

## Some recent interpretations of *Mathematical Psychics*

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I trust that I shall not suffer like the plaintiff in the old state of the English law who lost his case because in describing an article which had been stolen from him he spoke of a "ham" where he should have used the words "part of a ham."

—F. Y. Edgeworth<sup>1</sup>

### I. Introduction

The above quotation comes from Edgeworth's introduction to the reprint of part of his paper "The Determinateness of Equilibrium."<sup>2</sup> It is therefore rather ironic that the conventional interpretation of *Mathematical Psychics* has recently been questioned by a number of writers. For example Walker (1973, p. 139) asserts that "the indefiniteness of Edgeworth's early thought allowed him to entertain simultaneously two different notions of the equilibrating mechanism of a competitive market" and that "at no point . . . did he describe recontracting in the way that is now thought to characterise his work" (p. 147). Furthermore Tarascio (1972, p. 193) has argued that the diagram used by Edgeworth (1881, p. 28) "has nothing in common with the conventional Box used today." He goes so far as to suggest (p. 193) that "a geometrical manipulation of Edgeworth's diagram . . . results in a box diagram which is nonsensical in terms of the conventional box diagram." Tarascio's suggestion has been followed in papers by Jaffé (1974) and Weatherby (1976).<sup>3</sup> The acceptance of these assertions would mean that Edgeworth would have to be regarded as rather confused and inconsistent, merely suggesting a few 'lines of inquiry,'

1. *Papers* (1925), 2:314. For further discussion of Edgeworth's contribution to the theory of exchange and competition, see Creedy 1978.

2. He continued, "At any rate before condemnation is passed, reference should be made to the writer's essay on *Mathematical Psychics*." Unfortunately this plea has not always been heard sympathetically. Edgeworth was at one time a barrister-at-law!

3. The first two papers are very similar, though Jaffé (p. 343 n.3) mentions Tarascio only in a footnote.

rather than as a highly original theorist who developed some of the most basic results of modern microeconomics.<sup>4</sup>

The purpose of this article is, then, to examine these interpretations of what Stigler (p. 246) has called "probably the most elusively written book of importance in the history of economics." Section II below considers the arguments of Walker, while Section III considers those of Jaffé et al. First, it is perhaps useful to very briefly summarise the conventional interpretation of Edgeworth's contribution.

The models of exchange available before *Mathematical Psychics* were explicit in their assumption that all individuals took prices as 'given' (that is, parametric). Individuals were assumed to be unable to affect the rate of exchange at which their own transactions were conducted.<sup>5</sup> The equilibrium rate of exchange which would eliminate all excess demand and supply was then obtained as the solution to a set of simultaneous equations (using a given initial distribution of resources and preferences)—and no genuine 'competition' between individual buyers or sellers was necessary. Walras used his well-known *tâtonnement* process, and Jevons openly stated that he was unable to say anything about the process by which an equilibrium might actually be achieved.

Edgeworth then presented his *recontracting* model whereby buyers and sellers freely communicate with each other within the context of what is now called a cooperative game. The making and breaking of *provisional* contracts provides a free flow of information within the market; and if the number of competitors is sufficiently large the solution of such a game exactly corresponds to the Walrasian equilibrium. Again, the order in which the provisional contracts are made cannot be determined, although this does not affect the final solution. A major contribution of Edgeworth's model was that it explicitly introduced the role of *numbers* into the analysis of competition, showing that where there are few competitors the solution is indeterminate and depends on "the objectionable arts of higgling." It is now well known that these more subtle parts of *Mathematical Psychics* were long neglected until the important paper by Shubik (1959), which revealed the precise relationship between game theory and Edgeworth's model.

4. See Samuelson, p. 1277 n.19, and Schumpeter, p. 831, for panegyrics. Walker (p. 139 n.1) does not actually mention the main results, but suggests that the later game-theory interpretations are wrong. Very clear expositions of recent results can be found in Hildebrand and Kirman 1976 and Bacharach 1976.

5. Menger, in the *Grundsätze*, paid much more attention to factors like quality differences, imperfect information, etc., which would prevent equalisation of the rate of exchange. Edgeworth does not refer to Menger in 1881.

II. *The Interpretations of Walker*

It is this conventional view of Edgeworth that Walker challenges. First, it is argued that Edgeworth was inconsistent in his view that the uniformity of prices was the *result* of his model, rather than an initial assumption. Secondly, Walker argues that Edgeworth was inconsistent in his assumption that contracts are provisional.

*Uniformity of prices*

Walker first claims that the current interpretation of Edgeworth “has been made possible” because “his literary style was elusive, and his early thought on price formation was incomplete” (Walker, p. 139).<sup>6</sup> Walker suggests (p. 140) that Edgeworth simultaneously held two views about competition, and “writing in the Walrasian vein . . . neglected recontract.” As evidence of this assertion Walker then quotes Edgeworth 1881, p. 30, as saying, “you might suppose each dealer to write down his *demand*, . . . these data having been furnished to a sort of market-machine, the *price* to be passionlessly evaluated.” It is perfectly clear, however, that Edgeworth was simply contrasting this kind of *deus ex machina* with his own approach, involving as it did the “objectionable arts of higgling” (1881, p. 30). Indeed, just before the part quoted by Walker he says explicitly, “with this procedure . . . (that is, barter) . . . is contrasted the smooth machinery of the open market.” He says quite clearly that the model involving recontract is an improvement over Walras’ model, especially in showing how the result of his procedure is equivalent to one in which all agents are ‘price takers’ only in the special case of perfect competition.<sup>7</sup> Thus, “the advantage of this general method is that it is applicable to the particular case of imperfect competition; where the conceptions of *demand and supply at a price* are no longer appropriate.”

Walker (p. 140) then states that “when Edgeworth formally set out the properties of a perfectly competitive market, he introduced the equality of desired market demand and supply as a *postulate*, as an as-

6. There is, of course, no doubt that his style was elusive and his mathematics clumsy, but Walker (p. 139 n.2) also repeats the suggestion made by Stigler that Edgeworth did not, “tarry long at one point.” But Edgeworth often had his papers privately reprinted with additional sections, and regularly quoted from his own work. As Bowley notes (1928, p. 3), “Each time a problem was attacked afresh there were references to the results of former work, just as an Act of Parliament is to be interpreted by reference.” Presumably to suggest a lack of clarity, Walker (p. 142 n.1) notes that Edgeworth used the words *articles of contract* in two different senses, but he fails to point out that Edgeworth consistently used italics for the meaning given in 1881, p. 28.

7. The quotation reproduced in Walker (p. 143) from Edgeworth 1925, 2:311, and meant to show a change in Edgeworth’s thought, is simply a repetition of the same point made in 1881.

sumed condition of a market model, not as a derivative consequence of recontract.” To refute this suggestion it is only necessary to quote Edgeworth 1925, 2:311: “The proposition that there is only one price in a perfect market may be regarded as *deducible* from the more axiomatic principle of recontract (*Mathematical Psychics*, p. 40 and context).”<sup>8</sup> The importance of this result is that in large markets the recontracting process—where individuals obtain information through free communication—leads to the same unique rate of exchange as in a system where prices are parametric and individuals make choices without reference to others (though the equilibrium price has to be set by an omniscient *intellectus angelicus*).<sup>9</sup>

### *The nature of recontract*

A further part of Walker’s argument concerns the meaning of recontract. He suggests that Edgeworth assumed that exchange takes place at the formation of each coalition or contract.<sup>10</sup> In support of this he quotes Edgeworth as saying, “a landlord, on expiry of lease recontracts, it may be, with a new tenant” (Walker, p. 142; Edgeworth 1881, p. 17). It is, however, surprising that he does not quote the preceding part of the paragraph: “It is . . . *Pax* or *pact* between contractors during contract, *war*, when some of the contractors *without the consent of others* recontract. Thus an auctioneer having been in contract with the last bidder (to sell at such a price *if* no higher bid) *recontracts* with higher bidder.” While the statement about landlords is not very clear, there is no doubt about this earlier part. The phrases “*if* no higher bid” and “*without the consent of others*” can only mean that no actual exchange takes place. Thus the process of recontract involves no wealth changes until the *final settlement* (defined in 1881, p. 19) is reached. Edgeworth (1881, p. 19) also adds, “any individual is free to *recontract* with another independently of, *without the consent being required of*, any third party.” There would be no need to add the condition about consent if actual exchange were assumed to take

8. This was a note added in 1925, which refers to his original presentation. Also, in 1925, 2:453, Edgeworth repeated, “The existence of a uniform rate-of-exchange . . . is perhaps not so much axiomatic as deducible from the process of competition in a perfect market.”

9. See also Edgeworth 1881, p. 31 n.1. He explicitly stated that his model qualified the results of Walras and Jevons: “It must be carefully remembered that Prof. Jevons’ formulae of exchange apply not to isolated individuals . . . but . . . to individuals clothed with the properties of a market.” There are no grounds for Walker’s statement, “what is the associated equilibrium price? Edgeworth was unclear on this matter” (Walker, p. 141). See Section III, “The indifference map,” below. Walker’s comments at the bottom of page 141 are also not substantiated.

10. Walker says (p. 142), “they last until the end of the contract period, and he continues to talk about market days. This is considered further below.

place. Later (1881, p. 35) he adds, "it will generally be in the interest of the X of one couple and the Y of the other to rush together, leaving their partners in the lurch." Again this clearly shows that the coalition was only a provisional and conditional contract.<sup>11</sup> Furthermore it is difficult to see how a close reading of pp. 35–49 of Edgeworth 1881 could suggest that he had not fully worked through his recontract process.<sup>12</sup>

It is also worth noting that Edgeworth 1881 nowhere talks about 'market days,' whereas Walker confuses the discussion by the introduction of such 'days.' As Weintraub (p. 16) has pointed out, "the core . . . seems to embody a process dynamic . . . *there is not, however, any notion of time involved in this process.*"

Walker (p. 133) then attempts to show that in his later work Edgeworth used a "competitive recontract model, introducing disequilibrium transactions into the model." On the basis of his earlier argument that Edgeworth in 1881 was inconsistent and incomplete, Walker is then able to discount all later references by Edgeworth to his earlier work.<sup>13</sup> Walker's argument mainly rests, however, on a quotation selected from Edgeworth 1925, 1:40, which mentions two cases; one where "intending buyers and sellers . . . remain in communication without actually making exchanges," and the second where "hirings are made" (Edgeworth here is explicitly considering a labour market without unions), but "the parties encounter each other the following day, with dispositions the same as on the first day,—like combatants *armis animisque refecti?*" Walker argues (p. 145) that the second case is a non-*tâtonnement* process which Edgeworth used, "once he had a clear view of recontracting."

It is, however, perfectly clear that these two cases are *formally equivalent*. Furthermore Walker fails to note the very important sentence which states that "two kinds of higgling may be distinguished as appropriate respectively to short and long periods" (*Papers*, 1:40). The

11. Again in Edgeworth 1925, 2:313: "The dispositions and circumstances of the parties are assumed to remain throughout constant. But it is supposed that agreements are renewed or varied many times." Stigler (p. 247 n.25) also refers to "recontract, the institution which allows tentative contracts to be broken without penalty."

12. On first stating the main results Edgeworth (1881, p. 20) says, "*Conclusions* rather, the mathematical demonstration of which is not fully exhibited." Later (p. 35) when considering replication, he says, "for the sake of illustration (not of the argument)"; and more significantly, on page 38: "If this reasoning does not seem satisfactory, it would be possible to give a more formal proof; bringing out the important result that the common tangent to both indifference curves at the point  $\zeta\eta$  is the vector from the origin." These comments seem to indicate that his statements were much more than 'conjectures,' the term used by later game theorists.

13. For example, Walker (p. 145) says: "It can be seen . . . that he had come to want the competitive recontracting process . . . to be interpreted as a non-*tâtonnement* process." In Edgeworth 1925, 1:36, a new footnote is added to the paper on distribution, which repeats the argument of 1881.

reference to short and long periods is crucial here, and of course was written after the introduction of these concepts by Marshall. It is also the first time that Edgeworth uses the parable of market days to explain the process. It can therefore be argued that far from changing his views about recontract, Edgeworth was simply trying to show that his model may be of more than academic interest; that it could be applied to certain kinds of contracts—in this case labour contracts on a ‘daily’ or ‘short term’ hire<sup>14</sup>—where the ‘exchange’ does not really involve a *permanent* redistribution of wealth (the labourer has the same amount of labour to offer the next day and has gained information from his earlier contract).

### III. *The Interpretation of Jaffé et al.*

#### *The Box Diagram*

Tarascio, Jaffé, and Weatherby all point out that Pareto first published the Edgeworth Box Diagram in its familiar form,<sup>15</sup> but this can surely be no surprise. Indeed, Blaug (p. 311) merely states, “Instead of the now conventional box diagram introduced by Pareto . . . Edgeworth turns the axis around.” Pareto’s debt to Edgeworth has, however, long been recognised. Thus Pantaleoni (p. 548) notes, “Pareto . . . elaborated a system of general economic equilibrium . . . with materials only partially his own, . . . he would not have been what he has been without the author of *Mathematical Psychics* and the author of *Mathematical Investigations into the Theory of Value and Prices*.”<sup>16</sup>

The suggestion, noted above, that Edgeworth’s diagram bears no relation to the conventional Box Diagram is, however, more serious. The argument rests on the proposition that Edgeworth did not specify the initial endowments. Jaffé (p. 344) thus says, “Edgeworth’s contract curve diagram could only be converted into a box diagram of specific dimensions if the initial endowments, which Edgeworth deliberately chose to leave implicit, were explicitly specified.”

Although Edgeworth did not write the length of each axis in the dia-

14. Similarly, this may be applied to the case where goods are perishable within the day, but are ‘cropped’ daily in regular amounts. In his paper on distribution Edgeworth often talked of hiring a daily mountain guide. Again in 1925, 3:453, he mentions situations “continuously repeated under analogous conditions.”

15. Rather than Bowley (1924), who anyway acknowledged Pareto and claimed no originality. Weatherby (1976) actually suggests that the box was attributed to Edgeworth because of Marshall’s discussion in his appendix on barter (Marshall, 1:844), and the controversy, first reported at length in 2:790–98!

16. Pantaleoni, a great friend and correspondent of Pareto, also noted (p. 585): “In economics this work, I submit, consisted in bringing to a great perfection a machine which was to a very large extent already built up.” See also Shackle 1967.

gram (1881, p. 28)—for reasons discussed below—he was nevertheless quite explicit about the initial endowments in the text. Thus (1881, p. 20) he states, “let  $x$  and  $y$  be the portions interchanged, as in Prof. Jevons’ example. Then the utility of one party, say  $X$ , may be written  $\Phi_1(a-x) + \Psi_1(y)$ ; and the utility of the other party, say  $Y$ ,  $\Phi_2(x) + \Psi_2(b-y)$ ; where  $\Phi$  and  $\Psi$  are the integrals of Prof. Jevons’ symbols  $\phi$  and  $\psi$ .<sup>17</sup>

The person  $X$  therefore begins with an amount  $a$  of his own good (which Edgeworth also denotes by the letter  $X$ , for obvious reasons). Similarly person  $Y$  has an amount  $b$ , of good  $Y$  only. This is the initial situation which Edgeworth took from Jevons’ consideration of exchange, and which was used consistently throughout *Mathematical Psychics*.

Edgeworth then defined utility  $P$  of individual  $X$  as  $P = F(x,y)$ ,<sup>18</sup> and although the initial endowments are implicit in the function  $F$ , the reason for this presentation is fully discussed in his Appendix V (1881, p. 105). He showed that mathematically there is no point in writing, say,  $P = P(a-x, y)$ . Edgeworth preferred to write  $P = F(x, y)$  because he was primarily interested in deriving a theory of *exchange*, and it makes more sense to have the amounts exchanged as the arguments of the function. The later more pedagogic uses of the Box Diagram are usually concerned with *allocation* between individuals. Edgeworth (1881, p. 105) also defines the demand curve of person  $X$  for good  $Y$  (in terms of the offer of  $x$  of good  $X$ ). He transforms to polar coordinates  $P = p(\rho \cos \theta, \rho \sin \theta)$ , where  $\theta$  is the rate of exchange, and defines the demand curve as  $dP/d\rho = 0$ . Jaffé (p. 344 n. 4) seems to have missed this point when he quotes Edgeworth as saying, “it is not necessary . . . to specify this initial endowment [ $a$ ] explicitly.” Edgeworth was simply making a mathematical point about the equilibrium condition.

Similar considerations apply equally to the diagram, and once it had been constructed there was no need to write down the length of each axis since Edgeworth was primarily interested in discussing the choice by individuals of quantities *exchanged*. The emphasis on exchange, rather than allocation, also explains why Edgeworth’s diagram is rotated by  $90^\circ$  when compared with the conventional textbook figure; with amounts *exchanged* measured along each axis it would be idiosyncratic to place the origin anywhere other than in the southwest corner.<sup>19</sup>

17. On the next page he compares the definition of the contract curve with Jevons’ ratio of marginal utilities. All this is clearly understood in Marshall’s review; see Whitaker, p. 266.

18. The comma has been added here and below. Edgeworth actually wrote  $P = F(xy)$ . See also Whitaker, p. 266 n.8.

19. Stigler (p. 105) refers to ‘asymmetrical axes,’ but the asymmetry only applies to

*The indifference map*

A further rather surprising argument is that Edgeworth did not actually use a 'system' of indifference curves. Tarascio (p. 194) says, "Nowhere in *Mathematical Psychics* does one find an indifference mapping consisting of more than one curve per individual," and "the first person to use explicitly a system of indifference curves for each person was Irving Fisher in 1892."<sup>20</sup> Jaffé (p. 358) also states that Edgeworth mentioned "in passing 'an interior indifference curve'. . . in one place, and only in one place, in the *Mathematical Psychics*."<sup>21</sup>

Edgeworth of course did not draw a family of indifference curves; indeed he very rarely used diagrams and was much more concerned to emphasise that the results are obtained by the 'mathematical method.' It is, however, clear that the definition of the contract curve implies a 'system' of indifference curves. Thus a point on the contract curve is defined as where "their lines of indifference are coincident" (1881, p. 22). Furthermore, "these settlements are represented by an indefinite number of points" (p. 29). In Edgeworth's discussion of the introduction of additional traders (1881, pp. 35–38), the 'system' of indifference curves is explicit, both in the mathematics and in the discussion. On page 38 he says, "the common tangent to both indifference curves at the point  $\zeta\eta$  is the vector from the origin." But Edgeworth obviously felt that it would be otiose to actually draw these additional curves. This contention of Tarascio and Jaffé is even more surprising considering the statement (1881, p. 40), "let the indifference curves consist of families of concentric circles." Edgeworth's discussion in Appendix V is also relevant; for example, the problem of deriving the offer curve is, he says (p. 113) to "find the locus of the points where lines from the origin touch curves of indifference." Similarly, "for all points above Q where a curve of indifference of B touches the demand-curve of A" (p. 116).<sup>22</sup>

the formulation  $P = F(x,y)$ , not to the diagram. There is no asymmetry from the point of view of exchange.

20. Edgeworth's characteristically 'modest' review (1925, 3:39) is of interest here. After discussing Fisher's use of indifference curves he notes: "These conceptions, received by Dr. Fisher from his predecessors, have been greatly improved by him." Fisher's independent 'discovery' is, however, now recognised. Edgeworth was surprised and interested by Fisher's discussion of the integrability problem: "Soaring . . . beyond the furthest heights of his predecessors, . . . he attains . . . a conclusion which seems to us of unexpected value" (1925, 3:40). Pareto also discussed the same problem, of course. Indeed Pantaleoni (p. 584) notes that Pareto's first publication was in the theory of elasticity in solid bodies and "researches concerning the integration of the differential equations determining their equilibrium."

21. Jaffé (p. 351) also suggests, "when such curves are drawn, their intercepts on their respective 'axes of acquisition' acquire a theoretical significance apparently unsuspected by Edgeworth."

22. Also on page 113 Edgeworth refers to "the differential equation of our 'curves of



IV. *Conclusions*

It is therefore argued, in contradiction to the authors discussed above, that Edgeworth's recontract model was clearly worked out in 1881, that he consistently used the same concept of recontract, and that the current interpretation used in the literature on cooperative games is substantially correct. In 1928 Bowley produced a remarkable survey of Edgeworth's contributions to mathematical statistics. His statement (1928, p. 3) may also be applied to Edgeworth's work on exchange and competition: "The impression obtained is that his work is singularly consistent; the germs of the last analysis are to be found in his earliest papers."

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*indifference,* " and again on the same page, "if . . . the curves of indifference consist of a series of circles." Further, on page 114 is found, "the equation of any indifference curve"; and again, p. 116, "B's curves of indifference."

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