The "Cournot-Bertrand Debate": A Historical Perspective

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1. Introduction

The distinction between models in terms of quantity and models in terms of price is a classical and well-established one in modern oligopoly theory. The former had its origin in Antoine Augustin Cournot's *Recherches sur les Principes Mathématiques de la Théorie des Richesses* (1838), while the latter is the result of criticism of Cournot's book that Joseph Bertrand provided in a review published in 1883.[1]

The accepted version is that Cournot believed that producers in an oligopoly decide their policy assuming that other producers will maintain their output at its existing level, while Bertrand considered it more realistic to assume that producers act on the belief that competitors will maintain their price rather than their output.

According to W. Fellner (1949), "As is well known, the Cournot solution is based on the assumption that (in undifferentiated duopoly) each duopolist believes that his rival will go on producing a definite quantity irrespective of the quantity *he* produces. Obviously in these circumstances each duopolist believes that he can calculate the quan-

1. An English translation of Bertrand's text is provided in an appendix to this article.

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tity he should produce in order to maximize his profits" (57); Fellner adds that "the French mathematician Joseph Bertrand . . . essentially objected that Cournot had used output reaction functions instead of price reaction functions" (77).

Martin Shubik (1959) writes, "Bertrand objected to Cournot's analysis of the duopoly problem in terms of quantity as the strategic variable. He suggested a solution that depends on price variation" (80). While describing Cournot's oligopoly, James Friedman (1983) notes that "each firm makes a decision on output in a once-for-all way" (23); later, he adds that Bertrand "regards price as the decision variable, as price is used in Cournot's monopoly analysis of Chapters 5 and 6" (46).

More recently, A. Jacquemin (1987) states, "In his well-known critique of Cournot's model, Bertrand considers it more reasonable to assume that firms fix prices, not quantities" (66). Tirole (1988) explains that "the Bertrand (1883) paradox states that the unique equilibrium has the two firms charge the competitive price" (210).

In the recent *Handbook of Industrial Organization* (1989), Shapiro expresses the same belief, stating that "a natural objection to the Cournot quantity model is that in practice businesses choose price rather than quantities as their strategic variable. Indeed, the actual process of price formation in Cournot's theory is somewhat mysterious. Bertrand (1883), in his review of Cournot's book, was the first to criticize Cournot on these grounds, and his name has since been attached to simple pricing games, just as Cournot's is with simple quantity games" (343). While Bertrand is often referred to, he is never quoted, and probably never read, which perhaps explains why his criticism is "well-known."

The Cournot-Bertrand debate, as it appears in these texts, can be summarized by four main themes: a) each of Cournot's duopolists make the assumption that his rival will maintain his output, while b) each of Bertrand's duopolists makes the assumption that his rival will maintain his price; c) each of Cournot's duopolists manipulates his output in order to maximize his profits, while d) each of Bertrand's duopolists manipulates his price in order to maximize his profits.

In this article I will show that this widely accepted interpretation is quite erroneous. Actually, of the four themes listed above, only the first can be seen as historically correct. I will show in section 2 that the modern interpretation tends to give the wrong impression of Cournot's

work, where produced quantities can hardly appear as a strategic or decision variable. In section 3 Bertrand's criticism will be carefully translated and analyzed, and we will see how far it is from the standard view described above. In section 4 I shall try to trace how and why such an erroneous interpretation of both Cournot and Bertrand has become the generally accepted view today. Margaret Chevaillier's translation of Bertrand's review follows.

2. Prices and Quantities in the Work of Cournot

Chapters 5 to 9 in Cournot's *Researches* must be read to appreciate how he viewed the relation between price and quantity: Monopoly is dealt with in chapters 5 and 6. In chapter 7, duopoly and oligopoly are treated under the heading "Of the Competition of Producers." Chapter 8 ("Of Unlimited Competition") is about competition among many producers, and chapter 9 ("Of the Mutual Relations of Producers") deals with the case where two firms are producing goods that are complementary inputs for the product of a third party. These chapters are in a logical order based on the number of producers: one (monopoly), then two (duopoly) or several (oligopoly), and finally, many (competition). Cournot's view of this development is itself perfectly consistent, although not necessarily satisfactory in every respect. Throughout Cournot uses an analytic method based on profit maximization.

This strong consistency in Cournot's approach must be kept in mind when the question of the decision variable is examined. In the two chapters concerning monopoly behavior, profit is maximized with regard to the product price, but not with regard to the quantity produced, as is common today.

At the very beginning of chapter 5, the monopolist's behavior is described as the search for the best price: "For convenience in discussion, suppose that a man finds himself proprietor of a mineral spring which has just been found to possess salutary properties possessed by no other. He could doubtless fix the price of a liter of this water at 100 francs; but he would soon see by the scant demand, that this is not the way to make the most of his property. He will therefore successively reduce the price of the liter to the point which will give him the greatest possible profit" (56).

Later in this chapter he gives an equation with profit maximization as a general condition for monopoly,

$$D + dD/dp[p - d\phi(D)/dD] = 0 \quad (1)$$

where p is the price, $D = F(p)$ is the demand function, $\phi(D)$ is the total cost function. Of course, this formulation is equivalent to the usual modern formula, where marginal revenue is equal to marginal cost

$$p + D.dp(D)/dD = d\phi(D)/dD \quad (2)$$

where $p(D)$ is the inverse demand function. (To see why these formulations are equivalent, multiply equation 1 by dp/dD.)

This modern version of the profit-maximization equation is certainly easier to interpret than Cournot's, who could have adopted it: indeed he notes that "the complex function $pF(p) - \phi(D)$ can be regarded as depending implicitly on the single variable p, although generally the cost of production is an explicit function, not of the price of the article produced, but of the quantity produced" (57).

Why would Cournot insist that the profit function must be a function of price and not of output, an implicit and not a (more natural) explicit function? In his first model where there is no production cost, the water given by the spring is a fixed quantity; only sales can be modified, and only by manipulating the selling price. But when he introduces production costs, Cournot does not change his view of the role of price. In this case, the producer first sets a price to control demand, then produces to satisfy all demand at this best predetermined price. So, looking at Cournot's model of monopoly, one must assume that he quite intentionally considered price as the variable that is manipulated by the monopolist.

Consulting the chapters on oligopoly and competition, we find that the same hypothesis is maintained throughout, with the (quantitatively small and only apparent) exception of section 43 where the classical duopoly model is presented.[2] At the beginning of chapter 7, where duopoly is dealt with for the first time, Cournot makes an essential as-

2. When dealing with "unlimited competition," Cournot still seems to treat price as a decision variable. The equilibrium condition for the competitive firm is written deriving each firm's profit relative to p, as if small firms could manipulate it. This is obviously inconsistent, and the demonstration that Cournot gives (for the first time in an economic text) of the condition price being equal to marginal cost is incorrect (see 90).

sumption concerning his model, product homogeneity, and the obvious conclusion that only a single price can exist at a given moment: "Let us now imagine two proprietors and two springs of which the qualities are identical, and which, on account of their similar positions, supply the same market in competition. In this case the price is necessarily the same for each proprietor. If p is this price, $D = F(p)$ the total sales, D_1 the sales from the spring (1) and D_2 the sales from the spring (2), then $D_1 + D_2 = D$" (79). Cournot goes on with the hypothesis that the proprietors do not cooperate and notes that if they did, the outcome would be identical to monopoly.

He then proceeds to build on this model, and here it may seem that Cournot considers the quantity produced, instead of price as he had in the preceding chapters, as the important variable. But review the text again:

> Instead of adopting $D = F(\mathrm{p})$ as before, in this case it will be convenient to adopt the inverse notation $p = f(D)$; and then the profits of proprietors (1) and (2) will be respectively expressed by
>
> $$D_1 \times f(D_1 + D_2),\ D_2 \times f(D_1 + D_2),$$
>
> i.e. by functions into each of which enter two variables D_1 and D_2.
>
> Proprietor (1) can have no direct influence on the determination of D_2: all that he can do, when D_2 has been determined by proprietor (2), is to choose for D_1 the value which is best for him. This he will be able to accomplish by properly adjusting [the][3] price, except as proprietor (2), who, seeing himself forced to accept this price and this value of D_1, may adopt a new value for D_2. (80)

From this quotation it clearly appears that quantities are chosen as the variable because of convenience (see below); and that each owner will use price as a variable to control quantity, quantity control being itself a way of maximizing profit. This last point may appear to be confusing: for example, Shubik (1987) sees no reason for Cournot's view, when he writes that "it is at this point that Cournot switches from price to quantity of a homogeneous product as a strategic variable used by the competitors. His words and the mathematics do not quite

3. Bacon's translation is "This he will be able to accomplish by properly adjusting his price"; but this is not correct, since the original text is: "ce à quoi il parviendra en modifiant correctement le prix" (89). This has already been noted by Nichol (1934b), who proposed "suitably modifying the price" instead of "by properly adjusting his price" (note 18).

match. He says, 'This he will be able to accomplish by properly adjusting his price' " (710; see again my quotation of Shapiro 1989 in the introduction).

But keeping in mind Cournot's view of the respective roles of prices and quantities, price is manipulated to control the quantity demanded, and production is then fixed so as to serve the demand. This view is certainly difficult to accept, because it seems to be impossible for one of the duopolists to manipulate the market price in all circumstances. For instance, Nichol (1934b) has insisted that Cournot's model is consistent only if buyers, not sellers, determine the market price.[4] But if a duopolist actually assumes that his rival will not adjust his output, there is really no limit, except in the demand curve, to his ability to modify the price and his own output, whichever variable is used to set the other. To consider that price is manipulated, first leads to the realization that the assumption that the rival's output is given is unacceptable. But this is how Cournot conceived all markets, even the competitive one, a situation where the sellers name the price.

So it hardly seems justified to contend, as most modern literature does, that (produced) quantities, as opposed to price, are the strategic variable in Cournot's model of oligopoly. Moreover, in this framework, the opposition between price and quantity is meaningless. If the quantity of one of the duopolists is fixed (or considered as such), the other owner will behave like a monopolist with residual demand as regards his output and decide on the market-clearing price by considering the whole supply that will result; he can, as we have seen, maximize profit either with regard to price or with regard to quantity. The notion of a strategic variable is empty in Cournot's market, where sellers are supposed to have the power to modify the price easily.

Cournot himself indicated this later when he published *Principes de la Théorie des Richesses* in 1863. The nonmathematical summary of his earlier book is clear: the two proprietors were given as M and N, and their respective outputs, m and n. Cournot writes, "Let us admit, for a moment, that proprietor N has set arbitrarily, without regard to the price, the quantity n that he intends to deliver: then proprietor M will *set the selling price, that is, the total output (the sum of quantities*

4. "The perfect uniformity of price in Cournot's problem is not explained, *unless prices are directly determined by buyers*" (Spengler and Allen, 582).

m and n); that is, also, his output m, in such a way as to make the greatest possible income[5] (emphasis added).

However, and on this point no debate is possible, quantities are important in this model because they are the object of every seller's conjectures. This feature of Cournot's model of oligopoly is probably the reason why he found it "convenient" to use the quantity sold as the profit-maximization variable. To see why, remember that reaction functions appear at the beginning of chapter 7; they are formulated by setting the derivatives of the profit functions $\partial\pi_1(D_1, D_2)/\partial D_1$ and $\partial\pi_2(D_1, D_2)/\partial D_2$ equal to zero. These reaction functions, which can be written as

$$D_1 = R_1(D_2) \text{ and } D_2 = R_2(D_1)$$

can be represented on a graph with axes D_1 and D_2; visualized together their common point is the Cournot equilibrium; the convergence towards this point is easily seen on the graph. What would this be if selling price were used as the profit-maximization variable? No doubt it would be possible to give a formulation for the reaction functions. For each D_2, calculate the residual demand addressed to (1) and the best common price that would result, and vice versa. So to define in Cournot's framework "price-quantity reaction functions," write

$$p_1 = r_1(D_2) \text{ and } p_2 = r_2(D_1),$$
$$\text{where } r_1(D_2) = f(D_1 + D_2) = f(R_1(D_2) + D_2) \text{ and}$$
$$r_2(D_1) = f(D_1 + D_2) = f(D_1 + R_2(D_1)).$$

As the product is perfectly homogeneous, p_1 and p_2 are not differentiated prices, but different values given (perhaps successively) to the unique market price by the actions of (1) and (2). These equations cannot be replaced by equations having p_2 and p_1 as arguments instead of D_2 and D_1 because conjectures are on quantities (the link existing in a dynamic model between, say, $p_{1,t}$ and $p_{2,t-1}$ depends on the particular value of $D_{2,t-1}$). So if we were to draw a graph of these two "price-quantity reaction functions" it would be a three-dimensional graph (with dimensions p, D_1 and D_2) and the equilibrium point would not be

5. "Admettons, pour un moment, que le propriétaire N ait fixé arbitrairement, sans égard au prix, la quantité *n* qu'il entend livrer: alors, le propriétaire M fixera le prix de vente, c'est-à-dire la production totale (composée de la somme des quantités *m* et *n*); c'est-à-dire encore sa production *m*, de manière à se procurer le plus grand revenu possible" (1863, 62).

easy to see as these reaction functions would not cross anywhere (they lie in two different planes).

I am now ready to offer a hypothesis as to the "convenience" of using, in section 43, quantities instead of price as the maximization variable. In order to perform the stability analysis of his equilibrium and to illustrate it convincingly, Cournot needed the apparatus of the (usual) quantity reaction functions. This is linked to the fact that the conjectures are on quantities, but does not imply that quantity sold should be a strategic variable.

This hypothesis seems to be further strengthened by the fact that after this first paragraph of Chapter 7 (section 43), Cournot abandons the quantity-dependent profit maximization and returns to price dependency; indeed, the first phrase of section 44 is "From equations (1) and (2) [reaction functions] we derive first $D_1 = D_2$ (which ought to be the case, as the springs are supposed to be similar and similarly situated), and then by addition

$$2f(D) + Df'(D) = 0,$$

an equation which can be transformed into

$$D + 2p(dD/dp) = 0" \text{ (82)}. \tag{3}$$

The remainder of chapter 7, as well as chapters 8 (on competition) and 9, use price as the main variable. So only one section out of forty (26 to 65) in the book that deal with price theory (and only one out of seven in the chapter on oligopoly) is written with quantity as the apparently strategic variable.

I hope to have shown that the concept of a "strategic (or decision) variable" within the Cournot model of oligopoly is unnecessary and meaningless; Cournot himself considers that sellers use price to modify quantities demanded; and the use of quantities in section 43 as a profit-maximizing variable is due, as Cournot himself stressed, to the need for a convenient representation, but was not an essential assumption.

I conclude by focusing on Cournot's demonstration of why duopolists cannot maximize joint profits and act like a monopoly unless they make a formal contract (section 44). If they were in this situation, says Cournot, any of them could, with a momentary profit, change his output. Of course, he will be "punished for his mistake" (83) when the other duopolist reacts by changing his output rate, and so on; these successive reactions do not lead to the original coalition point, but nec-

essarily to the Cournot equilibrium as already demonstrated (section 43). Note from the last quotation that Cournot was perfectly aware that he attributed incorrect conjectures to his proprietors.

3. Bertrand's Criticism

In September 1883, *Le Journal des Savants* published a short text by Joseph Bertrand reviewing Walras's *Théorie Mathématique de la Richesse Sociale* and Cournot's *Researches*, forty-five years after the latter was first published. This review was not the first to deal with the *Researches;* Walras (1863) and Fontenay (1864) had already commented on it. Fauveau (1864 and 1867) had also criticized Cournot, although not in the form of a review.

In his short text, Bertrand presents some criticisms of the use of mathematics in economic reasoning and, concerning Cournot, considers that "removing the symbols would reduce the book to just a few pages" (1883, 500; see appendix). In many respects, it is impossible to believe that Bertrand seriously read Cournot (or Walras); for example, when dealing with Cournot's monopoly theory, he writes that demand ("debit") can be represented by $\phi(p)$. This is not Cournot's notation: he used $F(D)$ to symbolize the demand function and $\phi(D)$ for the cost function. This change in notation would be unimportant if, on the next page, Bertrand did not then use Cournot's own notations to show what the consequences of taxation are on monopoly pricing. There the expressions $\phi(p)$ (the demand function in Bertrand's notation) and $\phi(D)$ (Cournot's cost function) are nearly side by side with different and incompatible meanings. Bertrand writes with candor that "the reader is familiar with these letters and functions which figured in the previous pages" (502)!

After stating that Cournot's results sometimes seem unacceptable, Bertrand writes concerning duopoly that

> Such is the study made in chapter VII of the rivalry between two proprietors, who without having to worry about any competition, manage two springs of identical quality. It would be in their mutual interest to associate or at least to set a common price so as to make the largest possible revenue from all the buyers, but this solution is rejected. Cournot assumes that one of the proprietors will reduce his prices to attract buyers to him and that the other will, in turn, reduce

> his prices even more to attract business back to him. They will only stop undercutting each other in this way when either proprietor, even if the other abandoned the struggle, has nothing more to gain from reducing his prices. One major objection to this is that there is no solution under this assumption, in that there is no limit in the downward movement. Indeed, whatever the common price adopted, if one of the proprietors, alone, reduces his price he will, ignoring any minor exceptions, attract all the buyers and thus double his revenue if his rival lets him do so. If Cournot's formulation conceals this obvious result, it is because he most inadvertently introduces as D and D′ the two proprietors' respective outputs and by considering them as independent variables he assumes that should either proprietor change his output then the other proprietor's output could remain constant. It quite obviously could not. (503)

This text is not simple; it needs to be read with Cournot's chapter 7 "in the other hand." Following Cournot on this point, Bertrand first states that the coalitional solution would be the best. He next criticizes Cournot for rejecting this solution (above at the end of section 2). In the first place, in Bertrand's opinion, the Cournot solution does not appear to be an equilibrium: "there is no solution under this assumption, in that there is no limit to the downward movement" (503). The problem for which there is no solution is then quite explicitly: what happens when, starting from coalitional monopoly-like pricing, the proprietors begin to lower market price to gain more profit, according to their reaction functions? Bertrand believes that the fall in price will have no limit, because each owner could lower his price (the other price being constant) and in so doing take all the consumers. Such a critique of the Cournot model relies on a misunderstanding, because in this model, there cannot be two prices for the same good.

Therefore, it seems wrong to say that Bertrand suggested that producers in an oligopoly use price as their strategic variable, unless one accepts that a slip of language may constitute an argument; what he said is that in Cournot's model the owners could propose differentiated prices; and that is false.[6]

6. Schumpeter made the same point, writing that "Bertrand imputed to Cournot the hypothesis that each duopolist tries to undercut the other, which involves a misunderstanding of Cournot's argument and points towards a result that is, if anything, worse than Cournot's" (1954, 982 n. 31).

Then, Bertrand explains why he considers Cournot to be wrong. In the last two sentences of the text above, he attacks the type of conjecture that appears in this model (that the other producer's output will remain constant). Here again, we find some confusion: Bertrand attributes *to Cournot, not to the spring owners,* the belief that if D changes, D' will remain constant, and vice versa. He did not understand what Cournot meant by this assumption nor what the reaction functions could be. Not having grasped this essential feature of Cournot's approach (conjectures), Bertrand could not propose an alternative assumption. He remained content with the conclusion that "it quite obviously could not [be true]"; in no way did he assume that each owner would act believing that the other owner's price would stay constant.

Did Bertrand mean that, in duopoly, competition must lead to a situation where price is equal to marginal cost? Probably not, because he believed that competition would lead to no solution. Here I think that we must trust his wording to understand him, keeping in mind that he was an excellent mathematician for whom "no solution" certainly did not mean "a solution with zero profits," as he is interpreted now. He may have had the idea that in a price war, price will fall below marginal cost, or he may not have given any thought to the problem. What seems certain is that he wanted to criticize the Cournot equilibrium, to show that it is not a solution, and to *indicate that only the coalitional point could be an equilibrium.* In Bertrand's text, the term solution qualifies this point and no other.

In conclusion, Bertrand thought that in a duopoly the owners are led to associate or collude. He then criticized Cournot for believing that another attitude was likely and tried to show that this would not lead to any solution. In his demonstration, two obvious misunderstandings of Cournot appear: the belief that given Cournot's assumptions, price differentiation could happen, and a misreading of the meaning of conjectures.

That one of these misunderstandings is today considered the substance of Bertrand's criticism is ironic, notwithstanding the interest in oligopoly models incorporating conjectures in prices, or "Bertrand models." It is, of course, quite true that reading Bertrand can suggest to a normally intelligent mind that price differentiation should be considered in oligopoly problems, or that Cournot's conjectures are only one kind of conjecture, but Bertrand certainly did not intend to say that.

Although it is not my main task to discover if Bertrand had predecessors, it may be worthwhile to note that Ménard (1978) and Ekelund and Hébert (1990) see Fauveau as a forerunner of Bertrand's criticism. Fauveau's texts, *Considérations mathématiques sur la théorie de l'impôt* (1864), and *Considérations mathématiques sur la théorie de la valeur* (1867), both contained a criticism of Cournot's model of oligopoly; the former quotes the *Researches*, while the latter quotes the *Principes*, but the same objection was raised, and this objection was incorporated in Bertrand's opinion.[7] Fauveau attributed to Cournot the belief that (using Bertrand's notations) D' could remain constant after a change in D. Fauveau did not understand what Cournot's conjectures were.[8]

This mistake was actually repeated by Bertrand, as I have shown, and after him by Pareto and Moore (see below). There is no reason to believe that it has not been independently "discovered" by these four authors; in fact, Bertrand did not quote Fauveau; and it is likely that if he had read him, it was without sympathy. Bertrand was opposed to mathematical economics, while Fauveau tried to promote it. When Fauveau criticized Cournot, it was on purely technical grounds, while Bertrand's criticism was a broader attack against the import of the mathematical method in social sciences (see Lutfalla 1938). So it appears unlikely, but not impossible, that Bertrand was inspired by reading Fauveau.

4. The Emergence of the Bertrand Price-Conjectures Model Legend

Bertrand's review has probably not been widely read, but three important economists were soon to comment on it: Pareto, Edgeworth and Fisher; the first very critically, the second in a laudatory but superficial way; and Fisher seems to be, involuntarily, the source of the legend.

In his *Cours d'Economie Politique* (1896), Pareto, who later (1911) gave an erroneous version of Cournot's model of duopoly, notes that

7. It seems excessive to write, as Ekelund and Hébert (1990) do, that Fauveau *fully* anticipated Bertrand; for instance, there is nothing in his texts about collusion.

8. "L'auteur admet que chaque producteur peut fixer son débit sans que cela change les débits des autres producteurs: ce qui n'est pas vrai. Il ne peut, en effet, modifier son débit sans changer son prix de vente, ce qui fera varier les débits de ses concurrents" (Fauveau 1864 56–57).

"Cournot made a mistake . . . that Bertrand pointed out."[9] For Pareto, this mistake seems to be the belief that the spring owners would not sell the whole of their product. But Pareto specifies to the contrary, "However, it should not be believed that the fall [in price] would have no limit."[10] His judgement on Bertrand is rather negative: "This author himself made a mistake, proving he paid only very superficial attention to the reading of the book he criticizes"[11] (the error in question actually concerns the book by Walras and not that by Cournot!).

Later in *Economie Mathématique* (1911), Pareto was more severe. "J. Bertrand criticizes the theories of A. A. Cournot and L. Walras; but he himself falls into such errors which cannot possibly be explained coming as they do from such an eminent geometrician. The only possible explanation would be that he wrote his article without looking at the books he criticized, and that his memory served him incorrectly."[12] This comment seems quite appropriate, and it would have sent Bertrand into oblivion, if Pareto had been more widely read, or if Edgeworth had not had quite a different attitude towards Bertrand.

To understand Edgeworth's position, remember that in *Mathematical Psychics*, two years before the appearance of Bertrand's text, he had already commented on Cournot. Edgeworth's point of view is that equilibrium indeterminacy is inversely related to the number of traders. He recognizes that "this gradual 'extinction' of the influence of monopoly is well traced by Cournot in a discussion masterly, but limited by a particular condition, which may be called *uniformity of price, not (it is submitted) abstractedly necessary in the cases of imperfect competition*" (1881, 47). To Cournot's reasoning, he then opposes the statement that in imperfect competition "an indefinite number of *final settlements* are possible; that in such a case *different* final settlements would be reached if the system should run down from different *initial positions* or contracts" (47).

9. "Cournot a fait à ce sujet une erreur, qui a été relevée par Mr. Bertrand" (1896, 67).

10. "Il ne faut pas pourtant croire que la baisse n'aura pas de limite" (68).

11. "Cet auteur fait à son tour une erreur, qui prouve qu'il n'a donné qu'une attention fort superficielle à la lecture du livre qu'il critique" (67n).

12. "J. Bertrand a critiqué les théories de A. A. Cournot et de L. Walras; mais il tombe lui-même en de telles erreurs qu'il est absolument impossible de les expliquer de la part d'un géomètre aussi éminent. La seule explication possible serait qu'il a rédigé son article sans avoir sous les yeux les livres des auteurs qu'il critiquait, et que sa mémoire l'a mal servi" (334n).

This approach is, of course, very different from Bertrand's, but strangely enough Edgeworth will see the latter as a confirmation of his own views. In his famous text on "The Pure Theory of Monopoly," first published in Italian in 1897, Edgeworth gave a new formulation of his criticism of the Cournot duopoly determinacy. In this new formulation, the main feature is not the multiplicity of equilibria like that in the criticism in *Mathematical Psychics*, but the cyclical (or quasi-cyclical) character of duopoly equilibrium. In Edgeworth's paper both Marshall and Bertrand appear as authorities providing separate proofs of his theory.

> [The case of duopoly] is treated by Cournot as the first step in the transition from monopoly to perfect competition. He concludes that a determinate position of equilibrium will be reached. Cournot's conclusion has been shown to be erroneous by Bertrand for the case in which there is no cost of production; by Professor Marshall for the case in which the cost follows the law of increasing returns; and by the present writer for the case in which the cost follows the law of diminishing returns.
>
> In the last case there will be an indeterminate tract through which the index of value will oscillate, or rather will vibrate irregularly for an indefinite length of time. (1925, 117–18)[13]

In his own demonstration (1897), Edgeworth uses a model where the joint output of the two springs is not enough to sell the water at its marginal cost (zero); demand at a zero price would exceed supply. Each owner has his own selling price but the quantitative limitation has the result that no one will be driven out of the market if prices are not equal. This, according to Edgeworth, leads to perpetual movement.

13. This is not the first reference that Edgeworth made to Bertrand's argument. In two articles, appearing in *Nature* (1889) and in a paper in French in the *Revue d'Economie Politique* (1891), he had already referred to Bertrand's criticism; in the later paper, Edgeworth's solution is still indetermination and not yet oscillation. It is also of interest to note that in 1889 Edgeworth was not as negative towards Cournot as he was to be later. While stating emphatically, "I should have hesitated to assert that Cournot has made some serious mistakes in mathematics applied to political economy, but that the authority of the eminent mathematician Bertrand may be cited in support of that assertion," he added, "I hope to show on some future occasion that M. Bertrand's censures of Cournot and Prof. Walras are far too severe" (501). He seems to have changed his mind before he could do so.

The model developed in "The Pure Theory" (1897) can hardly be considered as a continuation or a confirmation of Bertrand's ideas. The assumptions are different; price-competition is introduced, not inadvertently as in Bertrand, but explicitly[14]; price-conjectures are implicitly introduced; and cyclical behavior happens because the supply limitation introduces a discontinuity in the reaction functions. Edgeworth's result could not be reached if he began with the assumption attributed to Bertrand, that there are no production costs.

On the other hand, Marshall's so-called demonstration is of yet again a different nature. He argued that "if the field of sale of each of the rivals were unlimited, and the commodities which they produced obeyed the law of Increasing Return then the position of equilibrium attained when each produced on the same scale would be unstable. For if any one of the rivals got an advantage, and increased his scale of production, he would thereby gain a further advantage, and soon drive his rivals out of the field. Cournot's argument does not introduce the limitations necessary to prevent this result" (1890, 485, n. 2; the note disappears in the fourth edition).

Now Edgeworth, as the champion of the idea of indeterminacy/instability, had good reason to refer to the authority of any predecessor; Marshall was an obvious choice, and Bertrand was famous as a mathematician. This probably explains why Edgeworth promoted him. Marshall himself does not seem to have had any direct knowledge of Bertrand's text; in the eighth edition of the *Principles* (1920; and perhaps in an earlier version after the fourth), he quotes Bertrand in a footnote (409) concerning an article by Henry L. Moore, "Paradoxes of Competition" (1906) (see below).

In 1898, when the English translation of Cournot's *Researches* first appeared, Fisher wrote a short text entitled "Cournot and Mathematical Economics," in which he gave his own view of duopoly. If he con-

14. A proof that Edgeworth was perfectly aware of the difference between Cournot's assumption and his own on this point is the note on page 122, "It may excite surprise that when Cournot treats of two monopolists dealing in two perfectly rival articles, he supposes the steps towards equilibrium to be made by varying one *quantity* while the other remains constant (ch. vii); whereas when he treats of two monopolists dealing in two articles perfectly complementary, he supposes that the steps are made by varying one of the *prices* while the other remains constant. An explanation may be found in the term 'perfectly.' If the articles are perfectly rival (that is, identical) there cannot well be supposed two prices." We can add that if Edgeworth had understood in these terms the criticism of Bertrand, he would not have concealed it.

sidered Cournot's treatment as "brilliant and suggestive" (126), Fisher nonetheless had some criticism to make: "The fault to be found with the reasoning is in his premise that each individual will act on the assumption that his rival's output is constant, and will strive only to so regulate his own output as to secure the largest profits" (126). Later Fisher admitted another possibility. "A more natural hypothesis, and one often tacitly adopted, is that each assumes his rival's *price* will remain fixed, while his own price is adjusted. Under this hypothesis each would undersell the other as long as any profit remained, so that the final result would be identical with the result of unlimited competition" (126). In a footnote, Fisher names Bertrand, Marshall, Pareto, and Edgeworth and seems to attribute to them this more natural hypothesis and its conclusion. However, Fisher actually did not agree with this view of the duopoly problem. "But, as a matter of fact, no business man assumes either that his rival's output or price will remain constant any more than a chess player assumes that his opponent will not interfere with his effort to capture a knight" (126).[15] It is, then, clear that Fisher suggested the symmetry between quantity- and price-conjectures and was the first to do so; and he also suggested for the first time a competitive outcome in the case of price conjectures. His formulation suggests that Bertrand could have been a precursor. But Fisher did not develop the idea and considered Edgeworth's treatment of duopoly to be better than Bertrand's.

Until the end of the thirties, Fisher's and Edgeworth's ideas were commonly accepted. A good example is Moore's article, "Paradoxes of Competition" (1906). This article is indeed quite penetrating; he intended to show how the choice of alternative assumptions could lead to different results, and thus analyzes the Cournot-Bertrand debate. Moore states at the beginning of his paper that the term competition "is a blanket-term covering more or less completely at least the following hypotheses" (213), which can be summarized as

I. profit maximization,
II. price unity,
III. producers will only have a negligible influence on product price,

15. Today this quotation seems ironic with regard to game theory, which developed largely from mechanical models such as Cournot's; game theorists still consider him a precursor.

IV. "the output of any one producer is negligible as compared with the total output" (214), and

V. "each producer orders the amount of his output without regard to the effect of his act upon the conduct of his competitors" (214).

Then Moore restates the conclusions of Cournot and Bertrand:

> Cournot's answers are: (1) The price will be lower than the monopoly price and higher than the price under perfect competition; (2) The amount of water supplied will be greater than the amount supplied under perfect monopoly; (3) Stable equilibrium will obtain. (217)
>
> Bertrand's own solution of the problem is: (1) There will be no limit to the fall in price; (2) The amount of water supplied will reach the amount of satiety, provided the resources of the springs are adequate; (3) Equilibrium is impossible. (217–18)

Note here that Moore understands the phrase in Bertrand "no solution is possible" as meaning a price equal to zero, and that seems contradictory with "equilibrium is impossible." Anyway, Moore further examines why the two French writers come to such different conclusions and concludes that their assumptions are different. "Cournot's hypotheses are I., II., V. Bertrand's are I., II., and the negatives of III., and IV" (219).

Actually, Cournot's hypotheses are I., II., and V., and the negatives of III. and IV., while for Bertrand we might perhaps say that they are I., and the negatives of all the others—if we insist on believing that he went so far in his reflections. It seems likely that Moore, to give weight to his thesis that different hypotheses lead to different conclusions, needed to create this artificial symmetry between Cournot's and Bertrand's views. In addition, Moore made the same mistake concerning conjectures as Bertrand did, and as Pareto did later in his *Economie Mathématique*. Moore attributes to Cournot the belief that when a duopolist fixes his output, the other's output will remain at its preceding level, while Cournot attributes this belief to the duopolists and considers it a mistake (1838, 83). Moore, in the last pages of his paper, contrasted Cournot with Edgeworth with whom he is in total agreement; better hypotheses (than Cournot's) will lead to the result that no determinate solution can emerge from the duopoly situation.

Summing up these early comments on the Cournot-Bertrand debate, it is quite clear that today's contrast between quantity-models and price-models was not an issue. The contrast then was between models with a determinate solution and models without a determinate solution, a result of the influence of Edgeworth and Fisher. In the twenties, there may have seemed to be a general agreement on this point. When publishing the English version of "The Pure Theory of Monopoly," Edgeworth added the comment that "Cournot had represented the transactions between two parties to be determinate in the same sense as competitive prices. . . . Now the demolition of Cournot's theory is generally accepted" (1925, 111).[16]

Edgeworth's influence in France during this period can be measured by the fact that Gaston Leduc (1927) is satisfied with a mere translation of Edgeworth's statement about Cournot's duopoly, a translation which he seems to have made himself.[17] However, Pareto was also influential, and in the 1938 reprint of Cournot's book, Georges Lutfalla gave an interpretation of the Cournot-Bertrand debate which is essentially the one proposed by Pareto.

In the third edition of *Economics of Welfare* (1929), in the chapter devoted to Monopolistic Competition, Pigou states that

> Cournot decided, as is well known, that the resources devoted to production under duopoly are a determinate quantity, lying between the quantities that would have been so devoted under simple competition and under simple monopoly respectively. Edgeworth, on the other hand, in an elaborate critique, maintained that the quantity is indeterminate. This latter view is now generally accepted by mathematical economists. The quantity of resources which at any moment it will be most profitable to A to employ in his business depends on the quantity which B is employing, and vice versa. The quantity employed by each, therefore, depends on his judgement of

16. Edgeworth was, at least in the last years of his life, very dogmatic on this point. In his 1922 review of "Lezioni di Economia Mathematica" by Luigi Amoroso, he made this surprising suggestion to his Italian colleague: "Altogether our author's teaching about duopoly cannot be regarded as part of accepted science. We should recommend the omission of this topic, if it were proposed to translate the work into English with the view of supplying the much-felt need of an introduction to mathematical economics" (405).

17. "Les conclusions de Cournot ont été en effet démontrées érronées par Bertrand, dans le cas où il n'y a aucun coût de production, et que la quantité susceptible d'être produite est illimitée . . . par Marshall dans le cas où le coût suit la loi de la productivité croissante . . , et par Edgeworth, dans le cas où le coût suit la loi de la productivité décroissante" (258).

> the policy which the other will pursue, and this judgement may be anything, according to the mood of each and his expectation of success from a policy of bluff. As in a game of chess each player will act on some forecast of the other's reply; but the forecast he acts on may, according on his mood and his reading of that opponent's psychology, be one thing or another thing." (267–68)

It was precisely in those years that a new interpretation of Bertrand's contribution was being elaborated. Chamberlin and Hotelling, simultaneously and apparently independently, were working on imperfect competition. Chamberlin, after obtaining his Ph.D. in 1927, published a portion of his thesis in the *Quarterly Journal of Economics* under the title "Duopoly: Value Where Sellers Are Few" (1929); it is (with slight differences) the future chapter 3 of *The Theory of Monopolistic Competition.* Hotelling's "Stability in Competition" appeared the same year in *The Economic Journal.* The second edition of Bacon's translation of Cournot appeared in 1929 as well, and *in this edition there is a reproduction of Fisher's comment.*

Hotelling's text is mainly about product differentiation, but in his introduction, he mentions the Cournot-Bertrand debate, speaking of Bertrand's "caustic review" (1929, 42). After presenting Cournot's model, Hotelling notes "Against this conclusion Bertrand brought an 'objection péremptoire.' The solution does not represent equilibrium, for either proprietor can by a slight reduction in price take away all his opponent's business and nearly double his own profits. The other will respond with a still lower price. Only by the use of the quantities as independent variable instead of the prices is the fallacy concealed" (43). Hotelling's last sentence ("instead of the prices") is quite unfounded in so far as it is supposed to represent Bertrand's view; it was probably inspired by Fisher.

Chamberlin built his exposition of duopoly theory on two main features of the models. First that each competitor may assume that the other will not react to his own actions, or he may anticipate that there will be some reaction and take it into account; and second, in the first scenario "his rival's policy may remain fixed with respect either to the amount he offers or to the price at which he offers it. The solution will be different in the two cases, as we shall see" (66). Then Chamberlin endeavors to present Cournot's theory, starting with the case of no conjectures and quantity policy: "In the first place, let us suppose each seller to determine upon the supply which is most profitable for himself

in the light of his rival's present offering, and assuming it not to change. It was in this way that Cournot conceived of the problem" (66). Chamberlin then comes to the second possibility: "let us suppose each seller to assume his rival's price (instead of his supply) unchanged" (69). After elaborating on the model, Chamberlin notes,

> the downward movement will continue until their entire joint output is disposed of, i.e., until the price is exactly zero in the present instance. This is the first of several possible solutions where prices are adjusted (and where indirect influence is ignored). . . . It is from this point of view that Cournot's theory was first the subject of attack. Thus Bertrand refuted him by arguing that there would be no limit to the fall in price (he assumed, evidently, that there was no limit to the supply), since each producer could always double his output by underbidding the other. (71)

It is not quite clear what theory Chamberlin attributes to Bertrand; certainly the idea that each producer considers his rival's price as given, and here he is wrong. Perhaps Chamberlin also attributes to him the idea that the limit of the fall in price is zero, and, if so, he would be wrong again according to my interpretation. Exactly as in Fisher's note, the reader is not explicitly told here what Bertrand said, but it is strongly suggested that he was the first to criticize Cournot by using the argument of price conjectures rather than quantity conjectures.

It is easy to understand why Chamberlin's (and Hotelling's) views had a strong influence (see below). One reason is that Chamberlin gave an excellent presentation of duopoly and oligopoly, where assumptions are clearly stated and discussed, a presentation that every professional economist could probably have understood. This is in stark contrast to Cournot, who is clear but more difficult to read, and to Edgeworth, who is technically difficult and hardly clear. A second reason for Chamberlin's success is probably that he discussed Edgeworth's model (from "The Pure Theory") at length in the paper and rather convincingly reduced his arguments to nothing. A final reason for Chamberlin's impact is probably that his own theory of monopolistic competition gave an authoritative aura to the rest of the book (1932) of which the text on duopoly would become chapter 3.

Hotelling's paper certainly had a strong influence by itself. In the fourth edition of *The Economics of Welfare* (1932), Pigou modified

the chapter on monopolistic competition after reading Hotelling. The phrases quoted before were modified to take it into account: instead of "The latter view . . . a policy of bluff" (as above), it becomes,

> In more recent discussions there is apparent some measure of return towards Cournot. If, it is held, each of the two monopolists, in regulating his action, assumes that the other will not alter his output in consequence of what he does, the quantity of resources devoted to production by the two together is determinate at the amount calculated by Cournot. If each monopolist assumes that the other will not alter his price in consequence of what he does, then, in a perfect market, the quantity of resources devoted to production by the two together is determinate at the amount proper to simple competition. (266–67)

Pigou did not like this approach and maintained the Edgeworthian view, but at the time he was nearly alone in doing so.

The texts by Chamberlin and Hotelling had a very immediate and strong influence.[18] They were both quoted in Zeuthen's *Problems of Monopoly and Economic Warfare* (1930); chapter 2, entitled "Monopolistic Competition," seems to endorse Edgeworth's view, but actually adopts the Chamberlinian version of an equilibrium at marginal cost in the case of a price competition. "If now, like Edgeworth and a number of other authors, we assume the highest degree of mobility, so that only a slight reduction in price by one competitor will immediately give him all the sales in so far as his capacity allows it, it is quite right that there is no position of equilibrium, at any rate not at a price above costs" (26). Such a statement could lead to the belief that there was a Bertrand-Edgeworth-Chamberlin continuity in the criticism of Cournot, and this is as false as Edgeworth's insistence that Bertrand, Marshall, and he had attacked Cournot on similar grounds. Obviously this belief, and not Pigou's, became the more influential.

In the thirties, another important book adopted Chamberlin's view. Stackelberg's *Marktform und Gleichgewicht* (1934), although it remained untranslated, was largely known and commented upon. In chapter 5, dealing with the history of duopoly, Stackelberg points out

18. An interesting example of resistance to Chamberlin's ideas on oligopoly is Nichol (1934a, 1934b and 1935). His work was not read with much care, probably because of that opposition.

that "the French mathematician Joseph Bertrand criticized Cournot's tentative solution for simple duopoly, leading immediately to a second tentative solution. Bertrand's starting point can be easily characterized with our terminology. While Cournot started from the hypotheses, that both duopolists follow a quantity policy, Bertrand stated on the contrary that for them only a price policy was in question" (69).[19] Quoting and commenting on Bertrand, Stackelberg is rather critical towards him, and rightly considers that "nevertheless the Bertrandesque tentative solution could play an important role in economic theory because authorities like Marshall [?] and Edgeworth gave strength to Bertrand's approach."[20] While one can agree with this judgment, it is not clear why Stackelberg adopts this view (that Bertrand proposed that the duopolists' policy be a price policy), unless we note that he too had read and quoted Chamberlin as well as Fisher. Stackelberg says, "In January 1898 Fisher adopted the Edgeworthian vision of Bertrand's argument."[21] This sentence is not quite clear but indicates that concerning Bertrand, Stackelberg took his lead from Chamberlin rather than from Fisher.

From Chamberlin and Hotelling to Stackelberg, the symmetry between a (Cournotian) quantity policy and a (Bertrandesque) price policy was not yet thoroughly formulated, but gradually emerged as a bare fact. To my knowledge, the first to state it clearly was Stigler, in the "Notes on the Theory of Duopoly" (1940).

> Cournot and most of his followers have accepted this last condition, i.e., even if B's output changes fifty times in the course of movement to equilibrium, A will continue to treat it as independent of his own output each of the fifty times he (A) changes his output. Similarly, Bertrand, in offering an alternative theory of duopoly,

19. "Der französische Mathematiker Bertrand unterzog den Lösungsversuch Cournots für das einfache Dyopol einer Kritik, die zugleich zu einem zweiten Lösungsversuch führte. Die Ausgangsposition Bertrands lässt sich mit Hilfe unserer Terminologie leicht charakterisieren. Während Cournot von der Annahme ausging, dass die beiden Dyopolisten Mengenpolitik treiben, behauptete demgegenüber Bertrand, dass für sie nur Preispolitik in Frage komme" (69).

20. "Trotzdem hat der Bertrandsche Lösungsversuch eine grosse Rolle in der Wirtschaftstheorie gespielt, wohl deshalb, weil Autoritäten wie Marshall und Edgeworth dem Bertrandschen Gedankengang zugestimmt haben" (71).

21. "Im Januar 1898 schliesst sich Irving Fisher der Edgeworthschen Fassung der Bertrandschen Argumentation an" (75).

> postulated that A would assume that B's price was independent of A's price—even if B's price were cut a hundred times in retaliation. (527)

The quotations given in my introduction are reproductions of this statement, so we can date the modern theory, or rather legend, of the Cournot-Bertrand debate to 1940. After this time most writings on the topic of oligopoly seem to agree with Stigler.[22] Searching for dissenting views, I note that Schumpeter was not an admirer of Bertrand (see note 3). He explained Bertrand's reputation in the following way. "The outstanding instance [of hostility to mathematical economics displayed by eminent mathematicians] is J. Bertrand's attack upon this nascent branch of the mathematical sciences in the *Journal des Savants*, September 1883. It was eagerly seized upon, as an authoritative condemnation, by people who understood neither mathematics nor economic theory, and hence received more attention than it deserved" (1954, 958 n. 9).[23]

Summary and Conclusion

I hope to have shown in these pages that Bertrand's criticism of Cournot's theory of duopoly has been overrated, and cannot be the real foundation of the classification of duopoly models in quantity models and price models. Bertrand's arguments were unclear and mainly resulted from his misunderstanding Cournot. They were used by Edgeworth to reinforce his own version of duopoly, incorporating price indetermination and instability. They were later used by Chamberlin and Hotelling to promote the study of differentiated oligopoly. In the thirties and even later it was felt that there was a serious need for clarification in the field of imperfect competition, and the new vision of Bertrand's criticism had the great merit of simplicity. The growing use of game theory is another reason why the supposed symmetry between Cournot's and Bertrand's approaches may have seemed valuable and remained unquestioned.

22. Paul Chamley, writing in 1944, had a view that we could qualify as "mixed." He considered that Bertrand had misunderstood Cournot's hypotheses, but also that Bertrand attributed to duopolists a "price policy." He had certainly read Bertrand, but was influenced by Chamberlin and Zeuthen (see 27).

23. One may wonder whom Schumpeter was thinking of, as Edgeworth seems to be the author of Bertrand's reputation, and he certainly does not fit the description given.

Appendix
Review by Joseph Bertrand of Two Books (translated by Margaret Chevaillier)

Théorie Mathématique de la Richesse Sociale. By Léon Walras, Professeur d'Economie politique à l'Académie de Lausanne. Lausanne, 1883.

Recherches sur les Principes Mathématiques de la Théorie des Richesses. By Augustin Cournot. Paris, 1838.[1]

The titles of these two books seem to herald great new avenues for the science of Adam Smith. Yet there has been a singular lack of interest in these two authors. Cournot was an eminent scholar, a learned writer who became a master in the art of deduction. Mr. Walras, who proudly claimed to be a follower of Cournot, once said of him, "He is the first scholar to have attempted to apply mathematics to political economy, in a work published in 1838, which has never been reviewed in France." The learned professor at the University of Lausanne then added "It is my wish to draw attention to the author of this major work, which, I repeat, has received little, if any, criticism and which, I dare say, has not met with the recognition it deserves."

The criticism, leveled publicly at Cournot's compatriots, prompted me to read a truly forgotten work whose small number of readers have not always rated it highly despite the author's commanding reputation. In his preface, Cournot wrote, "The title of my work foreshadows not only theoretical research but also indicates that I intend to apply the forms and symbols used in mathematics." The forms and symbols used in mathematics demand precision, assume rigor, and leave no room for indulgence. Mathematicians take the formulae used to be true or false, the definitions vague or precise, and reasoning rigorous or absurd. So does Cournot. Of the several essays which precede his own, Cournot only mentions one: *Les Principes d'Economie Politique* by Canard, a short work published in year X [1801] which won its author an award from the Institut. Cournot added, "these so-called principles are so totally wrong and their application is so erroneous that even the recognition afforded by this eminent body could not prevent this work from falling into oblivion. It is easy to understand why economists such as Say and Ricardo remained hostile to the use of algebra."

Although he was a professor of mathematics, citizen Canard was either oblivious to, or had forgotten how to use, elementary calculus. Knowing that the price of a commodity increases with the number of buyers, with their needs and disposable income, and that the price falls with the number and eagerness of the sellers,

1. First printed in French in *Journal des Savants* (1883, 499) and in *Bulletin des Sciences Mathématiques et Astronomiques* (1883, 293).

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he immediately expressed it algebraically as follows: $B + Ax$ is the standard increasing function for the variable x, and $B' - A'x$, the standard decreasing function—this was the starting point and foundation of his whole theory.

How did be become a prize winner at the Institute? Which commission nominated him for a prize? I have not been inquisitive enough to find out.[2]

The problems which Cournot addresses cannot be solved by reasoning alone, yet this learned author never entertains the possibility of using factual investigation. Not that he discounts the importance of such methods, but merely that he believes that his role in the intellectual division of labor lies elsewhere. Cournot examines laws, leaving others to examine figures. His formulae, written only in letters, bristle with unknown functions; he would consider it outside his field if he were to be more specific. Practical economists must feel that it would be of little value to study such formulae, be they true or false, so they escape from this study by merely closing the book. If Cournot's theory of wealth, despite the author's intellectual stature, his influence, and the quality of his other works, has failed to attract any serious attention over the past half century, it is because the ideas are lost under the profusion of algebraic signs. Removing the symbols would reduce the book to just a few pages which would nearly all contain judicious comments and assertions worthy of interest. Does Cournot wish to study the laws relating to the struggle which determines the current market price for each commodity, a very difficult problem which Canard solved so incorrectly? Cournot points out that for a given commodity, the selling price necessarily varies with sales; by calling it p, the demand function becomes $\phi(p)$; as $\phi(p)$ is a function whose derivative is negative, the producer's total revenue will be the product $p \cdot \phi(p)$; it is this product, if the commodity costs nothing, which has to be made a maximum. Without knowing or inquiring any further, the derivative can consequently be made to equal zero by using the rules of calculus. Hence,

$$p\phi'(p) + \phi(p) = 0$$

is the equation the seller has to solve. He also has to check that the second derivative is negative and check the inequality

$$p\phi''(p) + 2\phi'(p) < 0.$$

Such is the mathematical theory of monopoly for a commodity which costs nothing and on which no taxes are imposed. Those who wish to apply this theory merely have to identify the function $\phi(p)$. The learned author points out that if the seller cannot satisfy all the buyers he will, by raising the price, have to reduce demand so as to equal, but not exceed, the possible level of production. Given this

2. The second class of the Institute (Moral and Political Sciences) had suggested the following questions. Is it true that in an agricultural country, any type of tax is borne by landowners? If this is so, then do the same landowners bear indirect taxes in addition? Canard won the prize. Like the man with forty ecus, he pronounced that this was not true but only by turning the required solution into "one link in a chain of consequences" which Cournot correctly pointed out was wrong.

situation, what would happen if a tax were to be levied on the commodity? More often than not, the price would rise; it may, in some cases remain unchanged, but the tax would never make the price fall. All these assertions by Cournot are correct, but was it really necessary to use algebra to prove this? Let us examine closer the case of a spring which cannot satisfy the level of demand that would maximize gain. When a tax is imposed on each liter sold, the producer might find that once he has paid the tax, it is in his interest to reduce his level of production, thereby pushing up the price, which until then had maximized his gain. Indeed the rise in price affords him the same gain on each bottle sold as prior to the imposition of the tax. However, the loss is not the same, since on the unsold bottles the producer gains what he used to pay to the Treasury. Nevertheless, the fall in sales may offset the gain made both from charging a higher price and paying less tax: in that case, the proprietor of the spring bears the whole tax without either altering the price or the level of production.

Cournot adds, "From this it seems that the only condition limiting the Treasury in setting a tax is that it should not take up the proprietor's entire net income. Yet this result would be inexact and can be proved so in at least one case."

Cournot defines this case algebraically where the function $\phi'(D)$ increases with the variable D, and where $p' - p^\circ > i$, p° and p' are, respectively, the roots of the equations

$$F(p) + [p - \phi'(D)] \cdot F'(p) = 0 \text{ and} \qquad (1)$$
$$F(p) + [p - \phi'(D) - i] \cdot F'(p) = 0.$$

The reader is familiar with these letters and functions, which figured in the previous pages, yet even a mathematician may require a less scholarly explanation. Without offering any such explanation, Cournot continues, "In fact, if Δ is the necessary limit of production and Π the value of p derived from the relation $F(p) = \Delta$, it would be necessary for the hypothesis that $\Pi > p'$ and *a fortiori* $> p^\circ + i$, i being equal to $\Pi - \phi(\Delta)/\Delta$. We should therefore have

$$\Pi > p^\circ + \Pi - \phi(\Delta)/\Delta \text{ or } p^\circ < \phi(\Delta)/\Delta.$$

But the last inequality cannot hold true if $\phi'(p)$[3] is (according to the hypothesis) a function which increases with D; for then p° being smaller than Π, the demand D° corresponding to p° is greater than Δ, and $\phi(D^\circ)/D^\circ$ greater than $\phi(\Delta)/\Delta$; p° would, therefore, be less than $\phi(D^\circ)/D^\circ$. This value of p° would, therefore, cause the producer to make a loss and consequently could not be the root of equation (1)."[4]

This requires a translation.

The following question is raised. Taking the producer's situation as given by the terms of the problem, can the state, through a tax levied on every liter, appropriate

3. There is a misprint in Bertrand's text. This should read $\phi'(D)$ instead of $\phi'(p)$ (translator's note).

4. This quotation from Cournot is not in inverted commas in Bertrand's original text (translator's note).

the total net gain without the latter decreasing and without therefore pushing the price up. According to one of the assumptions expressed algebraically, production costs for each liter increase with production. If a tax on total production takes up the whole profit on the final units produced, which cost more than the others, then the producer would make a loss. Logically the producer would stop producing these units, and since the commodity would become scarce, the price would rise, which contradicts the assumption. It is, therefore, contradictory to conclude that total gain can be taken up by a fixed tax without production falling. Moreover even if costs are not increasing, before he hands over his total benefit to the Treasury, the proprietor will try to defend his position, even if that involves raising prices. It is hard to explain why Cournot who claims that his theory can be proved *in at least one case,* seems to have doubts about what might happen in other cases. Mr. Walras wrote of the chapter from which these examples and formulae were taken: "In chapter V of his "Recherches sur les Principes Mathématiques de la Théorie des Richesses" Mr. Cournot expresses the theory of monopoly mathematically, which is the clearest and most precise form. Unfortunately economists deemed it unnecessary to become acquainted with this theory, and as far as monopoly is concerned, they have been left in a state of mental confusion which in their case takes the form of a marvelous verbal confusion."

The condemnation is severe. However, the calculations which we have partially quoted are not clear to everybody; the results seem to be of minor importance; occasionally, I must admit, they appear unacceptable.

Such is the study made in chapter VII of the rivalry between two proprietors, who without having to worry about any competition, manage two springs of identical quality. It would be in their mutual interest to associate or, at least, to set a common price so as to make the largest possible revenue from all the buyers, but this solution is rejected. Cournot assumes that one of the proprietors will reduce his price to attract buyers to him, and that the other will in turn reduce his price even more to attract buyers back to him. They will only stop undercutting each other in this way, when either proprietor, even if the other abandoned the struggle, has nothing more to gain from reducing his price. One major objection to this is that there is no solution under this assumption, in that there is no limit to the downward movement. Indeed, whatever the common price adopted, if one of the owners, alone, reduces his price, he will, ignoring any minor exceptions, attract all the buyers, and thus double his revenue if his rival lets him do so. If Cournot's formulation conceals this obvious result, it is because he most inadvertently introduces as D and D′ the two proprietors' respective outputs, and by considering them as independent variables, he assumes that should either proprietor change his output then the other proprietor's output could remain constant. It quite obviously could not.

On other occasions, Cournot introduces abstractions into the terms of his problems which are expressed so formally as to avoid any responsibility falling on him as a mathematician. Is one not always free to formulate a problem as one wants? In this way, Cournot by translating the very complex problem of the freedom of

trade into formulae and having proved mathematically that the exporting nation increases its revenue while the importing nation decreases its revenue, then adds "Abstraction is made, after this real fall in revenue has been deducted, of the advantage accruing to those consumers who are induced to purchase by the lower price, of the fact that they can then spend their revenue more to their liking."

Let us assume, for example, that the price of linen falls by half in the nation that is said to be worse off; those who used to wear cotton garments in winter will be able to replace them by linen garments, and by thus spending their revenue *more to their liking,* they will be able to reduce their mortality. Cournot admits that this is an advantage; but since he cannot assess it in his formulae, he simple states that he will take no account of it. Are we entitled to criticize him for that?

Geometrical representations often replace formulae in Mr. Walras's book, his reasoning is more accessible, and his findings closer to something applied. He has achieved greater success in a shorter time. "If you were to consider the state of the question in France and England alone," Walras once wrote to the eminent scholar Stanley Jevons with whom he was at one on many points, "all we could share would be a reputation for being utopian dreamers. But the situation is different elsewhere, especially in Italy, where the whole spirit and scope of this new method has been grasped amazingly intelligently and quickly."

I will not review here the large number of important and complex questions that Mr. Walras has treated, nor give my verdict on the conclusions that have divided the most distinguished authorities, but merely discuss one principle which is said to be fundamental.

Let us imagine a market where, on the one hand, people appear with stocks of a commodity (A) which they are prepared to trade in order to obtain a commodity (B), and, on the other hand, people appear with stocks of a commodity (B) which they wish to convert into commodity (A). A price will be arrived at; m(A) will be exchanged for n(B). What factors determine this price? To solve this problem, Mr. Walras, whom I summarize, postulates that each individual bearer of one of the commodities leaves nothing to last minute impressions but comes to the market having decided exactly what to do in every possible case. For brevity, let us now replace commodity (B) by money and assume that commodity (A) is wheat in a market where growers who wish to obtain the highest price meet buyers who wish to pay the lowest price. Under this assumption, each buyer will give instructions to an agent, telling him for instance, if the price is twenty francs, buy one hundred hectoliters for me; if it is twenty-five francs, only buy sixty hectoliters; at thirty francs, I only want ten hectoliters; at thirty-five, I do not wish to buy anything. The table when complete would give, for each price, the corresponding figure for purchases. The sellers also give their instructions, and the amount each individual is prepared to sell at each price is known.

There is a very simple solution; the learned professor postulates that if we bring together all the buyers' order books for each successive price, we calculate the sum of the quantities demanded, then by bringing together all the sellers' order books, a similar table could be drawn up. The resulting tables could be replaced by curves

where the abscissas are the selling prices. The abscissa of the point at which the two curves intersect is the price which Mr. Walras calls equilibrium price. It is this price which tends to prevail.

Such is Mr. Walras's theorem; here is a demonstration of it. Let us assume that the two curves intersect at a point whose abcissa is twenty-five. If when the market opens, the price announced is twenty-five francs per hectoliter, demand at that price equals supply and the transactions will be carried out smoothly; each seller finds a buyer and vice versa. However, no further sale will be possible since when the price goes above twenty-five francs, there will be no more buyers, and when it falls below twenty-five francs there will be no more sellers. If the price had been fixed initially at over twenty-five francs, it would become apparent, after a few transactions that supply exceeds demand; consequently, the price would have been forced down. A price below twenty-five francs would, on the contrary, cause the price to rise, and in both cases, the price approaches the equilibrium level.

I believe I have, without detracting from its clarity, summarized the Lausanne professor's reasoning.

I will now raise an objection. By replacing the group of buyers with a single buyer who, at each price, wishes to acquire as many hectoliters as all the actual buyers together, the terms of the problem have been changed. One cannot replace all the sellers with a single seller either. As a demonstration, let us postulate that two buyers want one-hundred hectoliters each; the former is willing to pay twenty francs but will buy nothing if the price rises, the latter wants to buy, whatever the price. Let us assume, moreover, that the first time the price reaches twenty francs, the agent responsible for all the sales orders had sold a hundred hectoliters. The market will react differently depending on which of the buyers has carried out his transaction and withdrawn from the market. The remaining presence tends to push prices down in one case and up in the other case.

It should be noted that, without their intentions having changed, the curves representing the buyers' orders at the different price levels must necessarily vary for each of them while the market lasts. The problem is solved by the intersection of the resulting curves, which shift constantly, and we can easily demonstrate the variation that is necessary in the abscissa of their point of intersection. Let us assume, for example, that one of the buyers has given his agent the following instructions: buy one-hundred hectoliters when the price is twenty francs, buy sixty hectoliters when it is twenty-five francs, and only fifty hectoliters when it reaches thirty francs. The initial price is twenty francs; out of the one-hundred hectoliters he wants to buy, the agent can only purchase fifty; the price then rises to thirty francs where it remains at. What should the agent do? Buy fifty hectoliters at a price of thirty francs? Certainly not, since fifty hectoliters at twenty francs and fifty hectoliters at thirty francs represent one-hundred hectoliters at a price of twenty-five francs, but, at that price, the buyer only wants sixty hectoliters. The agent will have to base his decision on the condition that the average price between his latest purchase and the fifty hectoliters already purchased, corresponds, on the buyer's order book, to the total amount purchased on his behalf. He is confronted

with the same problem every time the price changes, and after each transaction, the curve representing the buyer's orders has to be recalculated and replotted. Should the new curve be used to obtain the equilibrium price? If the answer is yes, then Mr. Walras's theorem is no longer geometrically valid since the final result depends on accidental circumstances, which he postulated had been removed. Yet how can the answer be no? How can one admit that a newcomer to the market, who has become aware of the current state of affairs, should not be entitled to make use of these principles? In order to anticipate prices, he might just as well inquire into the orders given at the market during the previous month.

A final argument will dispel any remaining doubts. Let us assume that, on the basis of the known intentions of the buyers and sellers, the equilibrium price one hour before the market opens, calculated with the help of the above theorem, is twenty-five francs per hectoliter. A new buyer arrives; when the price is below twenty-five francs he wants to purchase everything available, but at twenty-five francs or, *a fortiori*, above twenty-five francs, he does not want to buy anything. His presence, if one believes Mr. Walras, will have no influence whatsoever. Indeed, his presence drives up infinitely the demand curve for the points whose abscissa is below twenty-five francs, but does not change anything for the others. The intersection, on which the result depends, will remain the same and will still correspond to the abscissa twenty five. Is such a conclusion acceptable? Twenty-five francs will be neither the only nor the first price to prevail, assuming that it does; for prices will tend to fluctuate around twenty-five francs; every time the price falls below that level the new buyer will come forward, and the sellers having sold to him all or part of their merchandise, will no longer offer at twenty-five francs what they had offered at the beginning. I assume that one of them had brought one-hundred hectoliters to the market; when the price was twenty-five francs he wanted to sell them all; at twenty-four francs he wanted to sell just eighty hectoliters; the price happens to be twenty-four francs, so the buyer we mentioned earlier purchased his eighty hectoliters: only twenty hectoliters remain to be sold, therefore when the abscissa reaches twenty-five, the ordinate of the seller's curve has suffered a fall equal to or greater than eighty, and the buyer's curve remained the same. The point of intersection of the two curves has shifted, and since one of them has infinite ordinates when the abscissa falls below twenty-five, the curves will intersect on the other side. Moreover, according to the same rule which we question, the intervention of the new buyer should push the final price up.

My intention is not to review Mr. Walras's book. I would find much in it to praise but equally as much to criticize. To finish off, I merely wish to point out a definition by which the learned scholar alters the well known, established meaning of a word. Such initiative can be allowed on the condition that new meaning is clearly defined. I do not think that this condition has been met, and yet the word *scarcity* as Mr. Walras interprets it, plays a very important role in his reasoning.

The ingenious author, whom I take the liberty of summarizing, postulates that the proprietor of a quantity (a), of a given commodity, derives a certain utility from owning the commodity, i.e., a certain satisfaction of his needs and his wants,

which increases with each successive unit acquired so that when the quantity owned raises from x to $x + dx$, the advantage for him is represented by $\phi(x)dx$. Owning a can be represented by the integral $\int_0^a \phi(x)dx$. There is no necessary relation between the price determined by market conditions and the function ϕ which varies from one individual to another. If we call the price of each unit bought or sold p, then it is clear that by paying pdx, the increment dx, which for him represents a satisfaction measured by $\phi(x)dx$, the buyer whom we are describing will make a good deal if $\phi(x)$ is more than p, and a bad deal if $\phi(x)$ is less than p. He will have to buy or sell part of the merchandise he owns depending on which ever of those conditions is satisfied, and stop buying or selling when $\phi(x) = p$. If $x = \alpha$ is the root of this equation, then α is what Mr. Walras calls the scarcity of a commodity for the person being studied.

The most serious defect of this definition, even if one neglects the inconvenience of using a widely used, well-known word, is that it becomes meaningless when it is applied to traders who should, on the contrary, be studied in this type of problem. A wheat merchant buys millions of hectoliters and knows what they have cost him. He sells at the prevailing price, when he can make a profit; sometimes when he expects prices to fall, he sells at a loss to avoid an even greater loss; and when he expects prices to rise, he holds onto his stock. He never determines his behavior in terms of the advantages he may derive from the different parts of his stock.

The two theories I have just summarized both play an important role in Mr. Walras's considerable work. Dropping those theories would cast doubt on more than one argument, but many others would remain intact. I refrain from reviewing them.

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