



OXFORD JOURNALS
OXFORD UNIVERSITY PRESS

Cournot and Mathematical Economics

Author(s): Irving Fisher

Source: *The Quarterly Journal of Economics*, Vol. 12, No. 2 (Jan., 1898), pp. 119-138

Published by: [Oxford University Press](#)

Stable URL: <http://www.jstor.org/stable/1882115>

Accessed: 29-04-2015 00:41 UTC

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at
<http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Oxford University Press is collaborating with JSTOR to digitize, preserve and extend access to *The Quarterly Journal of Economics*.

<http://www.jstor.org>

THE
QUARTERLY JOURNAL
OF
ECONOMICS

JANUARY, 1898

COURNOT AND MATHEMATICAL ECONOMICS.

“Cournot’s genius must give a new mental activity to every one who passes through his hands.”—MARSHALL.

THE appearance in English* of Cournot’s *Principes Mathématiques* offers a suitable occasion for a review of that remarkable work and of the later developments of economic method which it foreshadowed. In the six decades since the original work was published, a decided change has taken place in the modes of conceiving and treating economic problems. For good or for ill the mathematical method has finally taken root, and is flourishing with a vigor of which both its friends and enemies little dreamed. Sixty years ago the mathematical treatise of Cournot was passed over in silence, if not contempt. To-day the equally mathematical work of Pareto is received with almost universal praise. In Cournot’s time “mathematical economists” could be counted on one’s fingers, or even thumbs. To-day they muster some thirty active enthusi-

**Researches into the Mathematical Principles of the Theory of Wealth.* By Augustin Cournot, 1838, translated by Nathaniel T. Bacon. In the “Economic Classics” series. Macmillan, 1897.

asts and a much larger number of followers and sympathizers. In 1838 there seems to have been no institution of learning besides the Academy at Grenoble, of which Cournot was rector, where "mathematical economics" were employed or approved. In 1898 there are at least a dozen such institutions, and in England alone half that number, Oxford and Cambridge among them. It is in France, the prophet's own country, where he is still without honor. When Cournot wrote, no journal existed in which such investigations as his could find a welcome. To-day the *Economic Journal*, the *Journal of the Royal Statistical Society*, the *Giornale degli Economisti*, and the *Nationaloekonomisk Tidsskrift* receive such material with more or less regularity; while, within the last eight years alone, twenty other journals have occasionally published economic articles containing mathematics. Opponents of the new method no longer venture to ignore or ridicule it, but, in academic circles at least, seek to acquaint themselves with its history and present aims as matters of necessary and professional information. In recognition of such wide-spread interest the latest Dictionary of Political Economy devotes some forty articles to the history, writings, methods, and terminology of the "mathematical school."

It may fairly be claimed that Cournot was the principal founder of this school.* For this reason, if for no other, his book is an "economic classic," and as such deserves careful study. But its interest is not simply historical. The bulk of its reasoning and conclusions has never yet been superseded. Those who now read it for the first time will find it as new and fresh as any modern investigation. As the original work has long been out of print and scarce in the antiquarian market, the present edition serves the double purpose of translation and second edition. Moreover, thanks to the painstaking work of the

* Cf. Walras, *Théorie Mathématique de la Richesse Sociale*, 1883, p. 9.

translator, it far surpasses the original in typographical accuracy, a prime requisite in a mathematical work.

Exclusive of the preface and appended bibliography, the text occupies 166 pages. Of this material, the last two chapters, making 45 pages, have only an historical interest. As we shall see, they are vitiated throughout by fallacious conceptions of income. About 18 other pages (namely, §§ 34, 39, 42, 46 (2d par.), 48, 49, 52, 54, 64, 65, 72, 73) may be omitted without loss of continuity and without great loss of substance. The remaining 103 pages are almost uniformly excellent, and will repay very thorough study by all who care for exact ideas and demonstrations in Political Economy. Furthermore, the reader will find, before he has gone far, that very thorough study is indispensable to a mastery of the subtle author. Jevons, though himself a mathematical economist, confesses, with characteristic candor, "I have by no means mastered all parts, . . . my mathematical power being insufficient to enable me to follow Cournot in all parts of his analysis."* The general trend of reasoning and the final conclusions will be patent to the most non-mathematical reader, and I have heard a distinguished economist of that description say that he found the book easy reading. But a slight acquaintance with the notations of the Differential Calculus is necessary for interpreting the formulæ, and considerable familiarity for deriving them in some cases. To lift the beginner in the Calculus over the last sort of difficulties, I have appended to this article a series of notes.† The work is, of course, not wholly mathematical. Of the 103 pages above mentioned, containing the essential parts, only about 70 are mathematical. The reader who will make up his mind at the outset to work his way through these pages at half-speed or quarter-speed need not chafe over

* *Theory of Political Economy*, preface to 2d edition, p. xxix of 3d edition.

† See the Appendix.

necessary hindrances and delays, and will not regret the extra time required.

In his preface Cournot defends his method of treating economic science. Few better statements exist of the aims and merits of "deductive" and "mathematical" economics. While welcoming all study of facts, Cournot insists on a framework of *theory* in which those facts fit. A very few facts (such as that demand increases with a decrease of price) suffice to determine the main outlines of that theory, though its exact form depends on the specific circumstances of each particular case. He answers the alleged objection to a mathematical treatment that economic problems lack the data for numerical solution:—

Those skilled in mathematical analysis know that its object is not simply to calculate numbers, but that it is also employed to find the relations between magnitudes which cannot be expressed in numbers and between *functions* whose law is not capable of algebraic expression. . . . Thus . . . theoretical mechanics furnishes to practical mechanics general theorems of most useful application, although in almost all cases recourse to experience is necessary for the numerical results which practice requires.*

Entering upon the book itself, we find that it naturally falls under three heads. The introductory chapters, treating of value, "absolute and relative," and of the foreign exchanges, are quite apart from the rest of the book. Chapters IV.—X. inclusive discuss the determination of prices under different conditions as to monopoly and competition, taxes and bounties. This portion of the work is the most distinctive and the most widely celebrated. The remaining two chapters give an ambitious but erroneous theory of "Social Income."

Chapter I. is devoted to defining wealth, which term Cournot uses in the sense of value in exchange. He carefully distinguishes this idea from *utility*, with which he conceives the economist has no direct concern. Here,

* Page 3.

of course, he differs materially from modern mathematical economists, beginning with Jevons and Walras. To prevent all misunderstanding, Cournot points out that, under his definition of wealth, the destruction of spices by the East India Company, though opposed to the general good, was a "real creation of *wealth* in the commercial sense of the word." What relations exist between wealth thus conceived and the welfare of the human race Cournot regards as too difficult a problem to admit of present solution. Yet he does not disparage efforts towards that end.

Chapter II. deals with "Changes in Value, Absolute and Relative," — a subject of engaging interest in these latter days of conflicting monetary standards. The reader will be filled with surprise and admiration at Cournot's anticipations of modern thought on this difficult topic. The *values* of a system of commodities are compared to the positions of a system of particles. The value of each commodity is expressed relatively to other commodities, just as the position of each particle is expressed by reference to the other particles. When a change occurs in the relative values or positions, the question arises, Which term of the comparison has suffered an *absolute* change? Clinging to physical analogy, Cournot cites the remarkable passage in Newton's *Principia* in which an "absolute space" is supposed as a background for mechanical motion, distinct from the "relative space" made up of the system of moving points. He does not despair of distinguishing statistically absolute and relative changes, and observes that in case all commodities except one, such as gold or silver, preserve the same relative values, the probability is greater that the one commodity has changed than that all the others have changed. Although the whole discussion lacks one of its modern elements,—the idea of utility,—it must nevertheless be regarded as more profound and worthy of serious consideration than most contemporaneous treatments of the same theme.

Of the third chapter, on foreign exchange, Jevons says, rather dubiously, that it is "highly ingenious, if not particularly useful."* Its utility, however, seems commensurate with the utility of the subject of which it treats. It is a correct first approximation, based on the hypothesis of a regularly recurring annual indebtedness between nations or "centres of exchange." So far as it fails to explain the complex facts of the exchange market, the failure is due to this arbitrary hypothesis, which neglects "dynamic" causes. When a completer theory is developed (so far as I am aware, none such exists as yet), it will establish laws governing the *oscillations* of exchanges and the part played by foresight and speculation in such transitions.

Supposing only two centres of exchange, (1) and (2), and supposing (1) to be annually indebted to (2) the sum of $m_{1,2}$ francs, and (2) to (1) $m_{2,1}$ francs, if $c_{1,2}$ is the rate of exchange at (1) on (2), "or the amount of silver given at the place (2) in exchange for a weight of silver expressed by 1 and payable at the place (1)," then $c_{1,2} = \frac{m_{2,1}}{m_{1,2}}$. If three centres are taken instead of two, the formula becomes more complicated; but it is still possible to derive the six rates of exchange in terms of the six sums of indebtedness between the three centres, and so on for any number of centres. The limits set by the "specie points" are discussed, and the case of exchange between gold and silver countries without a par of exchange is briefly touched upon.

With Chapter IV. the main portion of the work, the theory of prices, begins. Cournot assumes that the demand for an article, in the sense of the quantity of it annually consumed, varies with (*i.e.*, is a "function" of) its price. The relation between price and demand is delineated by the now familiar "demand curve," which

**Theory of Political Economy*, 3d edition, p. xxix.

Cournot was the first to introduce. The character of this relation — *i.e.*, the form of the demand curve — depends on “the kind of utility of the article, on the nature of the services it can render or the enjoyments it can procure, on the habits and customs of the people, on the average wealth, and on the scale on which wealth is distributed.”* As is well known, Walras and later writers have gone a step deeper into the analysis, and have shown how to deduce the *general* demand curve used by Cournot from a system of *individual* demand curves, and have in turn deduced the individual curves from systems of relations between the “utility of the article” and its quantity, and from the “nature and habits of the people” and the “scale on which wealth is distributed.” In doing this, they have not superseded Cournot, but have simply laid bare the foundations on which he built.

Given the law of demand, Cournot first supposes a complete monopoly of the article in question, and shows what price will yield the maximum profit. He points out (§ 30) that *fixed charges*, or costs which do not vary with the output, have no influence on price,—a theorem whose truth and importance are often overlooked to-day, except, perhaps, in America, where it has been made conspicuous both in railway experience and theory.† Only the *running expenses* figure in the determination of rates. Cournot shows that an increase of what would now be called marginal cost always causes an increase in the price under a monopoly, but that the rise of price is sometimes more and sometimes less than the amount of the increase of cost. The criterion for distinguishing the two cases is deduced and discussed.

In passing from the study of perfect monopoly to that of perfect competition, Cournot considers also the intermediate case of a few, say two, competitors. The operation of self-interest in this case will, Cournot contends,

*Page 47.

†See Hadley, *Railway Transportation*, p. 265.

cause an equilibrium price to emerge, which will be lower than if the two rivals had combined, but higher than if a third competitor should enter the field.

Cournot's treatment of this difficult problem is brilliant and suggestive, but not free from serious objections. The fault to be found with the reasoning is in his premise that each individual will act on the assumption that his rival's output is constant, and will strive only to so regulate his own output as to secure the largest profits. He is regarded as oblivious of the consequences of his action on the tactics of his rival, and as assuming that the price which will be charged by that rival will be neither more nor less than that necessary to take off the fixed output imputed to him plus the output decided upon by himself. Under these conditions, Cournot's conclusions will hold true. But the conditions are not those which actually apply to competition between two producers. A more natural hypothesis, and one often tacitly adopted, is that each assumes his rival's *price* will remain fixed, while his own price is adjusted. Under this hypothesis each would undersell the other as long as any profit remained,* so that the final result would be identical with the result of unlimited competition. But, as a matter of fact, no business man assumes either that his rival's output or price will remain constant any more than a chess player assumes that his opponent will not interfere with his effort to capture a knight. On the contrary, his whole thought is to forecast what move the rival will make in response to one of his own. He may lower his price to steal his rival's business temporarily or with the hope of driving him out of business entirely. He may take great care to preserve the *modus vivendi*, so as not to break the market and provoke a rate war. He may raise his price, if ruin-

* Cf. Bertrand, *Journal des Savants*, 1883, p. 503; Marshall, *Principles*, i., 2d edition, p. 457; Pareto, *Cours d'Économie Politique*, i. p. 67; Edgeworth, *Giornale degli Economisti*, June, 1897, p. 24.

ously low, in hopes that his rival, who is in the same difficulty, may welcome the change, and follow suit. The whole study is a "dynamic" one, and far more complex than Cournot makes it out to be. The completest treatment of this intricate and neglected problem is contained in Professor Edgeworth's brilliant articles in the *Giornale degli Economisti*.*

Passing on to the case of "unlimited competition" (Chapter VIII.), Cournot shows that the price is, in this case, equal to the "marginal cost of production." Cournot himself does not use this term nor any other verbal description of the magnitude involved. He confines himself to mathematical symbolism. $\phi(x)$ being the total cost, to a particular producer, of producing x units, $\phi'(x)$ will be equal to the price. Since $\phi'(x)$ is the rate of increase of cost per unit of increased product,—i.e., "marginal cost,"—Cournot must be counted among the anticipators of Jevons, Menger, and Walras. These anticipators now appear to be Bernouilli, Anderson, Ricardo, Von Thünen, Rae, Cournot, Dupuit, and Gossen.

If we plot the relation between the product of each individual and his resulting marginal cost, we have a system of individual supply curves. These may be combined into a single general supply curve, which Cournot uses. He shows, what is now familiar to every student, that the intersection of this general supply curve with the general demand curve determines price. It is significant of the slow growth of economic science that these graphic pictures of supply and demand, now in almost universal use in text-book and class-room, were ignored or forgotten by Cournot's contemporaries, and were only restored in 1870, when independently obtained by Fleeming Jenkin. With his name, rather than with Cournot's, they are generally associated to-day.

In the same chapter Cournot enunciates two other prin-

* 1897, June, October, and November.

ciples which have become classic, though, like that just mentioned, they are seldom duly credited to him. One is in regard to the law of diminishing returns (p. 91), and the other is that a tax on an article subject to "unlimited competition" will raise the price by an amount less than the tax itself (p. 93).

Cournot next considers the "mutual relations of producers" or the connections between complementary materials, such as copper and zinc, which enter jointly into the production of a composite, such as brass. Cournot was apparently the first to investigate such "joint demand." His study here, unlike the rest of his work, is confined to a special case; namely, that where the component articles enter in perfectly definite proportions into the joint article. He shows, among other things, that the control by a single monopolist of both copper and zinc will result in a lower price of brass than the control of copper by one monopoly and zinc by the other. That is, in the case of complementary commodities, it is better for the consumer to be at the mercy of one monopolist than two. An important application, Professor Edgeworth points out, is to railway rates, where, as is well known, lower fares follow the consolidation of connecting lines. But, although Cournot's conclusions are in the main consonant with facts, his analysis of motives in the minds of the two monopolists is subject to much the same objection as above expressed in the case of two competitors.* Turning to the more trustworthy case of unlimited competition, Cournot develops several interesting results, among them that a tax levied on one of the two component articles will raise the price of that article and of the composite article, but will *lower* the price of the other component.

In introducing the subject of import duties or bounties, "without pretending, which would be absurd, to contradict the opinion which has been very generally formed, of

* See Edgeworth, *Giornale degli Economisti*, 1897, June and October.

the advantages for the community procured by improvements in the means of communication or by the extension of markets," * Cournot suggests that the extreme position of free traders is untenable. In following out this contention, Cournot commits a mathematical blunder which invalidates his main thesis; namely, that a tariff on imports may, under certain peculiar circumstances, lower prices of the goods imported. Formulæ (6) on page 122 are erroneous † for reasons explained in the appended notes (No. 50). The correct formulæ may be transcribed from those given by putting zero for ϵ . With this change it will be seen that Cournot's arguments on pages 123 and 124 are quite destroyed.

This singular error supplies one of many examples of a serious fault in our talented author,—gross carelessness. In spite of extraordinary acuteness and precision of mind, Cournot was neglectful of his duties as verifier and proof-reader. The translator, Mr. Bacon, has convicted him of some thirty-five inaccuracies. Though most of these are obvious misprints, some are clearly due to hasty and heedless mathematical transformations. Fortunately, only two of them affect the economic conclusions drawn. The first has just been mentioned, and the second will soon appear. It was not ignorance or unfamiliarity with mathematics which caused these slips. The evidence, internal and external, is decisive against this view. Rather was it his very facility in employing the mathematical apparatus which led Cournot to omit the essential labor of reviewing his reasoning and of checking his results by common sense. The impossibility of formulæ (6), page 122, appears from the simplest inspection; for it is *a priori* evident that δ (the effect of the tax) ought to vanish when u (the tax) vanishes, which it does not do.‡

* Page 121.

† Cf. Edgeworth, in Palgrave's Dictionary, article "Cournot," and Berry and Sanger, quoted by Edgeworth, *Economic Journal*, 1894, p. 627.

‡ Cf. Arthur Berry, quoted by Edgeworth, *Economic Journal*, 1894, p. 627.

Most readers of Cournot have trusted his mathematics, but been puzzled by his conclusions. Professor Bastable tries to explain the matter by the influence of some prejudice on his judgment.* But such an explanation does not seem to be required. Cournot's "curious views" are in large measure due to the mathematical error above mentioned. Moreover, his whole book stamps him as the most dispassionate of truth-seekers. He expressly disclaims any feeling in favor of protection : —

If we have tried to overthrow the doctrine of Smith's school as to barriers, it was only from theoretical considerations, and not in the least to make ourselves the advocates of prohibitory and restrictive laws.†

Again, in his preface, he says : —

I am far from having thought of writing in support of any system, and from joining the banners of any party ; I believe that there is an immense step in passing from theory to governmental applications ; I believe that theory loses none of its value in thus remaining preserved from contact with impassioned polemics ; and I believe, if this essay is of any practical value, it will be chiefly in making clear how far we are from being able to solve, with full knowledge of the case, a multitude of questions which are boldly decided every day.

The two concluding chapters on "Social Income" are the most unsatisfactory in the book. They form one of those innumerable and futile attempts to define the income of a community and analyze its variations. Cournot here loses his accustomed perspicuity. He first describes social income as the sum of individual incomes, the latter term being regarded as self-explanatory. He then redefines it as the sum of commodities "for consumption." He thinks he bridges over the gap between these two descriptions of

* "The treatment of the topic [the benefits of a tariff] in so defective a manner by an able and critical investigator suggests the belief that some disturbing cause must have influenced his judgment, and his evident desire to discover a scientific foundation for protectionism furnishes us with a very probable explanation of his curious views." *International Trade*, 2d edition, p. 179.

† Page 171.

income on the theory that the price of any commodity "for consumption" consists of parts ascribable to the different agents of production. This being the case, if D be the entire consumption of a "commodity for consumption," and p the price, "the product pD will express the sum to the extent of which this commodity co-operates in making up the social income." If p_0D_0 be the value of this product at one time, and p_1D_1 that at another, the difference between them, $p_0D_0 - p_1D_1$, expresses the diminution of social income (assuming for illustration that p_1D_1 is the lesser of the two products). This diminution occurs in the incomes of the various persons contributing to the *production* of the commodity in question; and Cournot argues that the incomes of all other persons may be considered unchanged, for perturbations in the prices of other commodities are apt to occur as much in one direction as in the other (pp. 129-132.)

According to this reckoning, a dearth of a necessity of life may cause an *increase* of social income if the price rises faster than the quantity consumed falls! To overcome this difficulty, Cournot distinguishes between the "nominal" reduction of income just described ($p_0D_0 - p_1D_1$) and a *real* reduction of income. He attempts to describe this real reduction of income without describing any "real income." The real reduction is found by taking into account the sacrifices that *consumers* of the commodity suffer in paying higher prices. Although it was already shown that the incomes of consumers, as a whole, may be considered as unchanged, still those who continue to buy after the price has risen have to pay the rise $p_1 - p_0$ on their purchase D_1 , thus expending $(p_1 - p_0)D_1$ more income for precisely the same return. Hence they "are really in just the same situation as to fortune as if the commodity had not risen and their incomes had been diminished by $(p_1 - p_0)D_1$." Adding this virtual loss of income for consumers to the loss already shown for

producers, — namely, $p_0 D_0 - p_1 D_1$, — Cournot obtains $p_0(D_0 - D_1)$ as the *total* real loss. He confesses, however, that, even with this amendment, he has not taken account of the loss to consumers who have ceased to buy the commodity because of the increased price, or of part of the loss (in the shape of reduced purchases) to those who do buy, but buy less. He pleads in extenuation of this omission: “But this kind of damage cannot be estimated numerically. . . . Here comes in one of those relations of size which numbers can indicate, indeed, but cannot measure.” Had Cournot reached the conception of “consumers’ rent,” he would have seen that numbers can measure as well as indicate the damage in question.*

In the final chapter Cournot applies his ideas of income to international trade, and attempts to show in particular that a protective tariff may, under special circumstances, increase the national income. Inasmuch as the idea of income is so arbitrary and faulty, little or no importance attaches to such speculations.†

Such, in brief, are some of the main outlines of Cournot’s economic doctrines. It is not possible, however, to reproduce the striking and ingenious observations with which his pages bristle, or to reflect the strong, clear style in which those observations are expressed. To feel Cournot’s power and stimulus, the reader must actually “pass through his hands.” He will scarcely fail to come away with a “new mental activity.”

* Cf. Edgeworth, *Economic Journal*, 1894, p. 628. If the price rises from OT to OT' (see Fig. 6, in Cournot), the loss to consumers, as estimated by Cournot, is the rectangle TS' , whereas the loss of consumers’ rent is the trapezoid $STT'S'$. That is, the loss due to consumers giving up consumption, which loss was neglected by Cournot, is the triangle of which SS' is hypotenuse. Evidently, this triangle may be very large. Cournot’s erroneous views on social income are treated at length by Pareto, *Giornale degli Economisti*, 1891, vol. iv. pp. 1-14.

† In one of them Cournot falls again into mathematical error. The inequality near the end of p. 158 — namely, $E < E - (D_b - D'_b)$ — is incorrect. It implies that $D_b < D'_b$, which contradicts what was said on p. 155, line 9. The author seems to have forgotten that D'_b does not here mean quantity consumed, but quantity produced (see p. 151, § 88, line 9).

And yet it is not surprising that the book seemed a failure when first published. It was too far in advance of the times. Its methods were too strange, its reasonings too intricate, for the crude and confident notions of political economy then current. It was quite inevitable that it should be neglected and forgotten until such kindred spirits as Jevons and Walras pointed out its virtues. Cournot accepted the situation philosophically, and tried to make his theories more palatable by divesting them of the mathematical form. He published his *Principes de la Théorie des Richesses* in 1863, and in 1876, the year before his death, his *Revue sommaire des Doctrines Économiques*. Both contain new matter. The second is said to retain the more successfully the strength and virtues of the *Principes Mathématiques*. I have seen only the first. Of this Jevons * said, with justice, that it "does not compare favorably in interest and importance with" the *Principes Mathématiques*.

In the seventies the main work began to show signs of coming to life. Walras quoted and praised it in his *Éléments d'Économie Politique*,† published in 1874, and in later works.‡ In 1875 an Italian translation appeared in the excellent series of Boccardo, *Biblioteca dell' Economista*. In 1879 Jevons, in the preface to the second edition of his *Theory of Political Economy*, described the contents of the book, a copy of which he had found as early as 1872. With these sponsors the work was brought into prominence, and studied with care. Among those who served to extend its fame were Launhardt, Auspitz and Lieben, Lexis, Marshall, Edgeworth, Cossa, Pantaleoni, Pareto, and Barone. Marshall testifies that his mode of formulating economic problems was

most affected by mathematical conceptions of continuity, as represented in Cournot's *Principes Mathématiques de la Théorie des*

* *Theory*, 3d edition, p. xxx.

† *E.g.*, preface and p. 423.

‡ *E.g.*, *Théorie Mathématique de la Richesse Sociale*, 1883, p. 9.

Richesses. He taught that it is necessary to face the difficulty of regarding the various elements of an economic problem, not as determining one another in a chain of causation, *A* determining *B*, *B* determining *C*, and so on, but as all mutually determining one another. Nature's action is complex; and nothing is gained in the long run by pretending that it is simple, and trying to describe it in a series of elementary propositions.

Under the guidance of Cournot and in a less degree of Von Thünen, I was led to attach great importance to the fact that our observations of nature, in the moral as in the physical world, relate not so much to aggregate quantities as to increments of quantities.*

Edgeworth admires "Cournot's masterly analysis of the dealings between a monopolist seller and a number of buyers competing against each other," † and makes frequent quotations and comments in numerous articles on Taxation, International Trade, and Money.‡

Seligman, writing on taxation, says: —

The authors who have in some respects, and within a limited field, done the best work in the study of incidence of taxation, are precisely those who have hitherto generally been overlooked [namely, mathematical economists]. . . . Of these, by all means the ablest and most suggestive is Cournot.§

Cournot's influence and eminence have not been confined to economics. He was something of a man of affairs, as is evident from the positions which he occupied.|| His literary work was many-sided. In addition to editing and translating, he published several works of note, both on Mathematics, pure and applied, and on Philosophy. Louis Liard, a competent critic, in an extensive review of a half-dozen philosophical writings of Cournot, says: —

Pendant plus de quarante ans, il a mis au service de la philosophie une science profonde de géomètre, des connaissances encyclopédiques,

**Principles*, p. xiv.

† Address before Section F of British Association, 1889, *Nature*, September 19, 1889, p. 499.

‡ *E.g.*, *Economic Journal*, 1894, pp. 624, sq.; 1897, pp. 53, 69, 227, 229; *Giornale degli Economisti*, 1897, June, October, and November.

§ *Shifting and Incidence*, 1892, p. 80. || See Introduction to Translation.

une pénétration peu commune d'analyse, d'éminentes qualités d'invention, et une rare indépendance de pensée.*

Liard goes on to remark that, had Cournot been endowed with less modesty and more assurance, he would have been the recognized head of an independent school of philosophy, intermediate between those of Kant and Comte.

Of Cournot's mathematical writings, Todhunter speaks with praise; † while Bertrand, though very dubious as to the value of mathematical economics, wrote of him:—

Savant distingué, écrivain habile, esprit original et élevé, dans l'art des déductions, Cournot était un maître. M. Walras se fait honneur d'être son disciple.‡

Yet it is as economist rather than philosopher or mathematician that Cournot is to-day most remembered. He is fulfilling Jevons's prophecy that he would "occupy a remarkable position in the history of the subject." § Although some score of writers had preceded him in attempting to apply mathematical processes to political economy, he was the first to win substantial results. He alone of the early writers exerts to-day a powerful influence on economic thought. It is with him, therefore, that any survey of modern mathematical economics should begin.

Between 1838 and 1871, the date of publication of Jevons's *Theory*, some thirty mathematico-economic writings appeared. But their authors were, for the most part, ignorant of Cournot and of each other. The movement first got coherence and impetus from Jevons and Walras, when almost simultaneously they and Menger, the founder of the Austrian school, discovered, or at least rediscovered, the principle of marginal utility. The freshness and fruitfulness of these new ideas, the fact that

* "Un géomètre philosophe," *Revue des Deux Mondes*, 1877, iv. p. 102.

† See Jevons, *Theory*, 3d edition, p. xxiv.

‡ *Journal des Savants*, 1883, p. 499. § *Theory*, 3d edition, p. xxviii.

three writers reached them independently and at the same time, attracted wide attention. Jevons's lucid style and his eminence in other lines than economics aided greatly among English readers. A vigorous controversy ensued over the method proper to economic study.* Both the works of Jevons and Walras went through three editions. A small band of writers, including Edgeworth, Marshall, and Wicksteed in England, Pantaleoni, Pareto, and Barone in Italy, Westergaard and Madsen in Denmark, D'Aulnis de Bourouill, Cohen Stuart, and Mees in Holland, Launhardt and Lehr in Germany, Auspitz and Lieben in Austria, began to build on the foundations thus laid.

But the progress of the new methods during this period was small compared with that which followed the appearance of Marshall's first volume. This work, which immediately took rank among the foremost treatises, has spread the mathematical ideas far and wide. Many who had never heard of mathematical economics began to give it serious consideration. Naturally, the old disputes broke out afresh. Marshall's diagrams and formulæ were called dangerous, falsely accurate, academic playthings. But Marshall's moderate and judicial tone in treating of the utility of mathematics, his relegation of all his mathematics to foot-notes and appendix, won him readers, and at the same time showed plainly *lacunæ* in the text wherever mathematical notes were subjoined. The despised diagrams were examined. The reader's prejudices melted away as he discovered their extreme simplicity, and found them throwing light into many dark corners of economic theory. To-day few economists can be found who regard diagrams as useless curiosities or as waste of valuable page space. When Professor Hadley's book appeared, with the diagrams in the text itself, scarcely a murmur of objection was raised.

The quickening which Marshall gave the new current

* See Cairnes, *Logical Method of Political Economy*, 2d edition, p. vi.

may be roughly measured by the fact that, since his first edition, writings employing mathematics have appeared at the rate of eighteen a year; while between Jevons and Marshall the rate was six, and between Cournot and Jevons only a little over one. All this is interesting when compared with the confident predictions of the opponents of the new "school": "There is, therefore, no future for this kind of study; and it is only waste of intellectual power to pursue it."*

It goes without saying that this growth will not cease with the year 1898. All indications point to an increased volume of writing and an increased number of writers. Behind the distinguished group of mathematical economists now in or beyond their prime stand a much larger number of youthful followers whose period of productivity has only just begun. Among them are Berry, Bortkewitsch, Johnson, Sanger, Wicksell, and Yule.

There are, of course, many who still believe the whole study a delusion. There are others who admit that curves are useful, but deny the utility of formulæ. Probably in the entire world of economic students the opponents of the method still outnumber the friends. But, within the narrower circle of those who lead economic thought, the opposite seems to be distinctly true. There is not space here to enter upon the merits of the controversy. Fortunately, this is unnecessary; for the field has been recently and ably covered in another American journal by Professor Pareto.†

One objection, however, to the introduction of mathematics into economic study ought not perhaps to go unmentioned. Mathematical economists are sometimes accused of forming an exclusive guild and withdrawing themselves from the practical world of commerce and labor. We are told that economists ought not to be recluses, but men of affairs, especially in these days of social upheaval.

* Ingram, *History of Political Economy*, 1888, p. 182.

† *Journal of Political Economy*, September, 1897.

Mathematical economics is useless in a political mass meeting. But in every science there must be a differentiation of technical investigation from popular text writing or teaching. Is physics less practical because the X-rays are studied in the seclusion of the laboratory by highly mathematical methods, of which the work-a-day world has no conception? If the experience of other sciences is a guide, the best way to make economic theories practical is to make them perfect. "Our speculations can scarce ever be too fine, provided they be just."* The profound technical treatise is a prerequisite of the good popular manual. What is not as clear as crystal to the writer of the first will never be clearer than Newfoundland fog to the reader of the second. Had Mill employed more mathematics, he would never have befuddled business men over an impossible "wages fund." As Newcomb, the astronomer, first pointed out, wages are a *flow*, and not a fund. Were economists more imbued with mathematics, we should have fewer such quantitative absurdities as "the excess of the statistics of family purchases over those of family consumption represents the supplies kept on hand," or "marginal utility is that portion of the supply which has the least utility," or "quantity of money cannot affect its value, since the latter is determined by cost of production," as if the cost could not vary with the quantity; nor would so many authors treat "supply" as a fixed quantity instead of a relation between two variables, or be so incapable of conceiving the utility of one commodity as a function of the quantities of two or more commodities. When such errors are eliminated, not only will economic science be more perfect as a science: it will be divested of those crudities which have made it too often a laughing-stock when applied to the hard and stubborn facts of the actual world.

IRVING FISHER.

YALE UNIVERSITY.

* Hume, *Essay on Commerce*.