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EDGEWORTH'S CONTRIBUTION TO THE THEORY OF EXCHANGE

JOHN CREEDY

University of Durham

It will be interesting to watch the development of his theory
[Marshall (1881, reprinted in 1975, p. 267)]

I

INTRODUCTION

It is now almost 100 years since the publication of Edgeworth's *Mathematical Psychics*. In an article published 76 years later Stigler was still able to write, "His exposition deserves the closest scrutiny in spite of the fact that few economists of his time or ours have attempted to disentangle and uncover the theorems and conjectures of the *Mathematical Psychics*, probably the most elusively written book of importance in the history of economics" (1957, reprinted in 1965, p. 246).[1] A further two years later Shubik then published his seminal paper on "Edgeworth market games", which used the recent results from the theory of games to elucidate Edgeworth's analysis of the number of "recontracting" competitors and the formation of prices. As a result of considerable activity during the last twenty years the modern judge is now (usually) much more benign. Thus Samuelson states, "what is now seen to be magnificent about Edgeworth's *Mathematical Psychics* is his 40 pages of discussion on indifference contours, exchange, recontracting, supply and demand, contract curves and (deepest of all) the core" (1974, p. 1279).[2]

It therefore seems an appropriate time to review Edgeworth's contribution to the pure theory of exchange. Just as Edgeworth's work is of crucial importance to the development of economic analysis, an appreciation of his thought on exchange is crucial in considering the rest of his life's work. Many of his views on taxation, monopoly and international trade are directly influenced by his earliest work in economics. This paper concentrates, however, on his direct contribution to exchange and competition theory.[3] In order to

[1] Ironically Stigler later stated, "The proof of the need for indefinite numbers has serious weaknesses" (1965, p. 248). When *Mathematical Psychics* was reprinted, Schneider remarked, "For about fifty years economic theory has neglected this contribution. How many detours and mistakes would have been avoided if this masterpiece had formed the basis for continuous development" (1935, p. 236).

[2] This praise is not of course unanimous. The criticisms of Walker (1973), Jaffé *et al.* (1974) have been considered elsewhere, in Creedy (1978).

[3] The remark by Hutchison (1953, p. 114) that, "In the light of . . . (Edgeworth's) . . . warnings and distinctions it would not appear that he could have attached any wide significance to his own dynamic model of recontracting", can surely not be taken seriously.

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appreciate the originality of this contribution it is first necessary to consider the state of the theory before 1881.[4]

The works of Jevons (1871) and Walras (1874) were well known to Edgeworth, although Jevons was by far the greatest influence.[5] Following the "high period" of utilitariarism in England, total utility was assumed to be a sum of the separate utilities of each good available after exchange, with positive but decreasing marginal utility.[6] The marginal utility theory could then be summarised by Jevons' famous "equations of exchange", whereby, "the degrees of utility of commodities will be in the inverse proportion of the magnitudes of the increments exchanged" (1970, p. 143).[7] The conditions on the utility function guaranteed, furthermore, that demand curves sloped downwards, that an increase in income produced an increase in the consumption of *all* goods (that is, no inferior goods), and also ruled out any complementarity.[8]

Although Jevons and Walras had clearly indicated how the quantities demanded could be obtained as the solution to a set of simultaneous equations with prices *taken as given* (parametric), it is important to stress that no theory of price formation as such existed. In particular, Jevons' famous "law of indifference" that, "there can only be one ratio of exchange of one uniform commodity at any moment" (1970, p. 132)[9] was part of the definition of a perfect market. Thus price taking was axiomatic, "higgling" between buyers and sellers was not necessary. Jevons also explicitly stated that his theory was limited to the static equilibrium conditions, "It is a far more easy task to lay down the conditions under which trade is completed and interchange ceases, than to attempt to ascertain at what rate trade will go on when equilibrium is not attained" (1970, p. 138).

Edgeworth's approach in *Mathematical Psychics* was, however, from a

[4] It is, of course, difficult to know the direct influences. In (1881, p. 34) he states that some proofs in his early *Mind* paper (1879, July) "were offered . . . without acknowledgement, because without knowledge, of the cumulative proofs already adduced by Prof. Jevons". In the *Memorials* (1925) he says that, ". . . Jevons highly praised the then recently published *Economics of Industry*" (1925, p. 66). In *Memorials* (1925, p. 371) a letter from Marshall to Jevons (dated 30 June, 1979) says that the *Economics of Industry* is "nearly finished" and "one of the first bound copies will find its way to Hampstead". This suggests that the two near neighbours did not meet until late in 1879. Edgeworth did not know of Marshall until Jevons recommended him.

[5] Edgeworth also refers to Gossen and Cournot, but strangely not to Menger (1871). Blaug argues (1968, p. 299), "Here was a revolution that was not generally admitted to have taken place until more than a generation after the event". See also Black *et al.* (1972).

[6] Thus the cardinal utility function was additively separable, with positive first, and negative second, derivatives.

[7] The simultaneity of the equations for the two goods was made clear by Jevons. See (1970, p. 143).

[8] The assumption that all goods were substitutes led to many mechanical analogies which would not be appropriate with complementarity. (See also Edgeworth (1925, III, p. 38.))

[9] Jevons noted, "In this principle we have one of the central pivots of the theory" (1970, p. 137). He also explicitly ruled out the possibility of consumer's surplus being extracted, "the last increments in an act of exchange must be exchanged in the same ratio as the whole quantities exchanged" (1970, p. 139). Again perfect knowledge was explicitly assumed, "A market, then, is theoretically perfect only when all traders have perfect knowledge of the conditions of supply and demand" (1970, p. 134).

quite different point of view. He was particularly concerned to examine the circumstances in which markets would be expected to be "perfect", and where a determinate and uniform price would *result* from a simple and stylised process of barter.[10] Thus, "Here it is attempted to proceed without postulating the phenomenon of uniformity of price by the longer route of *contract-curve*" (1881, p. 40).[11] In so doing Edgeworth explicitly introduced the role of the number of traders into the analysis of competitive markets, and showed the conditions under which competition between buyers and sellers (through the recontracting process) would lead to a *final settlement* which is in fact equivalent to one in which all individuals act independently as price takers. In this respect Stigler is correct in stating, "Edgeworth was the first to attempt a systematic and rigorous definition of perfect competition" (1965, p. 241).

In *Mathematical Psychics* Edgeworth's concentration on competition was so great that his new analytical contributions to utility theory were given a very terse treatment indeed, and much of the discussion is in an appendix.[12] These aspects will first be discussed in section II of this paper, while his work on contract and competition is examined in section III. Like Edgeworth, "we can only practice temperance not abstinence, in the matter of symbols" (1925, II, p. 458), although it has been decided to change the original, and potentially confusing, notation.[13]

II

Utility and Indifference Curves

There seems little doubt that Edgeworth's inspiration and first interests in "the calculus of pleasure" came from his lifelong interest in ethics (particularly the work of Sidgwick[14]), and the natural sciences. Both subjects led to the combination, through the principle of maximisation, which produced the utility theory of demand. Edgeworth expressed the hope that, " 'Méchanique Sociale' may one day take her place along with 'Méchanique Celeste', throned each upon the double-sided height of one maximum principle" (1881, p. 12).[15]

[10] As noted in (1925, II, p. 315), "The theory of exchange is founded on the principle of barter".

[11] This interpretation has been questioned by Walker (1973). In (1925, II, p. 453) Edgeworth said, "the existence of a uniform rate of exchange between any two commodities is perhaps not so much axiomatic as deducible from the process of competition in a perfect market". See also Creedy (1978).

[12] Many would agree with Marshall (1975, p. 267) that, "His readers may sometimes wish that he had kept his work by him a little longer till he had worked it out more fully, and obtained that simplicity which comes only through long labour".

[13] For example, Edgeworth uses X and Y to denote people *and* goods, and uses twice as many letters as necessary for the utility functions. The changes will be explained in footnotes where they first occur.

[14] In reviewing a memoir of Sidgwick (1925, III, p. 149) Edgeworth combines high praise of Sidgwick with a statement of his view of the role of "authority", ". . . we ought to defer even to the undemonstrated dicta and opinions of the wise, who have a power of mental vision acquired by experience".

[15] Edgeworth (1881, p. 9) refers to, "The particular hypothesis adopted in these pages, that Pleasure is the concomitant of Energy".

Although, "at first sight as hopelessly incalculable as whatever is in life capricious and irregular—as the smiles of beauty and the waves of passion" (1881, p. 14) Edgeworth thought that, "*the conception of man as a pleasure machine* may justify and facilitate the employment of . . . mathematical terms in social science" (1881, p. 15).

After summarising Jevons' additive utility function (1881, p. 20) Edgeworth immediately introduced his general form where, "Utility is regarded as a function of the two variables, not the sum of functions of each" (1881, p. 104), and left further discussion to an appendix. Unfortunately his method of handling the function mathematically has led to some confusion. Following Jevons he considered individuals A and B trading in two goods X and Y. Person A starts with an amount a of good X, but no Y; while B begins with no X but with an amount b of good Y. The person A exchanges an amount x of good X for an amount y of good Y, and Edgeworth writes his total utility after exchange as $U_A = U_A(x, y)$. The total utility of B is written $U_B = U_B(x, y)$.[16] Thus, "the two coordinates . . . represent the quantities of the two commodities exchanged, the *quid* and the *pro quo*" (1925, II, p. 291).

Although it is now more usual to write the arguments of the function as the amounts *consumed* (that is, held after exchange),[17] rather than the amounts exchanged, Edgeworth showed that in obtaining the equilibrium conditions for efficient exchange it is not necessary mathematically to specify the initial holdings.[18] Edgeworth had some difficulty in examining the convexity of indifference curves,[19] however, and it is possible that the awkward treatment of the signs of various derivatives also misled him.[20]

Schumpeter surprisingly calls the generalisation of the utility function, "an obvious improvement" (1952, p. 127), but it is remarkable how slow was the acceptance of the general form.[21] Many of the leading economists continued to use the additive form well into the twentieth century, and, combined with the considerable extra complexity, this may have been related to Marshall's statement that, "Professor Edgeworth's plan of representing U and V as

[16] Edgeworth (1881, p. 20) uses person X exchanging a quantity x of his good, and having total utility $P = F(x, y)$. Person Y (written as B above) has total utility $\pi = \phi(x, y)$. The commas have been added here, as Edgeworth actully wrote $P = F(xy)$. See also Marshall (1975, p. 266), who corrects the notation.

[17] See Edgeworth (1925, II, p. 463) where he says, in the course of a review of Johnson's (1913) paper, that for some purposes the "modern" use may be preferred.

[18] This is discussed further in Creedy (1978), in relation to the criticisms of Jaffé (1974). Stigler (1965, p. 105) refers to "asymmetrical axes". In (1881, p. 105) Edgeworth does write $P = F(-x, y)$.

[19] Blaug (1968, p. 311), discussing indifference curves, says that, "Edgeworth assumed without proof that these curves are concave". But see Edgeworth (1881, pp. 35–36 and 140–141).

[20] In this case there are the assumptions $\partial U_A/\partial x < 0$ (rather than using $\partial U_A/\partial(a-x) > 0$) and $\partial U_A/\partial y > 0$. Edgeworth (1881, p. 34) says that $\partial^2 U_A/\partial x^2$ is negative, but in fact it should be positive; although $\partial^2 U_A/\partial y^2 < 0$. Samuelson (1974, p. 1279) says that Edgeworth assumed that the cross derivative was positive, thereby necessarily assuming complementarity, but given the problem with the signs it is not really clear what Edgeworth intended.

[21] See Stigler (1965, p. 105) and Blaug (1968, p. 330). Even Fisher (1927) later used independence to try to measure marginal utility, along with Pigou (1910) in his method of measuring elasticities.

general functions of x and y has great attractions to the mathematicians, but it seems less adapted to express the everyday facts of economic life than that of regarding, as Jevons did, the marginal utility of apples as functions of x simply" (1961, p. 844).[22] It is worth noting that much modern theory, especially in intertemporal allocation problems, uses the additive form for reasons of tractability; and much empirical work still assumes additivity.[23]

Edgeworth later said, "the whole rigid system bursts up in a universal *debâcle*, as we relax the assumption that the (marginal) utility of one commodity is independent of that of others" (1925, III, p. 38). But Schumpeter is surely correct to, "... doubt whether Edgeworth was fully aware of the theoretical difficulties this improvement was to cause" (1952, p. 127). To be "fully aware" would have been very remarkable indeed!

An immediate implication of the generalised utility function is that it allows for complementarity. Edgeworth did not explicitly consider this in 1881, although after stating his assumption that $\partial^2 U_A/\partial x \partial y < 0$ he notes in parentheses (1881, p. 34), "Attention is solicited to the interpretation of the third condition."[24] The first formal definition is attributed to Auspitz and Lieben (1889),[25] and was used by Edgeworth in his paper on the pure theory of monopoly (1897),[26] and also by Pareto in his *Manuale* (1906).[27] This defines commodities i and j to be complements (substitutes) if $\partial^2 U/\partial q_i \partial q_j > 0\ (<0)$; with $U = U(q_1 \ldots q_n)$. The first major criticism came from Johnson (1913), who pointed out that the criterion was not invariant with respect to monotonic transformations of the utility function. Johnson's treatment was extended by Allen (1934) and by Hicks and Allen (1934), so that the modern definition involves *compensated* price changes.[28] Thus q_i is defined as a *net* substitute (complement) for q_j if $\delta q_i/\delta p_j > 0\ (<0)$, where the use of δ indicates that the price change is compensated (i.e. constant utility).

[22] But many critics have argued that Marshall was inconsistent here, especially concerning the so-called Giffen good. See Stigler (1965, pp. 324–384), and more recently the defence by Gramm (1970).

[23] See the recent paper by Deaton (1974), which examines the empirical implications of additivity.

[24] Stigler (1965, p. 131) says, "The utility theory of complementarity had to wait for Edgeworth's generalisation of the utility function. In fact it had to wait a little longer, for Edgeworth glossed over this problem". He also says (1965, p. 192) that Edgeworth, "was so punctilious in acknowledging predecessors that his tone suggests independence of discovery". (See 1925, I, p. 117.)

[25] Samuelson's comment (1974, p. 1279) is, "Not bad work for bankers". Edgeworth had a very high opinion of Auspitz and Lieben. When discussing their treatment of what is essentially the optimum tariff, he praises their, "Beautiful and original reasoning" (1925, II, p. 295).

[26] In the context of the tax treatment of monopolists. The analysis was later generalised by Hotelling (1932), and Schultz (1933). Hotelling's famous "general welfare" paper (1938) used his generalisation of Edgeworth's tax result. In (1929, reprinted in 1953, p. 467) Hotelling actually began his paper with, "After the work of the late Professor Edgeworth one may doubt that anything further can be said on the theory of competition among a small number of entrepreneurs". But Hotelling went on to make a seminal contribution to the theory of spatial competition!

[27] This is often referred to, for obvious reasons, as ALEP complementarity.

[28] See the essay by Samuelson (1974), written on the 40th anniversary of the "Hicks–Allen revolution in demand theory". More recently Chipman (1977) has shown that the "ALEP" definition still has empirical implications.

If uncompensated changes are considered the goods would now usually be called *gross* substitutes, but there is no symmetry between gross substitutes and complements as only the matrix of substitution elasticities is assumed to be symmetric.[29]

It should be noted that Edgeworth accepted Johnson's point, saying that, "many of the deductions which have been made respecting commodities correlated in the way of demand presuppose constancy in the (marginal utility) of money . . . and employ only the older definition" (1925, II, p. 465).[30]

In the sentence which follows Edgeworth's introduction of the general utility function, he raised the question of the equilibrium which may be reached with, "one or both refusing to move further", answered that, "contract" only supplies part of the answer so that, "supplementary conditions . . . supplied by competition or ethical motives" will be required, and then wrote the equation of his famous contract curve (1881, pp. 20–21).[31] He then presents three alternative, and all rather clumsy, derivations of the equation of the contract curve. The indifference curve (the "line of indifference") is introduced in the course of the first derivation, although a diagram is not drawn until seven pages later. At the end of three pages (1881, pp. 21–23) Edgeworth states, "No doubt the one theory which has been thus differently expressed could be presented by a professed mathematician more elegantly and scientifically" (1881, p. 24).[32] It is really no wonder that *Mathematical Psychics* remained obscure for so long, and the dissemination of the indifference curve method perhaps owes more to Fisher's (1892) independent discovery, their adoption by Pareto (1906) and Johnson (1913), and the later presentation in Bowley's superb *Groundwork* (1924).[33]

Edgeworth considers the utility functions of the two individuals as plotted in a three-dimensional graph, with total utility as the, "ordinate drawn from any point on the plane xy (say the plane of the paper) to the surface" (1881, p. 21). The problem of obtaining the equilibrium values of x and y which, "cannot be varied with the consent of the parties to it" was clearly stated as follows:

> It is required to find a point (xy) such that, in *whatever direction* we take an infinitely small step, (U_A) and (U_B) do not increase together, but that, while one increases, the other decreases (1881, p. 21).

[29] If the matrices of price and substitution elasticities are respectively $\{e_{ij}\}$ and $\{s_{ij}\}$, and the vectors of budget shares and income elasticities are respectively $\{w_j\}$ and $\{e_j\}$; then Slutsky's equation is simply $e_{ij} = s_{ij} - w_i e_j$.

[30] Edgeworth regarded "his" definition as "more intrinsic"—but could not argue that it was superior to Johnson's. Strangely Stigler (1965, p. 134) found it "difficult to see the purpose" in Johnson's or in the Hicks/Allen definition of complementarity, on the dubious grounds that "they cannot be applied introspectively".

[31] The nature of contract and re-contract will be discussed later in section III. The presentation here distinguishes between the optimum conditions and the means by which an optimum may be achieved.

[32] Later (1881, p. 96) he said, "the professed mathematician . . . (was) . . . sure to find many errors in these pages should they be so fortunate as to come under his notice". In fact there are no major errors, only the inelegance which is not unusual in highly original contributions.

[33] Marshall's appendix on Barter in the *Principles* is also important.

The locus of such points (now referred to as Pareto optimal points) ". . . it is here proposed to call the *contract curve*".

To consider a movement for person A, the total derivative of U_A is given by:

$$dU_A = \frac{\partial U_A}{\partial x} dx + \frac{\partial U_A}{\partial y} dy \tag{1}$$

and, "it is evident that (A) will step only on one side of a certain line, the *line of indifference*, as it may be called" (1881, p. 21).[34] This is of course because A will only consider positive values of dU_A; thus the equation of an indifference curve is[35]

$$\frac{\partial U_A}{\partial x} dx + \frac{\partial U_A}{\partial y} dy = 0 \tag{2}$$

The *marginal rate of substitution* of x for y, dx/dy, is therefore $-\frac{\partial U_A}{\partial y} \Big/ \frac{\partial U_A}{\partial x}$, or the ratio of marginal utilities. Modern discussion (after Hicks and Allen, 1934) is now predominantly couched in terms of this concept, although it is worth noting that Edgeworth did not consider it explicitly in 1881. This also explains why he continued unnecessarily to assume diminishing marginal utility.[36] To obtain the contract curve Edgeworth then asked,

> If we enquire in what directions (A) and (B) will consent to move *together*, the answer is, in any direction between their respective lines of indifference, in a direction *positive* as it may be called *for both*. At what point then will they refuse to move at all? When their *lines* of *indifference* are coincident (1881, p. 22).

The modern textbook would of course say that the marginal rate of substitution between the two goods must be the same for each individual, otherwise an opportunity for at least one individual to move to an indifference curve of greater total utility exists—given the convexity of the indifference curves. Thus

$$-\frac{dx}{dy} = \frac{\partial U_A}{\partial y} \Big/ \frac{\partial U_A}{\partial x} = \frac{\partial U_B}{\partial y} \Big/ \frac{\partial U_B}{\partial x} \tag{3}$$

and the equation of the contract curve is

$$\frac{\partial U_A}{\partial x}\frac{\partial U_B}{\partial y} - \frac{\partial U_A}{\partial y}\frac{\partial U_B}{\partial x} = 0 \tag{4}$$

[34] It is not correct to say that indifference curves were suggested by Marshall's curves (of 1879). See also Schumpeter (1954, p. 1065, n. 10). In fact Jevons' diagram and discussion (1970, p. 140) is more likely to suggest indifference curves, and Edgeworth himself says (1881, p. 26, n. 1) "The delicate relation between the conceptions—(Marshall's) *instability of trade* . . . and instability of *contract in general*—is not one of identity".

[35] Edgeworth actually writes the total derivative after transforming to polar co-ordinates (that is, he considers an individual moving a length ρ at an angle of θ). In writing the equation of the indifference curve he then transforms back again and writes dx and dy as discrete changes $(\xi - x)$ and $(\eta - y)$, with no explanation.

[36] For an explanation of this see Stigler (1965, p. 99, nn. 83 and 84). Edgeworth did write the differential equation of the indifference curve, as the first step in obtaining d^2y/dx^2 (1881, p. 36).

The discussion which then follows is worth quoting as a typical example of Edgeworth's elusive style.

> The conditions being here (i) that the pleasure-energy of (A) and (B) considered each as a function of (certain values of) the variables x and y should be functions of the *same* values: . . . that the charrioteer–pleasures should drive their teams *together* over the plane of xy; (ii) that the joint-team should never be urged in a direction contrary to the *preference* of either individual; that the resultant line of force (and the momentum) of the gross, the charriot, system should be continually intermediate between the (positive directions of the) lines of the respective pleasure forces . . . let us employ an *arbitrary function* to denote the unknown *principle of compromise* between the parties . . .. Then, by reasoning different from the preceeding only in the point of view, it appears that the *total utility of the system is at a relative maximum at any point on the pure contract-curve* (1881, pp. 24–25).[37]

Edgeworth's famous Box Diagram (1881, p. 28) is too well known to require discussion here.[38] The initial endowments of each individual determine the highest indifference curve which may be reached by each individual acting in isolation, and therefore the dimensions of the box. These curves are then placed in the box where the amount of y and x *exchanged* are measured northwards and eastwards respectively.[39] The contract curve is then obtained as the locus of tangencies as in equation (3), "the class of contracts to the variation of which consent of *both* parties cannot be obtained" (1881, p. 28).

The individual demand curve, showing the amount of x which person A is willing to give up for any y at a given price, was treated by Edgeworth in his Appendix V on Jevons' formulae of exchange. This is of course the form of "offer curve" used very effectively by Marshall (1879) although Edgeworth was the first to provide the "analytics" of the curve in terms of utility theory.

Transforming $U_A(x, y)$ to polar coordinates where ρ measures the length from the origin along a ray inclined at an angle of θ to the x axis; then, "tan θ expresses the *rate of exchange*. The demand curve of A is $(\partial U_A/\partial\rho) = 0$. For this locus expresses the utmost amount of dealing to which the dealer will consent at any given rate of exchange . . . his utility is a maximum at that

[37] Edgeworth then considered (in a rather obscure passage) the "impure contract curve . . . where the commodity of one party is a *discommodity* to the other" (1881, p. 26). This example of non-convexity was inappropriately quoted by Collard (1975, p. 356) as an example of a statement by Edgeworth on Altruism. Collard's paper mainly examines the footnote on p. 53 of Edgeworth (1881), where a "coefficient of effective sympathy" is introduced. A treatment of altruism was given by Boulding (1962) using the Edgeworth Box.

[38] It is well known that Pareto first published the Box diagram in its conventional form. A number of criticisms have been made by Tarascio (1972), Jaffé (1974), and Weatherby (1976)—but these are discussed in greater detail in Creedy (1978).

[39] Note that the construction of a box preceeds the Figure in Edgeworth (1881, p. 28), where only two sides are drawn. With *exchange* as the major interest it is clear that the origin should be in the south west corner of the diagram. The modern (pedagogic) presentation is usually concerned with *allocation* of fixed amounts.

rate" (1881, p. 105).[40] This technique has subsequently been extended to cover the analysis of production using isoquants; and the treatment of offer curves in international trade analysis has been extended to include the production function, endowments of factors of production, and tastes, all in the same diagram.[41]

It is now well known that Fisher (1892) independently "discovered" the indifference curve analysis, showed the conditions under which a cardinal concept of total utility may be avoided, and in so doing considered the integrability problem for more than two goods. Edgeworth had a very high opinion of Fisher, as seen from his statement that, "we may at least predict to Dr Fisher the degree of immortality which belongs to one who has deepened the foundations of the pure theory of economics" (1925, III, p. 41).[42]

However, both Fisher and Pareto[43] continued to use cardinal utility in some contexts, and it is worth noting that Edgeworth never "shirked"[44] from using a cardinal concept. Notwithstanding the considerable difficulties in its use, which Edgeworth regularly discussed,[45] his view is well summarised as follows

> The postulate here adopted that utility or welfare, "can be brought under the category of greater and less" rests primarily on the testimony of consciousness, the psychological observation that there are degrees of felt satisfaction. This personal experience is then extended by sympathy to the evaluation of other people's pleasures. Jevons's suggestion that the theory of utility is limited to the notions of a single mind . . . appears to us untenable. The contrary is postulated throughout large tracts of economic science; for instance, the theory of taxation and that of industrial conciliation (1925, II, p. 475).

Edgeworth's argument that for many economic problems the use of cardinal utility and of explicit inter-personal comparisons cannot be avoided remains

[40] Edgeworth unnecessarily switches to writing $U_A = U_A(a-x, y)$. Later (1881, p. 113) he succinctly states, "The problem under consideration may be expressed: Find the locus of the point where lines from the origin *touch* curves of indifference". Edgeworth's Figure (1881, p. 114) is drawn for the special case where the indifference curves are concentric circles.

[41] Aggregate indifference curves were often used in the analysis of trade with little question. They were examined critically by Leontief (1933).

[42] Referring to Fisher's discussion of ordinal utility Edgeworth says, "this appears to us a very remarkable result", and concerning integrability "He attains . . . a conclusion which seems to us of unexpected importance" (1925, III, p. 40).

[43] Pareto's contribution has, of course, been recorded by Hicks and Allen (1934). But Pantaleoni wrote in his obituary (1923, p. 584), "Pareto . . . elaborated a system of general economic equilibrium . . . with materials only partially his own, . . . he would not have been what he has been without . . . (Edgeworth and Fisher)."

[44] In (1881, pp. 7–8) he wrote, "such comparisons (between groups) can no longer be shirked, if there is to be any systematic morality at all. It is postulated by distributive justice".

[45] Thus in (1881, p. 8) "*Atoms of pleasure* are not easy to distinguish and discern; more continuous than sand, more discrete than liquid, as it were nuclei of the just-perceivable, embedded in circumambient semi-consciousness. We cannot *count* the golden sands of life; we cannot *number* the 'innumerable smiles' of seas of love; but we seem to be capable of observing that there is here a *greater*, there a *less*, multitude of pleasure-units, mass of happiness; and that is enough". Edgeworth's last book *Metretike* (1887) was concerned with measuring probabilities and utility.

true today, of course. From the classic works of Pigou and Meade, to the modern work on taxation (much of which stems from Edgeworth), it has been necessary to use cardinal utility.

Although some of the aspects of Edgeworth's contribution to the utility analysis of demand have been discussed at length here, it is important to note that in *Mathematical Psychics* they were all presented in the space of a few pages. His main contribution, and certainly the deepest, is his work on competitive markets. This is discussed in the following section.

III

Contract and Competition

The above discussion of Edgeworth's indifference curves and contract curve has concentrated on the *optimality conditions* for "efficient" exchange between two individuals: whether or not two isolated traders may in fact reach a settlement which satisfies those conditions has not been considered. In Edgeworth's presentation the two questions were pursued together, although it is important analytically to differentiate between them. As noted in section I, Edgeworth was not satisfied with examining the properties of a market "clearing" solution with parametric pricing, but considered a highly stylised but well specified process of competition which was co-operative in the sense that individuals form collusive groups.

When defining the properties of a "perfect" market Jevons explicitly assumed (in addition to his "law of indifference") complete knowledge and product divisibility. Edgeworth continued to assume divisibility but instead of assuming (initial) perfect knowledge he supposed that, "There is free communication throughout a *normal* competitive field. You might suppose the constituent individuals collected at a point, or connected by telephones—an ideal supposition, but sufficiently approximate to existence or tendency for the purposes of abstract science" (1881, p. 18). The knowledge of the other traders' dispositions and resources could then be obtained by the formation of tentative contracts which are not assumed to involve actual transfers,[46] and can be broken when further information is obtained. Edgeworth introduces this in typical style,

> "Is it peace or war?" asks the lover of "Maud", of economic *competition*, and answers hastily: it is both, *pax* or *pact* between contractors during contract, *war*, when some of the contractors *without the consent of others recontract* (1881, p. 17).[47]

[46] Again in Edgeworth (1925, II, p. 313), "The dispositions and circumstances of the parties are assumed to remain throughout constant. But it is supposed that agreements are renewed or varied many times". Stigler also refers to "recontract, the institution which allows tentative contracts to be broken without penalty" (1965, p. 247, n. 25). See also Kaldor (1934).

[47] Furthermore, "There is . . . no *combination* or precontract between two or more contractors that none of them will recontract without the consent of all" (1881, p. 19). Again in (1925, II, p. 369), "The particles of an economic system neither cohere as a solid,

Thus the role of the recontracting process is essentially to disseminate information between traders. It allows individuals who (with little information) initially agree to a contract which is not on the contract curve, to discover that an opportunity exists for improvement for at least one person without another suffering. The merit of Edgeworth's stylised process is therefore *not* to direct attention to the role of information, or other well known market "imperfections", but to concentrate on the role of the *number of individuals* in a market.[48] As shown in more detail below, a perfectly competitive market, in Edgeworth's model, is one in which the number of traders on both sides of the market is sufficient to ensure that the *recontracting process* will lead to a *unique settlement.*

Turning to the contract curve for two isolated traders, it is clear that along this curve, "the settlements are represented by an *indefinite number of points*" (1881, p. 29), so that the recontracting process would not be expected to result in a unique rate of exchange.[49] Thus, "it is in the interests of both parties that there should be *some settlement*, one of the contracts represented by the contract curve between the limits. But *which* of these contracts is arbitrary in the absense of arbitration" (1881, p. 29).[50] There is therefore nothing to ensure that individuals will trade on their demand (offer) curves,[51] and this led Edgeworth to make his often quoted remark that

> An accessory evil of indeterminate contract is the tendency, greater than in a full market, towards dissimulation and objectionable arts of higgling (1881, p. 30).

This basic two person model was extensively discussed in the context of wage bargaining, duopoly and bilateral monopoly, and was initially the source of some disagreement between Marshall and Edgeworth, where the former (not unusually) recruited the polemical abilities of a Cambridge

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nor collide with the independence of a gas. Their liquid movements are comparable to a dance in which youths and maidens move in unison; harmoniously, but subject to a change of partners". The conventional interpretation of recontract has been criticised by Walker (1973), but see Creedy (1978).

[48] Strictly, the number of independent traders, since indeterminacy is reintroduced by collective action (for example by Unions). Recontracting therefore provides costless sampling. It is necessary to distinguish between the information needed by individuals in an atomistic market, and by planners using a price mechanism.

[49] Note that if the traders start with some of both goods the range of indeterminacy is reduced.

[50] Thus, "The whole process groans and yearns, desiderating a principle of arbitration, an end of strife" (1881, p. 51). The second part of the *Mathematical Psychics* considers the implications of the utilitarian principle, where, "Any individual experiencing a unit of pleasure-intensity during a unit of time is to 'count for one' " (1881, p. 8). Edgeworth explicitly considered cases where individuals have different utility functions, however. This part of Edgeworth's work is also highly original, but seems to have been ignored in recent work on inequality and economic justice.

[51] Edgeworth contrasted, "this clogged and underground procedure" with the Walrasian system where, "You might suppose each dealer to write down his demand, how much of an article he would take at each price . . .; and these data having been furnished to a sort of market-machine, the *price* to be passionlessly evaluated" (1881, p. 30).

student.[52] The subsequent literature is, however, far too extensive to discuss here.[53]

Edgeworth quickly moved on to the introduction of further traders. His analysis of this problem, completed in just ten pages (1881, pp. 34–43), is without doubt the most difficult to follow in *Mathematical Psychics*. Indeed, as noted in section I above, it has taken many years for all the subtleties to be fully unravelled.[54] His approach was, however, clearly stated:

> It is not necessary to resolve analytically the composite mechanism of a *competitive field*. It will suffice to proceed synthetically, observing in a simple typical case the effect of continually introducing into the field additional competitors (1881, p. 34).

Edgeworth began by introducing "a second A and a second B", where the new traders are assumed to be exact replicas of the initial pair,[55] having the same tastes and initial endowments. This device enables the same box diagram as before to be used. The first point to note about this case is that in the *final settlement* which results from *recontract* both the A's (and of course the B's) must be treated equally. Since they are identical no individual A can have any advantage over his counterpart, and (with convex indifference curves) they would not settle for different allocations on the same indifference curve.[56] Furthermore the settlement must be on the contract curve. This was clearly stated by Edgeworth:

> It is evident that there cannot be equilibrium unless (1) all the field is collected at one point; (2) that point is on the *contract curve*. For (1) if possible let one couple be at one point, and another couple at another point. It will generally be the interest of the (A) of one couple and the (B) of the other to rush together, leaving their partners in the lurch. And (2) if the common point is not on the contract curve, it will be the interest of *all parties* to descend to the contract curve (1881, p. 35).

The next important question is whether the range of indeterminacy is reduced by the addition of these additional competitors. Edgeworth answered this by considering whether the limit of the old contract curve, say point C in Figure 1 (i), still qualifies as a *final settlement* (that is, to the variation of which the consent of all parties cannot be obtained). Because of the convexity of indifference curves it is clear that any point along the ray $0C$

[52] Although Berry's intervention did not clarify matters. See Guillebaud's variorum edition (1961, II, pp. 791–798). A misunderstanding arose partly because Marshall used a special case where the marginal utility of one good was constant for both parties, but did not provide an explanation. Edgeworth (1925, II, p. 317, n. 1) later explained: if $U_A = U_1(a-x)+\alpha y$ and $U_B = U_2(x)+\beta y$ then the contract curve is given by $U_1'(a-x)/U_2'(x) = \alpha/\beta$, x is constant and the "curve" is a straight line parallel to the y axis.

[53] A useful survey is given by Stahl (1972). A notable contribution was the paper by Hicks (1930), of which he later said, "It was this . . . which set me up as an economic theorist" (1973).

[54] Very clear statements can be found in Hildebrand and Kirman (1976) and Bacharach (1976).

[55] This assumption is made, "for the sake of illustration (not of the argument)" (1881, p. 35), and was later relaxed.

[56] A weighted average of the two bundles would get both onto a higher indifference curve.

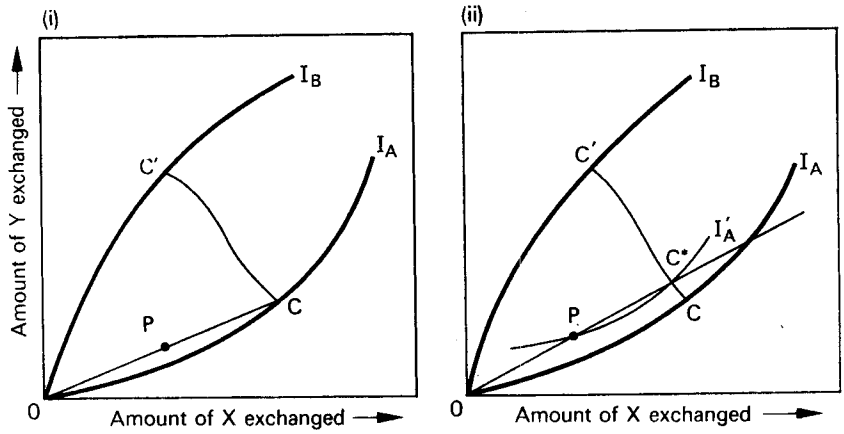

Figure 1. Additional (Identical) *A*'s and *B*'s.

represents an improvement over *C* for the two *A*'s. With an additional *B* it is now possible for the *A*'s to reach a tentative contract which makes B_1 as well off as possible, but leaves B_2 in isolation. Consider the point *P* (Figure 1 (i)), which is half way along 0*C*; remembering that both *A*'s *begin* with endowments of $(a, 0)$, where the first element is the amount of good *X* and the second element is the amount of good *Y*. Similarly both *B*'s begin with $(0, b)$. If point *C* has co-ordinates (x, y), a contract which places the two *A*'s at *P*, giving them $\{(a-x)+x/2, y/2\}$, B_1 at *C* (giving him $(x, b-y)$), and B_2 at 0 (with $0, b$)) is feasible.[57] It is however, important to notice that points between *P* and *C* cannot be achieved by this kind of coalition, since the two *A*'s and one *B* do not have sufficient resources to move further than half way towards *C*.[58]

This kind of contract would not, however, remain for long because there remains an opportunity for B_2, who has been left at the origin, to form an agreement with one of the *A*'s.[59] This was clearly noted by Edgeworth:

> When this relation is satisfied the system of three might remain in the position reached; but for (B_2) who has been left out in the cold. He will now strike in, with the result that the system will be worked down to the contract-curve again; to a point at least as favourable for the (*A*'s) as (*P*). Thus the (*A*'s) will have lost some of their original advantage by competition (1881, p. 37).

The point along *CC'* which is "at least as favourable" as *P* is shown in Figure 1 (ii) as *C**. Here *P* is still half way along the new ray from 0 to *C**, and the indifference curve, I'_A, of the two *A*'s passes through *P and C**. Since no point between *P* and *C** can be attained by the *A*'s, *C** must be the new

[57] As before, *x* and *y* denote the amount *exchanged*. The sum (over all individuals) of the first elements is $2a$, and that of the second elements is $2b$. Thus B_2 cannot *block* (or veto) the coalition between his counterpart B_1 and the two *A*'s.

[58] An attempt to move three-quarters of the way would give the *A*'s $\{(a-x)+x/4, 3y/4\}$ and the total amount of *Y* needed would be $2(b+y)$—which is obviously excessive.

[59] B_2 could immediately *re-contract* with one of the *A*'s to trade at, say, *C'*, thus breaking up the initial contract between the *A*'s and B_1.

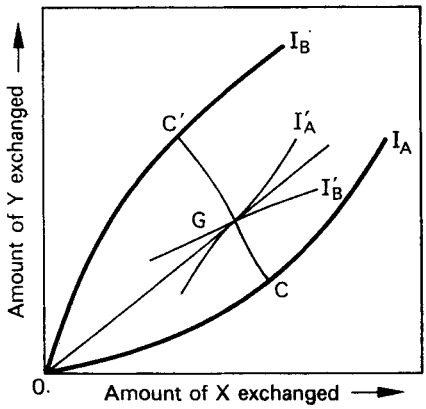

Figure 2. The competitive solution.

limit to the contract curve. Clearly the same kind of reasoning can be applied to the point C'. In modern terminology it would then be said that the *core* of the economy "shrinks" as a result of the introduction of additional traders.[60]

If a third pair of A's and B's are introduced there is clearly a possibility for the three A's to improve on C^* by forming a coalition with two of the B's. Here it is possible to reach two-thirds of the way from the origin to any point on the contract curve, so that C^* will move inwards although a certain amount of indeterminacy will remain. In the general case of a number, N, of each of the A's and B's; then the "attainable" ratio of $0P$ to $0C$ is $(N-1)/N$, which has a limiting value of unity.[61] The result of "working in" from both extremes of the contract curve would be a unique point on the contract curve where the common indifference curve of the A's is tangential to that of the B's. This is shown in Figure 2.

Edgeworth then states[62]:

> If this reasoning does not seem satisfactory, it would be possible to give a more formal proof; bringing out the important result that the common tangent to both indifference curves at the point (G) is the vector from the origin (1881, p. 38).

The significance of this result is that the vector from the origin is precisely the price vector which, if imposed, would achieve equilibrium in a market where all individuals were price "takers".[63] The equilibrium solution with

[60] Edgeworth's only reference to a "core" was not in this context, however. He referred to, "the 'controlless core' of human selfishness" (1881, p. 52).

[61] The consumption "bundle" of each of the N A's would be $\{a-x(N-1)/N, y(N-1)/N\}$. The B who is "left in the cold" would get $(0, b)$, while the remaining B's would each get $(x, b-y)$. The total consumption of good X is therefore Na, and that of good Y is Nb. This kind of provisional contract is therefore feasible. As N doubles from 10 to 20 the ratio $(N-1)/N$ increases from 0·9 to 0·95; and as N doubles again to 40 this ratio becomes 0·975.

[62] Edgeworth's statement of his result is often referred to as his "conjecture", however.

[63] Of course such large economies where each individual is insignificant are precisely those in which people are more likely to act as price takers. See also the section on Jevons below.

parametric pricing is therefore the same as that achieved by the co-operative recontracting game.

Edgeworth then suggested that similar results apply when some of the assumptions are relaxed. Thus, "when we suppose plurality of natures as well as persons, we have to suppose a plurality of contract-curves . . .. Then, by considerations analogous to those already employed, it may appear that the quantity of final settlements is diminished as the number of competitors is increased" (1881, p. 40). He then briefly considers different numbers of *A*'s and *B*'s, concluding that, "The theorem admits of being extended to the general case of unequal numbers and natures" (1881, p. 43).[64]

A considerable number of articles have been written since the late 1950s which have examined various aspects of the Edgeworth recontract model under different assumptions. This is not, however, the place to consider these contributions, especially as several useful surveys already exist.[65]

One "topical" question to which Edgeworth applied these results, and to which he clearly attached importance, was that of the power of trade unions to increase wages, saying that, "coordinators tend to introduce or increase indeterminacy; and the final settlements . . . are more favourable to the combiners" (1881, p. 43).[66]

Although Edgeworth openly acknowledged a considerable debt to Jevons (both personal and intellectual),[67] he was quite clear in his criticism of Jevons for not adequately discussing the importance of the number of competitors in relation to the "law of indifference". Thus

> In dealing with exceptional cases . . . a reference to . . . the presupposition of competition would have introduced greater precision, and suggested the distinction submitted in these pages, . . . namely, that exchange is indeterminate, if *either* (1) one of the trading bodies (*qua* individual or *qua* union) or (2) the commodity supplied by one of the dealers, be *indivisible or not perfectly divisible* (1881, p. 109).

In Edgeworth's model exchange between isolated couples would not necessarily be conducted always at the same rate of exchange.[68] He does, however, suggest that in Jevons' *Theory*, "it does not seem to be lost sight of.

[64] Some of the results do not hold in the general case; for example equality within the group of *A*'s no longer holds when there are unequal numbers of *A*'s and *B*'s.

[65] A small selection includes, Allingham (1975), Bacharach (1976), Bewley (1973), Green (1974), Hildebrand and Kirman (1976), Negishi (1962), Newman (1965), Telser (1971) and Weintraub (1977).

[66] Also, "one thing from an abstract point of view visible amidst the jumble of catallactic molecules, the jostle of competitive crowds, is that those who form themselves into compact bodies by *combination* do not tend to lose, but *stand to gain* . . . in point of utility" (1881, p. 44). Edgeworth was quoted approvingly by the Webbs (1919, p. 647).

[67] In a letter to Mrs Jevons after her bereavement he wrote, "I shall always remember with gratitude the kind encouragement and a peculiar intellectual sympathy which he extended . . ." (Quoted by Black (1962)).

[68] Streissler (1972) discusses Menger's attitude towards an assumption corresponding to Jevons' "law of indifference". For example, "A decisive term used by Menger to describe the determination of a price . . . is *price conflict*" (1972, p. 171). Nevertheless this was based on other imperfections than that of limitation of numbers (quality differences, technical change, imperfect knowledge, etc.).

His couple of dealers are, I take it, a sort of typical couple, clothed with the properties of 'indifference' whose origin in an 'open market' is so lucidly described" (1881, p. 109).[69]

It has recently been suggested that Edgeworth was much too generous to Jevons,[70] although the well balanced judgement of Black (1970) would seem the most sensible and given the originality of Jevons' work it is not surprising that, "There are indeed certain passages which admit of this interpretation, but equally there are others which do not" (1970, p. 267). As an example of one which supports the view that Jevons concentrated on price taking, "The equation (of exchange) still more accurately represents the position of an individual consumer with regard to the aggregate trade of a large community, since he must buy at the current prices, which he cannot in an appreciable degree affect"[71] (1970, p. 151).

When reading Jevons' statements on the indeterminacy which results from lack of divisibility of the commodity,[72] it is almost impossible not to see some correspondence with passages in *Mathematical Psychics*. For example,

> The disposition and force of character of the parties, their comparative persistency, their adroitness and experience in business, or it may be feelings of justice or of kindliness, will also influence the decision. These are motives more or less extraneous to a theory of economics, and yet they appear necessary considerations in this problem. It may be that indeterminate bargains of this kind are best arranged by an arbitrator or third party (1970, p. 159).

IV

Conclusions

Edgeworth was quite clear about the implications of his prediction that, "It does not seem very rash to infer . . . at least in the proximate future, a considerable extent of indeterminateness" (1881, p. 50).[73] The result would be, "To impair . . . the reverence paid to *competition*; in whose results—as if worked out by a play of physical forces, impersonal, impartial—economists

[69] Other references to Jevons concerning this aspect are in (1881, pp. 19, 30, 31, 39, 109, 115).

[70] For example Jaffé (1972, p. 121) says, "Edgeworth's erudite interpretation remains unconvincing". But there seems little point in asking whether Jevons *realised* the implications of small numbers, what is clear—and this is really all that Edgeworth suggested—is that Jevons was primarily concerned with the analysis of markets where price taking is appropriate, and each individual's endowment is a small proportion of the market stock.

[71] Furthermore in (1970, p. 155) in the section on "competition in exchange"—after applying the law of indifference to three traders he says in the discussion of the equations, "that the general result would be, that the smaller holder must more or less conform to the prices of the larger holder". Black's interpretation (1970, pp. 22, 267) of the "trading body", that it ensures continuity in aggregate dealings, seems entirely justified. Jevons' discussion (1970, p. 135) is very suggestive of cases like those which arise in quantitative assay—where individuals respond at different discrete levels but the aggregate relationship is smooth (and usually sigmoid).

[72] Jevons discussed the case of house sale. Bohm-Bowerk's example (1890) of a horse sale is of course also well known. Pantaleoni (1889) also discussed a house sale in this context, but strangely did not refer to Edgeworth.

[73] The prediction is based on the growing importance of trade unions.

have complacently acquiesced" (1881, p. 50). He later suggested that in a regime of monopolies the role of abstract economics would be much reduced and, "there would survive only the empirical school, flourishing in a chaos congenial to their mentality" (1925, I, p. 138).[74] It is of interest that Edgeworth did not attempt to "refine" the recontract model (although its implications were often quoted) and he devoted much more energy in later years to his work on statistical inference and index numbers. Pantaleoni, who had also done original work in the pure theory of competition, also remarked that,[75] "I would like to write 'finis' in this direction, and I request my students to interest themselves now only, as I do myself, in the research of the nature of the functions which we are handling" (1923, p. 585).[76]

The role of perfect competition as the standard model in economic analysis has, however, shown remarkable longevity.[77] In recent years a considerable amount of energy has been devoted (mainly in the U.S.) to the analysis of models where exchange is allowed to take place out of "equilibrium".[78] This work has, furthermore, involved mathematical expertise well beyond that attained by the "self taught barrister". Some of this work has also been subject to criticism on both sides of the Atlantic, especially in the early 1970s when virtually every "Presidential Address" was devoted to a criticism of abstract economics.[79] Here, it seems appropriate to allow Edgeworth the last word, in a quotation from his Presidential Address to section F of the *Royal Society* (1889).

> Scarcely has the powerful engine of symbolic language been applied, when the train of reasoning comes to a stop. The case is like that of the "swell" in *Punch*, who, about to enter a hansom, inquires solicitously of the driver whether he has got a good horse. "Yes, sir, very good 'oss". "Aw—then dwive to next door". However, our road, though short, is so slippery as to require every precaution (1925, II, p. 286).

References

ALLEN, R. G. D. (1934). A comparison between different definitions of complementary and competitive goods. *Econometrica*, vol. 2.

ALLINGHAM, H. (1975). *General Equilibrium*. Macmillan.

[74] It is not clear if this statement is meant to be derogatory. The interpretation depends on which individuals are included in the "empirical school" (i.e. how closely "historical" may be substituted for "empirical").

[75] In (1889) Pantaleoni discussed the question of the appropriate division of the surplus which arises from co-operation in production.

[76] Edgeworth was, however, not optimistic about estimating these schedules. He saw, "only a faint hope of obtaining what Jevons too confidently expected, statistical data for the relations between supply and demand" (1925, II, p. 285).

[77] Notwithstanding the Chamberlain/Robinson "revolution", and the Chicago Controversies.

[78] See, for example, the survey by Weintraub (1977). He concludes with some justification, "If answers are emerging only slowly, it is because the problems are difficult" (1977, p. 19).

[79] See the volume edited by Worswick (1972). The scarcity of good empirical work should also be noted, however. The indiscriminate use of ordinary least squares regression "packages" is all too common.

BACHARACH, M. (1976). *Economics and the Theory of Games*. Macmillan.
BEWLEY, T. (1973). Edgeworth's conjecture. *Econometrica*, vol. 41.
BLACK, R. D. C. (1962). W. S. Jevons and the economists of his time. *Manchester School*, XXX.
BLACK, R. D. C. *et al.* (1972). *The Marginal Revolution in Economics*. Duke University Press.
BLAUG, M. (1968). *Economic Theory in Retrospect*. Homewood, Ill.
BOULDING, K. (1962). Notes on a theory of Philanthropy in F. G. Dickinson (ed.). *Philanthropy and Public Policy*. N BER.
BOWLEY, A. L. (1924). *The mathematical groundwork of Economics. An introductory Treatise*. Oxford.
CHIPMAN, J. (1977). An empirical implication of Auspitz–Lieben–Edgeworth–Pareto complementarity. *Journal of Economic Theory*, vol. 14.
CREEDY, J. (1978). Some recent interpretations of *Mathematical Psychics* (*mimeo*).
COLLARD, D. (1975). Edgeworth's propositions on altruism. *Economic Journal*.
DEATON, A. (1974). A reconsideration of the empirical implications of additive preferences. *Economic Journal*, vol. 84.
EDGEWORTH, F. Y. (1879). The Hedonical calculus. *Mind*, July.
EDGEWORTH, F. Y. (1881). *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences* (L.S.E. Reprint 1935).
EDGEWORTH, F. Y. (1925). *Papers Relating to Political Economy* (3 vols.). Macmillan.
FISHER, I. (1892). *Mathematical Investigations in the Theory of Value and Prices*. (Reprinted in 1926.)
FISHER, I. (1930). A statistical method for measuring marginal utility. *Essays in Honour of J. B. Clark*.
GRAMM, W. P. (1970). Giffen's paradox and the Marshallian demand curve. *Manchester School*, vol. 38.
GREEN, J. (1974). The stability of Edgeworth's recontracting process. *Econometrica*, vol. 42.
HICKS, J. R. (1930). Edgeworth, Marshall and the indeterminateness of wages. *Economic Journal*.
HICKS, J. R. (1973). Recollections and documents. *Economica*.
HICKS, J. R. and ALLEN, R. G. D. (1934). A reconsideration of the theory of value. *Economica*.
HILDEBRAND, W. and KIRMAN, A. (1976). *General Equilibrium Analysis*. North-Holland.
HOTELLING, H. (1929). Stability in competition. *Economic Journal*, vol. 39.
HOTELLING, H. (1932). Edgeworth's taxation paradox and the nature of supply and demand functions. *Journal of Political Economy*, vol. 40.
HOTELLING, H. (1938). The general welfare in relation to problems of taxation and railway rates. *Econometrica*.
HUTCHISON, T. W. (1953). *A Review of Economic Doctrines, 1870–1929*. Oxford.
JAFFÉ, W. (1972). Leon Walras's role in the "marginal revolution" of the 1870s. Chapter 7 of Black *et al.* (1972).
JAFFÉ, W. (1974). Edgeworth's contract curve: A propadeutic essay in clarification. *History of Political Economy*.
JEVONS, W. S. (1970). *Theory of Political Economy*. Penguin. (Edited by Black, R. D. C.).
JOHNSON, W. E. (1913). The pure theory of utility curves. *Economic Journal*, vol. 23.
KALDOR, N. (1934). A classificatory note on the determinateness of equilibrium. *Review of Economic Studies*, vol. 1.
LEONTIEF, W. (1933). The use of indifference curves in the analysis of foreign trade. *Quarterly Journal of Economics*.
MARSHALL, A. (1975). Review of Edgeworths *Mathematical Psychics*, reprinted in Whitaker, J. K. (1975).
MARSHALL, A. (1961). *Principles of Economics*, 9th Variorum edition, Edited by C. W. Guillebaud, Macmillan.
NEGISHI, T. (1962). The stability of a competitive economy: A survey article. *Econometrica*, vol. 30.
NEWMAN, P. (1965). *The Theory of Exchange*. Prentice-Hall.
PANTALEONI, M. (1889). *Pure Economics*. Macmillan (translated by T. Boston Bruce).
PANTALEONI, M. (1923). Obituary of V. Pareto. *Economic Journal*.
PIGOU, A. C. (1910). A method of determining the numerical value of elasticities of demand. *Economic Journal*, vol. 20.
PIGOU, A. C. (1912). *Wealth and Welfare*. Macmillan.

PIGOU, A. C. (ed.) (1925). *Memorials of Alfred Marshall.* Macmillan.
SAMUELSON, P. A. (1974). Complementarity, an essay on the 40th Anniversary of the Hicks–Allen revolution in demand theory. *Journal of Economic Literature.*
SCHULTZ, H. L. (1933). Interrelations of demand. *Journal of Political Economy.*
SCHUMPETER, J. A. (1952). *Ten Great Economists.* Allen and Unwin.
SCHUMPETER, J. A. (1954). *History of Economic Analysis.* Allen and Unwin.
SHUBIK, M. (1959). Edgeworth market games, in Luce, R. D. and Tucker, A. W. (eds.). *Contributions to the Theory of Games, IV.* Annals of Math. Studies, 40. Princeton.
STAHL, T. (1972). *Bargaining Theory.* Economic Research Institute.
STIGLER, G. J. (1965). *Essays in the History of Economics.*
STREISSLER, E. (1972). To what extent was the Austrian school Marginalist? in Black *et al.* (1972).
TARASCIO, V. J. (1972). A correction; on the geneology of the so-called Edgeworth–Bowley Diagram. *Western Economic Journal.*
TELSER, L. (1971). *Competition, Collusion, and Game Theory.* Macmillan.
WALKER, D. A. (1973). Edgeworth's theory of recontract. *Economic Journal,* vol. 83.
WEATHERBY, J. L. (1976). Why was it called an Edgeworth–Bowley Box? A possible explanation. *Economic Inquiry.*
WEBB, S. and B. (1919). *Industrial Democracy.* Longmans.
WEINTRAUB, E. R. (1977). The microfoundations of Macroeconomics; A critical survey. *Journal of Economic Literature.*
WHITAKER, J. K. (1975). *The Early Economic Writings of Alfred Marshall,* vol. II, Macmillan.