the pure exchange case, but it is more problematic in the production version of the model. Walras eventually adopted a "no trade outside of equilibrium" condition for both the pure exchange and production models but this solution is entirely counter to his original purpose for introducing the tâtonnement process. In Jaffé's words: "It is, in fact, an abandonment of realism and with this abandonment the initial purpose of the theory of tâtonnement is lost from sight" (Jaffé, 1967, p. 12). These problems – and here is the point for the Keynesian story – led Pareto to completely abandon any discussion of the tâtonnement mechanism. There was a brief mention in the Cours, but it is totally absent from the Manual (Donzelli, 2006, pp.12-19). Thus if one considers the evolution of "Walrasian" general equilibrium theory from the early editions of the Elements to the Manual, the tâtonnement goes from being an important part of the story but modeled differently than (T), to being very problematic, to being entirely abandoned.

Moving forward in time to Hicks and Samuelson, Hicks discussed multiple market stability in Value and Capital (1946) by generalizing the stability condition for a single market. Samuelson (1941) argued that Hicks's conditions did not represent "true dynamic stability." Samuelson's tâtonnement adjustment mechanism (T) and his stability condition – negative real parts of the characteristic roots of the excess demand Jacobian matrix evaluated at equilibrium prices – became the standard tool for talking about the local stability in Walrasian general equilibrium theory. The literature on the local stability of the Walrasian

[14] Walras did not use the term "dynamic" for (any version) of his competitive price adjustment mechanism. For Walras, "dynamics" involved changes in the fundamentals of the analysis – tastes, technology, endowments, etc. – and the tâtonnement is not dynamic in this sense.

tâtonnement that appeared in a steady stream during the next twenty years often amounted to trying to find reasonable economic restrictions that would be sufficient for Samuelson's condition.[15] The analysis of global stability came later during the late 1950s as a result of applying Liapunov theory: the conical results were provided in Arrow and Hurwicz (1958) and Arrow, Block, and Hurwicz (1959). These papers proved that the Walrasian general equilibrium price vector (p*) would be unique and globally stable under a variety of specific restrictions (gross substitutes being the most important).[16]

Samuelson's initial papers on local stability were published in 1941 and 1942, but they were included in Foundations as chapter nine and ten. The stated purpose of Foundations was to provide a mathematical technique that could be used to generate "operationally meaningful" theorems for a wide range of economic models. The main focus was comparative statics. If we start with a model where equilibrium values of (say n) dependent variables (x*s) can be written as (usually differentiable) functions of (say m) independent variables (α_is):

$$x_i^* = f_i(\alpha_1, \alpha_2, \ldots, \alpha_m) \quad \text{for all } i = 1, 2, \ldots, n,$$

then comparative statics results show how the value of each dependent variable would change for a change in any one of the parameters. In other words, successful comparative statics exercises will be able to determine (or at least sign) the nxm terms: $\partial x_i^* / \partial \alpha_j$ for all $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. Foundations provided a mathematical framework for deriving such comparative statics results and demonstrated how the technique could be applied to a variety of economic models.

It is significant that Foundations was divided into two separate parts. The first part discussed economic models where the equilibrium was associated with the maximum or minimum of some function (what Samuelson called extremum problems). The examples in part I were the topics that would come to dominate microeconomic textbooks during the next few decades: consumer choice (demand) theory, cost and production, profit maximizing firm behavior, welfare economics, etc. Part II was also about comparative statics, but it examined models where the equilibrium was not associated with the maximum or minimum of any function. Comparative statics results are more difficult in such cases

[15] This result became standard condition for local stability throughout economic theory, not just Walrasian general equilibrium theory.

[16] This paragraph is only a very brief sketch of a massive amount of literature. See Weintraub (1991) for a detailed historical discussion of the stability literature during this period, including the work of many economists whose contributions had not previously been recognized.

because of the weaker mathematical restrictions in such problems (in particular, such models do not have second order conditions). For these non-optimization-based models Samuelson proposed the correspondence principle. It employed similar mathematical techniques, but used the stability of the model, rather than optimality, to obtain comparative statics results. The explicit motivation for discussing this class of models and subsuming them under the same formalism was the fact that Keynesian models (business cycle theories) are of this second, non-optimization-based, kind. As Samuelson explains in the introduction to Foundations:

> However, when we leave single economic units, the determination of unknowns is found to be unrelated to an extremum position. In even the simplest business cycle theories there is lacking symmetry in the conditions of equilibrium so that there is no possibility of directly reducing the problem to that of a maximum or minimum. Instead the dynamical properties of the system are specified, and the hypothesis is made that the system is in 'stable' equilibrium or motion. By means of what I have called the Correspondence Principle between comparative statics and dynamics, definite operationally meaningful theorems can be derived from so simple a hypothesis. (Samuelson, 1947, p. 5)

Important to the story here is the fact that the Samuelson's discussion of the stability of the Walrasian tâtonnement (in fact all of his explicit discussion of Walrasian general equilibrium) was contained in part II (the non-optimization-based Keynesian part) of Foundations. Chapter nine – which was Samuelson (1941) – starts out discussing the correspondence principle, moves to the stability of two-dimensional market models, then the stability of Walrasian multiple-market general equilibrium (his criticism of Hicks, his main stability result, etc.), and finally analyzes a 3-variable, 3-parameter, Keynesian model. For Samuelson, the analysis of Walrasian dynamics – in 1941 and in Foundations – was more like the analysis of a Keynesian model than the microeconomic theory in part I. Of course Samuelson, like others working on Walrasian models during this period, was assuming that utility maximizing consumers and profit maximizing firms were in some sense "behind" the excess demand functions of in the tâtonnement (T), but Walrasian dynamics as a topic of economic analysis was directly linked, by formal structure and in its dependency on the correspondence principle, to Keynesian economics.

Another important contributor to the neoclassical synthesis – perhaps even more self-conscious about forging a synthesis than Samuelson – was Oscar Lange. The goal of Lange's Price Flexibility and Employment

(1944) was to restate general equilibrium "in a way which explicitly takes account of money" (Lange, 1944, ii). The second paragraph of Lange's preface lists the economists who most influenced the study and it reads like a who's who of the neoclassical synthesis: Keynes (on the "substitution between money and goods"), Hicks (for providing the "most up-to-date formulation of the theory of general economic equilibrium"), and Samuelson (for the "dynamic theory of stability of economic equilibrium"). Key to Lange's analysis in Price Flexibility is Samuelson's version of the Walrasian tâtonnement (T). Prices which obey (T) exhibit price flexibility (p. 2). The purpose of his analysis was to investigate the relationship between price flexibility in this sense and "employment and economic stability" (p. 1). The book was thus an attempt to combine Walrasian general equilibrium theory – particularly the stability analysis of the Walrasian tâtonnement – with a Keynesian analysis of unemployment and economic stability (in the macro sense). Although the argument was far from tight, the often repeated theme in the book was that "flexibility of factor prices fails to assure full employment of factors of production" (p. 51) unless a number of additional conditions are satisfied. In classic Keynesian style, full employment in a rare event in a competitive market economy, even when the market economy is represented by a Walrasian general equilibrium system.

Lange also used Walrasian theory to make a Keynesian theoretical point in his paper on Say's Law (Lange 1942). He makes the distinction between Walras' Law (valid in a general equilibrium system) and Say's law (invalid in such a system). Again a combination of Keynes and the Walrasian formalism were being used make Keynesian political-economic points. Since this section is getting quite long, I will simply note that many others who participated in the theoretical literature of the neoclassical synthesis – Alvin Hansen (1949), Don Patinkin (1965), James Tobin (1958, 1969) and others – also combined elements of general equilibrium theory with elements of Keynesian economics and did so using much the same formula as Lange (though generally with more moderate politics).[17] The Walrasian model formed the theoretical backbone – with a strong emphasis on stability analysis – and the Keynesian influence entered on the money/liquidity and policy sides.

Some of the argument presented in this section regarding the close relationship between Walrasian stability and the neoclassical synthesis, has been presented in Roy Weintraub's Stabilizing Dynamics (1991). Weintraub explains that prior to the neoclassical synthesis terms like "equilibrium" and "stability" had a variety of different meanings – the discourse was not stabilized – with different economists and texts using the terms in different ways. He argues that one of the driving forces

[17] See Weintraub (1979, 1991) for more detail on these various economists (particularly Patinkin).

behind the ultimate stabilization that took place during the 1950s and 1960s (basically that "dynamic" meant that the system was specified explicitly in terms of differential or difference equations and "stability" meant convergence to equilibrium as $t \to \infty$) was the effort to reconcile the idea of general equilibrium with ostensibly "disequilibrium" phenomena of involuntary unemployment. As Weintraub explains:

> The literature associated with Frisch, Tinbergen, Hicks, and finally Samuelson was associated with understanding the conditions under which an equilibrium would be stable, so as to permit the conjunction of equilibrium theorizing and unemployment analysis. (Weintraub, 1991, p. 123)

This means that the neoclassical synthesis played an essential role in stabilizing dynamics (in general equilibrium theory and in economics more generally).

> The mathematization of equilibrium and stability, the papers from Samuelson on through Arrow and Hurwicz, stabilized that discourse … The restriction of 'dynamic' to 'dynamical system,' and the construction of 'stable' to 'locally stable equilibrium motion of a dissipative dynamical system,' permitted concurrence … on the meaning of the claim that unemployment was a disequilibrium position associated with a 'usually' stable competitive equilibrium. The neoclassical synthesis was literally unthinkable before the availability of the mathematization of equilibrium and stability. (Weintraub, 1991, p. 125)

Although I agree with Weintraub's basic argument on this matter, I want to emphasize a different aspect of the story. There were no "stabilized dynamics" before the neoclassical synthesis. The sequential tâtonnement of Walras was quite different from the (more Keynesian accommodating) tâtonnement process of the later literature (T) and discussion of the tâtonnement had all but disappeared from the Walrasian literature by Pareto's most mature work. The "price adjustment mechanism" was not a significant part of general equilibrium theory in the period immediately preceding the work of Hicks and Samuelson; it <u>became</u> a significant part of the Walrasian research program and it <u>became</u> so in part because of the neoclassical synthesis and the concerns of Keynesians economics. I also think it needs to be emphasized how <u>important</u> the topics of stability and Walrasian dynamics were to general equilibrium theorizing during the heyday of Arrow-Debreu theory. In fact it seems reasonable to say that stability theory (perhaps adding the closely related analysis of uniqueness) was "primarily" what general equilibrium theorists did; questions about existence and the central welfare theorems were quickly

and successfully settled, and serious computational general equilibrium was still over the horizon. For example, Arrow and Hahn's General Competitive Analysis (1971) – the canonical text in Arrow-Debreu general equilibrium theory – dedicates far more pages to stability than any other topic. The book has fourteen chapters (and a number of mathematical appendices). There is one chapter on consumer choice, one chapter on production theory, and one chapter on existence, but there are three chapters on stability analysis (two on the traditional tâtonnement and one on alternative ways of modeling general equilibrium dynamics). Add to this the fact that there is an entire chapter on uniqueness and yet another on "The Keynesian Model" and one starts to get some indication of how important stability analysis and its connection to Keynesian economics was for Walrasian general equilibrium theory at its peak. The purpose of this section has been to show that would not have been the case if Walrasian economics at its peak had not been a product of co-evolution with Keynesian economic theory.

2C. Reversibility, Path-Dependency and All That

This topic is related to the stability discussion in the previous section, but it can be separated from the way that Walrasian tâtonnement dynamics jelled together with Keynesian notions of disequilibrium and unemployment. One feature of Walrasian models (of any sort) and Keynesian models (of the IS-LM sort) is an absence of path-dependencies, irreversibilities, reference-dependence, endowment effects, or any other situations where the path or initial position influences/determines the characteristics of the equilibrium (or which equilibrium is) reached. Disequilibrium adjustment can be characterized in both models, but the process/mechanism by which the equilibrium is reached has no impact on the resulting equilibrium position.

The story on the microeconomic side is fairly familiar. In recent years a vast amount of empirical evidence from experimental and behavioral economics suggests that such path-dependencies and irreversibilities are pervasive features of actual human choice (in laboratories and in markets) – see for example Camerer and Loewenstein (2004), DellaVigna (2009), Kahneman (2003), Kahneman, Knetsch, and Thaler (1991), Knetsch (1989, 1992), or Thaler (1980) – but such effects are entirely absent from Walrasian choice theory. For Walrasian theory the individual consumer has well-ordered preferences (and thus a well-behaved ordinal utility function) and chooses the most preferred bundle (maximizes utility) from the affordable set. The consumer is assumed to have full information, infinitely fast computational ability, and to move immediately (actually timelessly) to the optimal bundle – as Nicholas Georgescu-Roegen once put it, the agent's behavior is like a "bird" that drops down instantaneously on the optimal bundle, rather than like a

"worm" that actually moves through the choice space in real time to arrive at the optimal choice (Georgescu-Roegen, 1968, p. 255). Of course if the behavior were worm-like the particular path taken might matter to the final choice (path-dependency) and reversing the parameter change that initiated the choice might not return the consumer to the initial position (irreversibility). Of course, this feature is common to many neoclassical-based models of individual choice and is not restricted to Walrasian choice theory, but – and here is the point – such path-dependencies and irreversibilities were common features of many of the demand theories the Walrasian program was competing against during the 1930s and 1940s.[18] Many of the different competing approaches to choice/price theory listed above in the discussion of interwar pluralism, were motivated by the idea that economic agents do not have stable and reversible preferences, infinitely fast computational abilities, act instantaneously, and so forth. These issues, present in the recent literature on experimental and behavioral economics, disappeared with the ascension of the Walrasian version of choice theory. The question is: How does all this relate to Keynesian economics?

The fact is that Keynesian economics – at least in the form it came to take during the neoclassical synthesis – was also characterized by hermetic separation of the equilibrium position from any dependency on, or influence from, the process/path by which that equilibrium is reached. In a recent paper on "What was Lost with IS-LM" Roger Backhouse and David Laidler (2004) discuss a number of problems associated with the passage of time and related issues that concerned macroeconomists during the interwar period, but disappeared from discussion once the profession came to accept the IS-LM framework. As they explain: "All of these matters had received widespread attention in the interwar literature, but the wholesale adoption of the static IS-LM framework form the 1940's onward led to their falling into neglect" (Backhouse and Laidler, 2004, p. 31). At the same time that the Walrasian program was rising to dominance in microeconomics and thus facilitating the profession's dismissal of many of the issues of time and path that concerned microeconomists during the interwar period, the rise to dominance of textbook Keynesianism facilitated a similar dismissal of time-related issues within macroeconomics. Micro and macro both stabilized around theoretical frameworks where the "dynamic structure of the world plays no role in determining the equilibrium toward which the economy converges" (Backhouse and Laidler, 2004, 32).

[18] See Hands (2006, 2008b, 2009) for a more detailed discussion of, and evidence for, this argument and how it relates to various issues in the history of demand theory (integrability in particular) as well as to recent research in experimental and behavioral economics.

So given the argument in the preceding paragraphs, it does seem that neoclassical synthesis micro and macro were very similar on the issues of path and time – and perhaps defeated competitors that allowed for path-dependency and irreversibility on both the micro and macro side – but how does this show that Keynesian ideas influenced Walrasian theorizing? To answer this notice how the literature on the Walrasian tâtonnement is based on an entirely different strategy for answering the question of how the competitive market reaches equilibrium than Walrasian theory employs when answering the question of how the individual economic agent reaches equilibrium (optimal choice). In the case of the individual economic agent (intra-agent equilibrium), Walrasian theory makes equilibrium instantaneous and avoids all issues associated with time, path, initial position, irreversibility, or the dynamic process of "getting there." The Walrasian consumer does not grope around in the choice space; they are essentially always in equilibrium. One can of course do comparative statics exercises on such models and compare one equilibrium to another, but no time passes (even logical or virtual time) between the two equilibrium positions, the path between the two points does not effect either equilibrium, and reversing the initial parameter change would simply take the consumer back to the original point. Notice how different this is from the Walrasian tâtonnement.

The tâtonnement is also timeless in the sense that no trade takes place until the equilibrium price vector (p*) is reached, but the variables are tracing out paths in "time." This "time" has a natural direction; it makes no sense to talk about "reversing" the dynamic system (T). In (inter-agent) equilibrium the motion simply stops – $dp_i / dt = 0$ for all $i = 1, 2, \ldots, n$. Since stability implies $\lim_{t \to \infty} p(t) = p^*$, and no trade takes place until p* is reach, this process may take a very long "time." It is analytical (or virtual) "time" and not real time, but there is still a big difference between this notion of "getting to" equilibrium and the instantaneous choice of the Walrasian agent. The "behavior" of the Walrasian auctioneer is conceptually quite different than the "behavior" of the Walrasian agent and in particular, the tâtonnement (T) allows for "stickiness" or "disequilibrium" in a way that is inconceivable for the Walrasian agent. If, contrary to the Walrasian models of the neoclassical synthesis, the equilibrium in the Walrasian market were modeled in the same way as the equilibrium of the Walrasian consumer there would be no tâtonnement "adjustment"; the competitive price system would always be in equilibrium. In fact in the late 1970s when the New Classical macro of Robert Lucas (1981) replaced Keynesian macro this is exactly the way general equilibrium was discussed. There was no "adjustment"; the economy was always in equilibrium in the same way that the Walrasian consumer is always in equilibrium. As Kevin Hoover explains: general

equilibrium in the Lucas model means that "self-interested economic agents successfully maximize their utility or profits subject to constraints on their budgets and, crucially, on available information" (Hoover, 1988, p. 42). The Lucas model is strictly Walrasian in that the representative agent does what Walrasian agents have always done. No tâtonnement is needed. As Lucas himself says: "the idea that an economic system in equilibrium is in any sense 'at rest' is simply an anachronism" (Lucas, 1981, p. 287). Yes, an anachronism of the neoclassical synthesis.

So in the end it seems that this discussion of individual versus market behavior has left us at a point similar to where we were at the end of section 2B. The conception of "dynamics" that stabilized in Walrasian general equilibrium theory during the 1950s was in part driven by a desire to find a notion of multi-market competitive equilibrium that was consistent with a version of Keynesian unemployment. What this section has added is that the stabilization was not only about equilibrium and stability in a competitive market, but also about the characterization of the behavior of the individual economic agent. Walrasian economics overcame (or circumvented, or suppressed, depending on your point of view) the path-dependency and irreversibilities that were a concern of a number of non-Walrasian theories of demand during the interwar period, and it also overcame these same issues concerning time and position in the theory of market adjustment, but the two "solutions" were quite different: and different in part because of Keynesian concerns on the market side. Later, freed from these Keynesian concerns, New Classical Walrasians such as Lucas endorsed a more consistent Walrasian view where equilibrium in agents and markets meant essentially the same thing. It seems reasonable to conclude that the combination of instantaneously optimizing agents and tâtonnement adjusting competitive markets that characterized Walrasian economics during the period of Arrow-Debreu high theory – present in Walrasian theory neither before, nor after, the synthesis – was a product of its co-evolution with Keynesian theory.

2D. Income Matters

As discussed in section 2A, the "Walrasian" demand theory that emerged triumphant during the neoclassical synthesis descended more from Pareto than Walras, and it was put in essentially its final form in Slutsky (1915) and Hicks and Allen (1934).[19] Out of all the various contributions

[19] One thing to note here. The term "final form" refers to the theory's conceptual formulation and not necessarily the mathematical machinery used in its derivation or presentation. The mathematical formulation of demand theory in much of the Arrow-Debreu literature was algebraic, not calculus-based as it was in the work of Slutsky, Hicks and Allen, or Samuelson. That said, the basic conceptual formulation (individuals maximizing well-ordered preferences subject to a linear budget constraint) and the main

and contributors during the half century stabilization of demand theory, Slutsky (1915) has traditionally been considered the key development. Over the last decade there has been a substantial amount of historical research on Slutsky and we now know quite a lot more about his life and work (e.g. Barnett 2004, Chipman 2004, Chipman and Lenfant 2002, Weber 1999a. 1999b), but the one question that does not seem to have been adequately answered is: Why does the Slutsky equation – the "fundamental equation of value theory" (Hicks, 1946, p. 309) – have such a pronounced role in microeconomic theory and economic education?[20] Since the 1950s the Slutsky equation has been generally considered to be the most important result in demand theory. The argument in this section will be that this is in part because of the neoclassical synthesis and the impact of Keynesian economics.

If one is thinking about which of the various theories of demand from the 1930s and 1940s would best "fit" with Keynesian macroeconomics, the Walrasian formulation has an obvious advantage: Walrasian demand functions have nominal income as arguments. If one is trying to meld consumer choice theory and Keynesian economics then one needs to be able to explain how changes in nominal income have real effects: how changes in income cause real changes in behavior. Income matters in Keynesian economics and income must matter in any demand theory that is going to live comfortably with Keynesian theory.[21] This feature of Walrasian demand theory certainly gave it a big advantage over various demand theories that did not have such income effects; for example Hotelling (1932) with no income term at all, or various versions of Chicago price theory where income effects are compensated away (Friedman 1953). Although this explains why Walrasian theory made the short list for the neoclassical synthesis – it fulfilled an important necessary condition – it does not provide much explanation for the particular emphasis on the Slutsky equation. But there is a synthesis-based story for that as well.

Perhaps at this point it would be useful to write down the Slutsky equation,

---

implications of the theory (prices changes decompose into substitution and income effects, negative substitution effects, zero degree homogeneity of demand functions, etc.) are exactly the same.

[20] The story I tell when teaching microeconomics to undergraduates involves Popperian novel facts. Giffen goods were thought to be a "falsification" of neoclassical demand theory, but Slutsky demonstrated both that the theory was consistent with upward sloping demand curves (thus saving it from falsification) and also made novel (non ad-hoc, content increasing) predictions; the theory could predict in advance the characteristics of the goods likely to be Giffen (inferior goods with large income effects). Although this story seems to impress undergraduates, there is not really any evidence that it had had anything to do with the profession's attachment to the Slutsky equation.

[21] This was pointed out in Hands and Mirowski (1998, p. 366).

$$\frac{\partial x_i^h}{\partial p_j} = S_{ij}^h - d_j^h \frac{\partial d_i^h}{\partial M^h}, \qquad \text{(S)}$$

and the main results: the own Slutsky substitution terms are strictly negative ($S_{ii}^h < 0$), the cross-substitution terms are symmetric ($S_{ij} = S_{ji}$ for all $i \neq j$), and the $n \times n$ matrix of Slutsky substitution terms $S^h = \left[S_{ij}^h\right]$, is negative semi-definite ($x^T S^h x \leq 0$ for all $x \neq 0$). The substitution terms (S_{ij}^h) show the change in the consumption of the good caused by only a change in relative prices; the remainder of the expression is the income effect which shows the change in the consumption of the good caused by a price-induced change in real income.

Notice that (S) is in many ways the perfect expression of the neoclassical synthesis. It decomposes observed changes in consumption into a micro part (the change based on only relative prices) and a macro part (the change based on a change in the purchasing power of money). According to the macroeconomic/monetary theory preceding (and following) the dominance of Keynesian macroeconomics we should keep these two things strictly separate: changes in relative prices are an issue for value theory while changes in the purchasing power of money is an issue for monetary theory, and a strict dichotomy should be maintained between these two types of economic theory. The Slutsky equation not only violates this strict dichotomy, it demonstrates the harmony of micro and macro in one simple expression.

As the Frank Knight quote in the epigraph demonstrates, this was precisely the criticism of the Walras-Hicks-Slutsky demand theory raised by certain members of the Chicago school during the 1940s and 1950s and one of their main reasons for advocating an alternative – more purely micro – theory of demand. As Knight explains:

> The treatment of the Slutzky school adopts the assumption that the price of X varies under the condition that the prices of all other goods (and the consumer's money income) are constant. Hence real income must change … It throws together two distinct effects upon consumption, the "price effect" and the "income effect." The treatment then proceeds to separate these by means of an ingenious analysis. The cleverness of it all must be conceded. But it is called for only because of an initial confusion in the statement of the problem which is wholly unnecessary and should clearly be avoided … The 'income effect' of Slutsky et al. is merely a particular case or mode of change in the purchasing-power

> of money, or the price level: and it is this problem as a whole that should be isolated and reserved for special treatment. (Knight, 1944, p. 299
>
> An approach to any particular price problem which "jumbles" effects of change in the purchasing-power of money (however caused) with effects of change in the relative value for purchasing different things is mere gratuitous confusion. (ibid., p. 300)

Friedman made similar remarks in his paper on the Marshallian demand curve. Friedman argued that "the separation of the theory of relative prices from monetary theory" was one of "Marshall's basic organizing principles" which led him to use "a constant purchasing power of money as a means of impounding monetary forces" (Friedman, 1953, p. 66). Friedman argued that Marshall, unlike Walrasian theorists, correctly "recognized the desirability of separating two quite different effects and constructed his demand curve so that it encompassed solely the effect that he wished to isolate for study, namely, the substitution effect" (ibid., pp. 64-5).[22]

So the bottom line is that the Slutsky equation was a product of the neoclassical synthesis (the translation of Slutsky's paper was published in 1952). Having nominal income as an argument in demand functions helped the Walrasian theory win out over its competitors, but in addition the elevation of the Slutsky equation to the most important result in demand theory makes perfect sense in the context of the neoclassical synthesis. Of course, like all of the other topics discussed in 2A-2C, the argument is certainly not that the Keynesian connection is "the" reason the Slutsky equation got so much attention; it is a reason, and an unrecognized one, but it is certainly not the only reason. All things considered though, the Slutsky equation does seem to be a beautiful way to demonstrate that "the broad cleavage between microeconomics and macroeconomics has now been closed" (Samuelson, 1964, p. 361).

3. Every Good Thing Must End (Or the Down Side of Co-evolution)

This section will extend the argument about the influence of Keynesian ideas on the rise and character of Walrasian economics to the question of the Keynesian contribution to the fall of Walrasian economics. The discussion here will be less detailed and more suggestive than the argument in the previous section. One reason is simply that this is

[22] Note Friedman is only being quoted here to make the point about the Keynesian connection to the Slutsky equation and Friedman's criticism of it; it is not an endorsement of Friedman's interpretation of Marshall.

already a long paper and a serious analysis of the fall would require a lot more space than is available here, but there are other reasons as well.

First, the problem is that the "fall" (if that is the proper term) of Walrasian microeconomics is still a work in progress and serious historical research requires distance that it simply not available at this time. It is still an open question about the degree to which there has been a "fall" at all. Although many consider it to be obvious that the profession has moved on (e.g. Colander 2006; Colander, Holt and Rosser 2004a, 2004b; Davis 2006, 2008; Rizvi 2003, and many others), it is also clear that Walrasian economics is still an active research program[23] and it remains the standard framework in microeconomics textbooks at every level.

Second, in addition to the question about the existence of the fall is a broader question about the future of neoclassical economics and rational choice theory more generally. Walrasian general equilibrium theory is a particular version of neoclassical economics and to some extent utilizes rational choice theory in its characterization of individual agents. But the extensive literature in experimental and behavioral economics that finds repeated systematic empirical violations of rational choice theory and/or the stability and reversibility that is characteristic of preferences in neoclassical models, clearly has implications for the future of Walrasian economics. If rational choice theory somehow comes to be replaced as the core organizing framework for the way that economists think about, model, and formalize individual behavior, then certainly Walrasian economics is in for an even bigger downturn than it has experienced during the last few years. Of course these issues are still in flux and that severely limits what one can say about the "fall" of Walrasian economics at the current time.

Finally, there is the question of the current status of Keynesian economics. Twenty-five or so years ago it was quite clear that Keynesian economics had fallen from grace within the economics profession (although not necessarily from policy makers and undergraduate instruction[24]), but now, with the recent financial crisis and world economic recession, this is not nearly so clear. It is certainly possible there will be a Keynesian revival in macroeconomics and finance within the profession, as there has been to some extent among policy makers and those within the financial community; perhaps Hyman Minsky

[23] Consider for example recent developments in post-SMD Walrasian theory such as the so-called Brown-Matzkin literature (Rizvi 2006) based on the work of Donald Brown, Rosa Matzkin, and others (e.g. Aloqeili 2005, Brown and Matzkin 1996, Brown and Shannon 2000, Chiappori and Ekeland 2004, Matzkin 2005).

[24] See the various papers in De Vroey and Hoover (2004).

(1975, 1986) will be on graduate reading lists in a few years rather than Robert Lucas (1981) and Eugene Fama (1970). Then again, perhaps not. The point is simply that we may be in a period of significant change within the discipline of economics – change that has very much involves both Walrasian and Keynesian theory – and that presents a serious challenge to an attempt to provide an historical analysis of the fall of Walrasian theory.

Taking into consideration all that was said in the previous three paragraphs, I do think it is possible to say a couple things about the "fall" of Walrasian microeconomics and its relationship to Keynesian theory and the neoclassical synthesis. First of all, whether Walrasian economics has taken, or will take, a terminal fall or not, it is clear that it has been substantially demoted during the last few decades. Second, whether Keynesian economics stages a comeback or not, during the late 1970s and 1980s it definitely fell from the position it held during the heyday of the neoclassical synthesis. Not only does it seem reasonable to accept these stylized facts of the history of modern economics, it also seems reasonable to agree about some of the causes for both of these events. On the Walrasian side it is clear that the failure of stability analysis – starting in the 1960s with the counterexamples by David Gale (1963) and Herbert Scarf (1960), and exacerbated by the SMD results which opened the floodgates for more counterexamples[25] – and the associated failure of to prove uniqueness raised serious challenges to the hegemony of the Walrasian program. As Alan Kirman explained, without "stability or uniqueness, the intrinsic interest of economic analysis based on the general equilibrium model was extremely limited" (Kirman, 2006, p. 257). With respect to Keynesian economics the inflation and supply-side oil shocks of the 1970s were clearly empirical factors, and the rise of monetarism and the failure of the microfoundations project certainly contributed on the theoretical side. It is not necessary to debate the details, or weight the relative significance, of any of these factors; for the purposes here all that is required is agreement that the Walrasian program has faltered during the last decade or so, the Keynesian program came to be seen as a failure by the 1980s, and that the things mentioned here on the theoretical and empirical side played some role in these negative developments. Given all this it seems quite clear that there are a few ways in which Keynesian economics – and the previous co-evolution of the two research programs – contributed to the decline of Walrasian economics.

The main point of this section will be that Walrasian theory ran into trouble at precisely the points where the Keynesian influence was most

[25] The literature here is quite extensive. See for example (Ingrao and Israel 1990; Kirman 1989, 2006; Rivzi 1998, 2003, 2006; Scarf 1981)

pronounced. Consider stability first. Almost all of the serious theoretical problems associated with the Arrow-Debreu research program revolve around the stability of the tâtonnement adjustment mechanism (T). The problem is that the tâtonnement process is only globally stable when very restrictive additional assumptions are imposed on excess demand functions (and local stability is only marginally easier): these include gross substitutes, the Weak Axiom of revealed preference holding on aggregate excess demands, a dominant diagonal on the excess demand Jacobian matrix, and several others (see chapters 11 and 12 of Arrow and Hahn 1971). These assumptions are restrictive in at least four different senses. First, they are over and above what is implied by the standard Arrow-Debreu assumptions on consumer and firm behavior (they are theoretically restrictive). Second, there is no obvious reason why the behavior they would require should actually be the behavior of consumers and firms in a competitive market economy (they are empirically restrictive). Third, they are much more restrictive than what is required for existence of competitive equilibrium (the main positive result of the research program). And fourth, they are only sufficient, not necessary, conditions. The goal of stability theory was to find out what stability implied – in the same way that Slutsky, Hicks, Allen, and others found out what income-constrained utility maximization implied – but all that could be found were a variety of different conditions that implied stability (not what stability implied). Of course there were also problems with uniqueness, but in every special case where the tâtonnement process is globally stable the equilibrium price vector (p*) is also unique (see chapter 9 of Arrow and Hahn 1971). And there were also problems with comparative statics, but again the problem is really about stability. The correspondence principle attempts to derive comparative statics from stability, but if the stability results are not available, or weak, or economically uninterpretable, then the comparative statics results inherit these same problems. As Arrow and Hahn explained:

> Thus what the "correspondence principle" amounts to is this: Most of the restrictions on the form of the excess-demand functions that are at present known to be sufficient to ensure global stability are also sufficient to allow certain exercises in comparing equilibria. It should be added that these same conditions also turn up in the discussion of the uniqueness of competitive equilibrium. All these restrictions share the characteristic that they are not necessary for the task for which they were invented; they are only sufficient and this explains why the correspondence principle "isn't." (Arrow and Hahn, 1971, p. 321)

As discussed in section 2C, the stability of the system of differential equations (T) was not part of the original Walrasian model and by

Pareto's Manual there was no discussion of multiple market stability at all. It was also not a part – and was considered one of the problems – in the macro-Walrasian general equilibrium theory that came after the Keynesian fall from grace. As Robert Lucas explains:

> Samuelson proposed a dynamic model of price adjustment in which the rates of change of prices offered in each market were related to the level of "excess demands" in all markets. Whatever the history or underlying objectives of this model of price dynamics … this theory introduced sufficient additional (to those needed to describe tastes and technology) parameters to the equilibrium system so that, given an initial shock to the system, a wide variety of paths were consistent with it eventual return to equilibrium.
>
> This introduction of additional … free parameters held out the promise that one could construct a theoretical system the stationary point of each which was a general equilibrium in the neoclassical sense but whose movements, out of equilibrium, might replicate the "Keynesian" behavior captured so well by the econometric models … The objective of the enterprise was widely agreed to be "unification" of the two types of theories into which Keynesian ideas were translated in the 1930s and 1940s.[26]

The idea that the stability of the tâtonnement process should be one of the most important issues for Walrasian general equilibrium theory was a product of the neoclassical synthesis and the effort to unify Keynesian and Walrasian economics. It is clear from the contributions of those like Lange and Samuelson in the 1940s, and it is also clear from its rejection in the post-synthesis Walrasian macro literature of those like Lucas. The stability of the tâtonnement ultimately became a very serious problem for Walrasian economics, and it was a problem that developed right at the particular point in the Arrow-Debreu theoretical edifice where the Keynesian co-evolution had left its greatest impact on Walrasian theory.

But this is not all there is to the story; there is yet another Keynesian aspect to the theoretical difficulties that developed within Arrow-Debreu general equilibrium theory. Not only was the stability of the neoclassical synthesis-inspired tâtonnement the main problem, "the stability problem" was itself a product of the income terms in the Hicks-Slutsky version of Walrasian demand theory. The effort to forge a seamless connection between Walrasian and Keynesian theory contributed to an emphasis on both tâtonnement dynamics and income effects. The income effects were the primary cause of instability in Walrasian models which

[26] Think of the H_i functions in (T) or the k_is in (T') as Lucas's "free parameters."

in turn became the most important theoretical difficulty for the Walrasian program.

To see the problem, recall the discussion from section 2D above and the Slutsky expression given in (S). As noted there, the Slutsky matrix is negative semi-definite,[27] but consumer choice theory imposes no sign restrictions on the income effects (goods could be normal or inferior). As noted above, the standard way of proving global stability was to find a Liapunov function. Applying the Liapunov result from this period to problem of Walrasian stability, we have that if there exists a function V[p(t)] defined over the price path p(t) generated by (T) with the following three properties (glossing over various mathematical complexities and assuming the numéraire good has been eliminated):[28]

a) $V\left[p(t)\right] > 0$ for all $p \neq p^*$,

b) $\dfrac{dV\left[p(t)\right]}{dt} < 0$ for all $p \neq p^*$,

c) $\dfrac{dV\left[p^*\right]}{dt} = V\left[p^*\right] = 0$,

then the equilibrium price vector (p*) is globally asymptotically stable. One popular Liapunov function was:[29]

$$V\left[p(t)\right] = \frac{1}{2}\sum_{i=1}^{n} z_i^2\left[p(t)\right].$$

Computing the time derivative of this Liapunov function we have:

$$\frac{dV\left[p(t)\right]}{dt} = z^T[p(t)]JZ\left[p(t)\right]z[p(t)] \quad \text{for all } p \neq p^*,$$

where the right-hand side is a quadratic form of the excess demand Jacobian matrix JZ. If the matrix JZ forms a negative definite quadratic form, then the expression on the right hand side will be negative for all nonequilibrium prices and equal to zero at equilibrium, which in turn implies the tâtonnement is globally stable.

---

[27] And will be negative definite if the numéraire row and column is eliminated as it normally would be for stability analysis.

[28] See Arrow and Hahn (1971, ch. 11).

[29] Arrow and Hurwicz (1958).

But the matrix JZ – remember it is the Jacobian of the excess demand function (demand minus supply) – will consist of three separate parts,

$$JZ = S - M - F \ ,$$

where the S and M matrices are from the market demand functions – they are the market equivalents of the substitution and income effects in (S) respectively – and the F matrix is the supply function Jacobian (see Mukherji, 1974, pp. 247-8). We know the S matrix is negative definite from the standard Slutsky results on demand functions and we also know that the traditional assumptions on the production side of the Arrow-Debreu model guarantee that the matrix F is positive semi-definite (Arrow and Hahn, 1971, p. 72). So both of these terms are signed the "right way" for stability. Neither the substitution effects on the demand side or the supply side are a problem. The only problem – the only matrix that is not signed the "right way" by the standard assumptions – is the matrix of income effects (M above). This means that the full burden of all of the stability "problems" in general equilibrium systems rests with the income terms on the demand side of the market excess demand functions. The various conditions that have been demonstrated to be sufficient for global stability during the late 1950s and 1960s all amount in various ways to getting around, having substitution effects dominate, or otherwise eliminating, the problematic income effects.

So it seems that Keynesian economics must bear some responsibility for the fall, or at least faltering, of Walrasian general equilibrium theory during the last few decades. Any neoclassically-inspired theory that could get along with Keynesian economics well enough to form a stable partnership would need to be able to account for unemployment/disequilibrium in an otherwise general equilibrium world and it would need to have nominal income matter to consumer choice behavior: thus (T) and (S). And yet these two aspects of the Walrasian theory of the neoclassical synthesis are right at the heart of the Walrasian program's later technical difficulties. The features of Walrasian theory that were most influenced by its co-evolution with Keynesian economics during the middle of the twentieth century were precisely the features most responsible for its decline at the end of the century.

4. Conclusion

This paper has tried to make a fairly simple point. What Keynesian and Walrasian economics evolved into – what they became – when they stabilized into textbook macro and (advanced) textbook micro during the 1950s and 1960s, was, at least in part, a result of the fact they were joined together in, and co-evolved within the context of, the neoclassical

synthesis. Even if it is assumed they each contained certain hard core concepts identifiable across time, there were particular features of each in the later period that emerged because those particular aspects had survival value for the synthesis they were both a part of, and thus, because of the influence of the other theory. Many historians of economic thought readily accept that Keynesian economics in its heyday was what it was at least in part because of its compact with Walrasian economics. My point was simply that the agreement involved commitment from both research programs and that influence flowed both ways.

In section two I listed four ways that Keynesian ideas contributed to the eventual success of Walrasian micro over its immediate theoretical competition and/or influenced the content of the theory in its final form. Frankly, taken in isolation, none of these points seems to be very significant, but I believe that taken together they provide a substantial amount of new insight into how the co-evolution of the two research programs manifested itself on the Walrasian side. One of the main arguments in this section was that the theoretical equilibrium of mid-twentieth century Walrasian economics – unlike a stable equilibrium generated by the Walrasian tâtonnement – was highly path dependent. In the third section I tried to show how the main theoretical problems of Walrasian theory at the end of the century – primarily stability, but also related issues such as uniqueness, comparative statics, and the SMD results – emerged at precisely the point within the Walrasian program where the Keynesian imprint was most visible.

Although I believe the story told here is an important explanation of both the rise and the fall (or faltering) of Walrasian general equilibrium theory, I noted repeatedly in the paper that I was in no way attempting to provide the only, or perhaps even the main, reason that Walrasian economics took the particular path that it did during the twentieth century. There are were many other forces at work in the life-history of Walrasian economics – other forces that, at particular moments, may have mattered more than the Keynesian connection. My point was simply that Keynesian theory mattered (something not generally recognized), not that it was the only thing that mattered.[30] In closing, I would like to note that I believe a number of political-economic factors – specifically the threat to liberal society posed by the great depression and the cold war – also played a significant role in the co-evolution of these two economic research programs.[31] These issues were not considered at all in this paper, but their influence in no way conflicts with the more conceptual, theoretical, and technical issues examined here. On the other hand, this can also be said for a variety of other factors not discussed here. It is a

[30] As I indicated in note 7, I have discussed a number of other factors in previous research.
[31] See Amadae (2003), Bernstein (2001), and Mirowski (2002) for discussion of some of these issues.

broad tent; I have just let in one more, new and important, contributing factor.

References

Allen, R. G. D. (1936), "Professor Slutsky's Theory of Consumer's Choice," Review of Economic Studies, 3, 120-29.

Aloqeili, Marwan (2005), "On the Characterization of Excess Demand Functions," Economic Theory, 26, 217-225.

Amadae, Sonja M. (2003), Rationalizing Capitalist Democracy: The Cold War Origins of Rational Choice Theory. Chicago: University of Chicago Press.

Armstrong, W. E. (1939), "The Determinateness of the Utility Function," The Economic Journal, 49, 453-467.

Arrow, Kenneth J., Block, H. David, and Hurwicz, Leonid (1959), "On the Stability of the Competitive Equilibrium II," Econometrica, 27, 82-109.

Arrow, Kenneth J. and Hahn, Frank H. (1971), General Competitive Analysis. San Francisco: Holden-Day.

Arrow, Kenneth J. and Hurwicz, Leonid (1958), "On the Stability of the Competitive Equilibrium I," Econometrica, 26, 522-552.

Backhouse, Roger E. (2003), "The Stabilization of Price Theory, 1920-1955," in A Companion to the History of Economic Thought, W. J. Samuels, J. E. Biddle, and J. B. Davis (eds.), Oxford: Wiley-Blackwell, 308-324.

Backhouse, Roger E. and Laidler, David (2004), "What Was Lost with IS-LM?" in The IS-LM Model: Its Rise, Fall, and Strange Persistence, M. De Vroey and K. D. Hoover (eds.), Durham, NC: Duke University Press, 25-56 [Annual Supplement to Volume 36 History of Political Economy].

Barnett, Vincent (2004), "E. E. Slutsky: Mathematical Statistician, Economist, and Political Economist," Journal of the History of Economic Thought, 26, 3-18.

Bernardelli, Harro (1952), "A Rehabilitation of the Classical Theory of Marginal Utility," Economica, 19, 254-68.

Bernstein, Michael, A. (2001), A Perilous Progress: Economists and Public Purpose in Twentieth-Century America. Princeton: Princeton University Press.

Bowles, Samuel and Gintis, Herbert (2000), "Walrasian Economics in Retrospect," Quarterly Journal of Economics, 115, 1411-1439.

Brown, Donald J. and Matzkin, Rosa L. (1996), "Testable Restrictions on the Equilibrium Manifold," Econometrica, 64, 1249-1262.

Brown, Donald J. and Shannon, Chris (2000), "Uniqueness, Stability, and Comparative Statics in Rationalizable Walrasian Markets," Econometrica, 68, 1529-39.

Camerer, Colin F. and Loewenstein, George (2004), "Behavioral Economics: Past, Present, and Future," in Advances in Behavioral Economics, Princeton: Princeton University Press, 3-51.

Chiappori, Pierre-Andre and Ekeland, Ivar (2004), "Individual Excess Demand," Journal of Mathematical Economics, 40, 41-57.

Chipman, John S. (2004), "Slutsky's Praxeology and His Critique of Böhm-Bawerk," Structural Change and Economic Dynamics, 15, 345-56.

Chipman, John S. and Lenfant, Jean-Sebastien (2002), "Slutsky's 1915 Article: How it Came to be Found and Interpreted," History of Political Economy, 34, 553-97.

Clarida, Richard; Gali, Jordi; and Gertler, Mark (2000), "The Scope of Monetary Policy: A New Keynesian Perspective," Journal of Economic Literature, 37, 1661-707.

Clower, Robert W. (1965), "The Keynesian Counter-Revolution: A Theoretical Appraisal," in The Theory of Interest Rates, F. Hahn and F. Brechling (eds.), London: Macmillan, 103-125.

Coddington, Alan (1983), Keynesian Economics: The Search for First Principles. London: George Allen & Unwin.

Colander, David (2000), "The Death of Neoclassical Economics," Journal of the History of Economic Thought, 22, 127-43.

Colander, David (ed.), (2006), Post Walrasian Macroeconomics: Beyond the Dynamic Stochastic General Equilibrium Model. Cambridge: Cambridge University Press.

Colander, David; Holt, Richard; and Rosser, Barkley (2004a), "The Changing Face of Mainstream Economics," Review of Political Economy, 16, 485-99.

Colander, David; Holt, Richard; and Rosser, Barkley (2004b), Changing Face of Economics: Conversations with Cutting Edge Economists. Ann Arbor: University of Michigan Press.

Davis, John B. (2006), "The Turn in Economics: Neoclassical Dominance to Mainstream Pluralism," Journal of Institutional Economics, 2, 1-20.

Davis, John B. (2008), "The Turn in Recent Economics and Return of Orthodoxy," Cambridge Journal of Economics, 32, 349-366.

Debreu, Gerard (1974), "Excess Demand Functions." Journal of Mathematical Economics, 1, 15-21.

DellaVigna, Stafano (2009), "Psychology and Economics: Evidence from the Field," Journal of Economic Literature, 47, 315-72.

De Vroey, Michel and Hoover, Kevin D. (eds.) (2004), The IS-LM Model: Its Rise, Fall, and Strange Persistence. Durham, NC: Duke University Press [Annual Supplement to Volume 36 of History of Political Economy].

Donzelli, Franco (2006), "Walras and Pareto on the Meaning of the Solution Concept in General Equilibrium Theory," University of Milano Economics, Business and Statistics Working Paper No. 2006-31. Available at SSRN: http://ssrn.com/abstract=939539.

Donzelli, Franco (2007), "Equilibrium and Tâtonnement in Walras's Elements," History of Economic Ideas, 15, 85-138.

Evans, Griffith C. (1930), Mathematical Introduction to Economics. New York: McGraw-Hill Book Co.

Fama, Eugene E. (1970), "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance, 25, 383-417.

Friedman, Milton (1953), "The Marshallian Demand Curve," in Essays in Positive Economics. Chicago: The University of Chicago Press, 47-99.

Frisch, Ragnar (1926) "On A Problem in Pure Economics," Norsk Matematisk Forenings Skrifter [translation by J. S. Chipman, published in Preferences, Utility, and Demand, Chipman, John S.; Hurwicz, Leonid; Richter, Marcel; and Sonnenschein, Hugo F. (eds.), 1971, New York: Harcourt Brace Jovanovich, 386-423].

Gale, David (1963), "A Note on Global Instability of Competitive Equilibrium," Naval Research Logistics Quarterly, 10, 81-87.

Georgescu-Roegen, Nicholas (1936), "The Pure Theory of Consumer's Behaviour," Quarterly Journal of Economics, 50, 545-93.

Georgescu-Roegen, Nicholas (1950), "The Theory of Choice and the Constancy of Economic Laws," Quarterly Journal of Economics, 50, 545-93.

Georgescu-Roegen, Nicholas (1968), "Utility," in International Encyclopedia of the Social Sciences, D. L. Sills (ed.), New York: Macmillan, 236-67.

Giocoli, Nicola (2003), Modeling Rational Agents: From Interwar Economics to Early Modern Game Theory. Cheltenham, UK: Edward Elgar.

Goodfriend, Marvin and King, Robert (1997), "The New Neoclassical Synthesis and the Role of Monetary Policy," in NBER Macroeconomics Annual 1997, B. Bernanke and J. Rotemberg (eds.), Cambridge, MA: MIT Press, 231-96.

Hands, D. Wade (1994), "Restabilizing Dynamics," Economics and Philosophy, 10, 243-83.

Hands, D. Wade (2006), "Integrability, Rationalizability, and Path-Dependency in the History of Demand Theory," in Agreement on Demand: Consumer Theory in the Twentieth Century, P. Mirowski and D. W. Hands (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 38 of History of Political Economy], 153-85.

Hands, D. Wade (2007), "2006 HES Presidential Address: A Tale of Two Mainstreams: Economics and Philosophy of Natural Science in the mid-Twentieth Century," Journal of the History of Economic Thought, 29, 1-13.

Hands, D. Wade (2008a), "Economics, Psychology, and the History of Consumer Choice Theory," Cambridge Journal of Economics [forthcoming].

Hands, D. Wade (2008b), "Back to the Ordinalist Revolution: Behavioral Economic Concerns in Early Twentieth Century Consumer Choice Theory" Available at SSRN: http://ssrn.com/abstract=1128950

Hands, D. Wade (2009), "Stabilizing Consumer Choice: The Role of 'True Dynamic Stability" and Related Concepts in the History of Consumer

Choice Theory," The European Journal of the History of Economic Thought [forthcoming].

Hands, D. Wade and Mirowski, Philip (1998), "Harold Hotelling and the Neoclassical Dream," in Economics and Methodology: Crossing Boundaries, R. Backhouse, D. Hausman, U. Mäki, and A. Salanti (eds.), London: Macmillan, 322-397.

Hanson, Alvin (1949), Monetary Theory and Fiscal Policy. New York: McGraw-Hill

Hicks, John R. (1934), "Leon Walras," Econometrica, 2, 338-48.

Hicks, John R. (1937), "Mr. Keynes and the 'Classics': A Suggested Interpretation," Econometrica, 5, 147-59.

Hicks, John R. (1946), Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory, 2nd edition, Oxford: Clarendon Press [1st edition 1939].

Hicks, John R. (1980-81), "IS-LM: An Explanation," Journal of Post Keynesian Economics, 3, 139-54.

Hicks, John R. and Allen, R. G. D. (1934), "A Reconsideration of the Theory of Value, Parts I and II," Economica, 2, 52-76, 196-219.

Hoover, Kevin D. (1988), The New Classical Macroeconomics. Oxford: Basil Blackwell.

Hotelling, Harold (1932), "Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions," Journal of Political Economy, 40, 577-616.

Ingrao, Bruna and Israel, Giorgio (1990), The Invisible Hand: Economic Theory in the History of Science. Cambridge, MA: MIT Press.

Jaffé, William (1967), "Walras' Theory of Tâtonnement: A Critique of Recent Interpretations," Journal of Political Economy, 75, 1-19.

Jaffé, William (1981), "Another Look at Léon Walras' Theory of Tâtonnement," History of Political Economy, 13, 313-36.

Kahneman, Daniel (2003), "Maps of Bounded Rationality: A Perspective on Intuitive Judgment," American Economic Review, 93, 1449--1475.

Kahneman, Daniel; Knetsch, Jack L. and Thaler, Richard (1990), "Experimental Tests of the Endowment Effect and the Coase Theorem," Journal of Political Economy, 98, 1325-48.

Keynes, John Maynard (1936), The General Theory of Employment, Interest and Money. London: Macmillan.

Kirman, Alan (1989), "The Intrinsic Limits of Modern Equilibrium Theory: The Emperor Has No Clothes," The Economic Journal, 99, 126-39.

Kirman, Alan (1992), "Whom or What Does the Representative Individual Represent," Journal of Economic Perspectives, 6, 117-36.

Kirman, Alan (2006), "Demand Theory and General Equilibrium: From Explanation to Introspection, A Journey Down the Wrong Road," in Agreement on Demand: Consumer Theory in the Twentieth Century, P. Mirowski and D. W. Hands (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 38 of History of Political Economy], 246-80.

Knetsch, Jack L. (1989), "The Endowment Effect and Evidence of Nonreversible Indifference Curves," American Economic Review, 79, 1277-1284.

Knetsch, Jack L. (1992), "Preferences and the Nonreversibility of Indifference Curves," Journal of Economic Behavior and Organization, 17, 131-39.

Knight, Frank H. (1944), "Realism and Relevance in the Theory of Demand," Journal of Political Economy, 52, 289-318.

Lange, Oscar (1942), "Say's Law: A Restatement and Criticism," in Studies in Mathematical Economics and Econometrics: In Memory of Henry Schultz, O. Lange, F. McIntyre, and T. O. Untema (eds.), Chicago: University of Chicago Press, 49-68.

Lange, Oscar (1944), Price Flexibility and Employment. Bloomington, IN: The Principia Press.

Leijonhufvud, Axel (1968), On Keynesian Economics and the Economics of Keynes: A Study in Monetary Theory. Oxford: Oxford University Press.

Lucas, Robert E. Jr. (1981), Studies in Business Cycle Theory. Oxford: Basil Blackwell.

Mantel, Rolf R. (1974), "On the Characterization of Aggregate Excess Demand," Journal of Economic Theory, 7, 348-353.

Mantel, Rolf R. (1977), "Implications of Microeconomic Theory for Community Excess Demand Functions," in Frontiers in Quantitative Economic IIIA, M. D. Intriligator (ed.), Amsterdam: North-Holland, 111-26.

Mas-Colell, Andreu; Whinston, Michael D.; and Green, Jerry R. (1995), Microeconomic Theory. New York: Oxford University Press.

Mayer, Thomas (2009), "On the Right Side for the Wrong Reason: Friedman on the Marshall-Walras Divide," in The Methodology of Positive Economics: Reflections on the Milton Friedman Legacy, U. Mäki (ed.), Cambridge: Cambridge University Press, 119-42.

Matzkin, Rosa L. (2005), "Identification of Consumer's Preferences When Their Choices are Unobservable," Economic Theory, 26, 423-43.

Minsky, Hyman P. (1975), John Maynard Keynes. New York: Columbia University Press.

Minsky, Hyman P. (1986), Stabilizing and Unstable Economy. New Haven, CT: Yale University Press.

Mirowski, Philip (2002), Machine Dreams: Economics Becomes a Cyborg Science. Cambridge: Cambridge University Press.

Mirowski, Philip and Hands, D. Wade (1998), "A Paradox of Budgets: The Postwar Stabilization of American Neoclassical Demand Theory," in From Interwar Pluralism to Postwar Neoclassicism, in M. S. Morgan and M. Rutherford (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 30 of History of Political Economy], 269-92.

Modigliani, Franco (1944), "Liquidity Preference and the Theory of Interest and Money," Econometrica, 12, 45-88.

Moore, Henry Ludwell (1914), Economic Cycles: Their Law and Cause. New York: Macmillan [Augustus M. Kelley reprint 1967].

Morgan, Mary S. and Rutherford, Malcolm (eds.),(1998), From Interwar Pluralism to Postwar Neoclassicism. Durham, NC: Duke University Press [Annual Supplement to Volume 30 of History of Political Economy].

Morgenstern, Oskar (1948), "Demand Theory Reconsidered," Quarterly Journal of Economics, 62, 165-201.

Morishima, Michio (1977), Walras' Economics: A Pure Theory of Capital and Money. Cambridge: Cambridge University Press.

Mukherji, Anjan (1974), "Stability in an Economy with Production," in Trade, Stability and Macroeconomics: Essays in Honor of Lloyd A. Metzler, G. Horwich and P. A. Samuelson (eds.), New York: Academic Press, 243-58.

Pasinetti, Luigi L. (2007), Keynes and the Cambridge Keynesians. Cambridge: Cambridge University Press.

Patinkin, Don (1965), Money, Interest, and Prices: An Integration of Monetary and Value Theory, 2nd edition, New York: Harper & Row [1st edition 1956].

Pareto, Vilfredo (1971), Manual of Political Economy. New York: Agustus M. Kelley [translated by Ann S. Schwier from the 1927 French edition].

Pearce, Kerry A. and Hoover, Kevin D. (1995), "After the Revolution: Paul Samuelson and the Textbook Keynesian Model," in New Perspectives on Keynes, A. F. Cottrell and M. S. Lawlor (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 27 of History of Political Economy], 183-216.

Pigou, Arthur (1933), The Theory of Unemployment. London: Macmillan.

Rizvi, S. Abu Turab (1998), "Responses to Arbitrariness in Contemporary Economics," in New Economics and Its History, John B. Davis (ed.), Durham, NC: Duke University Press [Annual Supplement to Volume 29 of History of Political Economy], 275-88.

Rizvi, S. Abu Turab (2003), "The Stabilization of Price Theory, 1920-1955," in A Companion to the History of Economic Thought, W. J. Samuels, J. E. Biddle, and J. B. Davis (eds.), Oxford: Wiley-Blackwell, 372-94.

Rizvi, S. Abu Turab (2006), "The Sonnenschein-Mantel-Debreu Results After Thirty Years," in Agreement on Demand: Consumer Theory in the Twentieth Century, P. Mirowski and D. W. Hands (eds.), Durham, NC: Duke University Press [Annual Supplement to Volume 38 of History of Political Economy], 228-45.

Robertson, Dennis H. (1952), Utility and All That and Other Essays. London: Allen & Unwin.

Robinson, Joan (1975), “What Has Become of the Keynesian Revolution?” in Essays on John Maynard Keynes, M. Keynes (ed.), Cambridge: Cambridge University Press, 123-31.

Samuelson, Paul A. (1938), "A Note on the Pure Theory of Consumer's Behaviour," Economica, 5, 61-71.

Samuelson, Paul A. (1941), “The Stability of Equilibrium: Comparative Statics and Dynamics.” Econometrica, 9, 97-120.

Samuelson, Paul A. (1942), “The Stability of Equilibrium: Linear and Non-Linear Systems,” Econometrica, 10, 1-25.

Samuelson, Paul A. (1944), “The Relation Between Hicksian Stability and True Dynamic Stability,” Econometrica, 12, 256-257.

Samuelson, Paul A. (1947), Foundations of Economic Analysis. Cambridge, MA: Harvard University Press.

Samuelson, Paul A. (1964), Economics. 6th edition, New York: McGraw-Hill.

Scarf, Herbert (1960), "Some Examples of Global Instability of the Competitive Equilibrium," International Economic Review, 1, 157-72.

Scarf, Herbert (1981), “Comment on: ‘On the Stability of Competitive Equilibrium and the Patterns of Initial Holdings: An Example’,” International Economic Review, 22, 469-470.

Schultz, Henry (1928), Statistical Laws of Demand and Supply: with Special Application to Sugar. Chicago: University of Chicago Press.

Schultz, Henry (1938), The Theory and Measurement of Demand, Chicago: University of Chicago Press.

Shafer, Wayne and Sonnenschein, Hugo (1982), "Market Excess Demand Functions," in Handbook of Mathematical Economics, Vol. II, K. J. Arrow and M. D. Intriligator (eds.), Amsterdam: North-Holland, 671-93.

Slutsky, Eugene E. (1915), "Sulla Teoria del Bilancio del Consonatore," Giornale degli Economisti, 51, 1-26. Translated in Readings in Price Theory, G. J. Stigler and K. E. Boulding (eds.), Homewood, IL: Richard D. Irwin, 1952, 27-56.

Snowdon, Brian and Vane, Howard R. (2005), Modern Macroeconomics. Cheltenham, UK; Edward Elgar.

Sonnenschein, Hugo (1972), "Market Excess Demand Functions," Econometrica, 40, 549-63.

Sonnenschein, Hugo (1973), "Do Walras' Identity and Continuity Characterize the Class of Community Excess Demand Functions?," Journal of Economic Theory, 6, 345-54.

Thaler, Richard H. (1980), “Toward a Positive Theory of Consumer Choice,” Journal of Economic Behavior and Organization, 1, 39-60.

Tobin, James (1958), “Liquidity Preference as Behavior Towards Risk,” Review of Economic Studies, 25, 65-86.

Tobin, James (1969), “A General Equilibrium Approach to Monetary Theory,” Journal of Money, Credit and Banking, 1, 15-29.

Uzawa, Hirofumi (1960), “Walras’ Tâtonnement in the Theory of Exchange,” Review of Economic Studies, 27, 182-194.

Van Daal, Jan and Walker, Donald A. (1990), "The Problem of Aggregation in Walras's General Equilibrium Theory," History of Political Economy, 22, 489-505.

Walras, Leon (1954), Elements of Pure Economics. Translated by W. Jaffé form the 4th definitive edition 1926, Homewood, IL: Richard D. Irwin [1st edition 1974, Augustus M. Kelley reprint 1977].

Weber, Christian E. (1999a), "Slutsky and Additive Utility Functions," History of Political Economy, 31, 394-416.

Weber, Christian E. (1999b), "More on Slutsky's Equation as Pareto's Solution," History of Political Economy, 31, 575-86

Weintraub, E. Roy (1979), Microfoundations: The Compatibility of Microeconomics and Macroeconomics. Cambridge: Cambridge University Press.

Weintraub, E. Roy (1985), General Equilibrium Analysis: Studies in Appraisal. Cambridge: Cambridge University Press.

Weintraub, E. Roy (1991), Stabilizing Dynamics: Constructing Economic Knowledge. Cambridge: Cambridge University Press.

Weintraub, E. Roy (2002), How Economics Became a Mathematical Science. Durham, NC: Duke University Press.