

MARSHALL AND EDGEWORTH

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I

INTRODUCTION

The year 1890 is usually regarded as an important one in British economics for two main reasons. The first and most obvious reason is that it marks the publication of Marshall's long-awaited *Principles of Economics*; but it also saw the formation of the British Economic Association, which in 1902 became the Royal Economic Society. A primary role of the Association was its publication of the *Economic Journal*, edited by Edgeworth, who took a very active part in its affairs from the first issue of the journal until his death in 1926. But 1890 was a notable year for Edgeworth for quite a different reason: it marks his first professorial appointment, as Tooke Professor of Political Economy at King's College, London. This position was achieved after a rather varied teaching 'career' and unsuccessful (and perhaps somewhat premature) applications for chairs in philosophy at King's College and University College, Liverpool, in 1880 and 1881 respectively. Edgeworth's first chair in economics proved to be only a stepping stone to the much more influential Professorship of Political Economy at All Souls' College, Oxford, which he held from 1891 until his retirement in 1922. On being appointed to the Oxford chair, Edgeworth received a congratulatory letter from Marshall which began 'Hurrah! Hurrah!! Hurrah!!!'¹

With Marshall, at Cambridge, as the acknowledged leader of British economics, Edgeworth therefore assumed the 'second position' at Oxford. This situation was to continue for over thirty years until both died within two years in the mid-1920s. Yet these two contemporary economists presented an interesting contrast, even in 1890. Marshall's position in economics was of course for ever established with the publication of his *Principles*, but his high, even international, reputation was until then based on relatively few substantial publications (see Whitaker, 1975, I, pp. xviii–xxi) and his evidence to the Commission on the Depression of Trade and Industry (1886) and the Gold and Silver Commission (1887); see

¹ This letter is held in the Special Collections section of the London School of Economics (LSE) Library, along with the other letters mentioned in this paper.

Marshall (1926). His career had been spent in Cambridge except for the period of 'exile' from 1877 to 1885, spent at the newly established University College, Bristol, and Balliol College, Oxford (1883–5). Edgeworth's activities since graduation were much more varied; indeed, he only turned seriously to economics in 1879. Yet in a relatively brief period he had published three monographs, acted as Secretary to the British Association Report on Index Numbers (which produced three volumes) and published a long string of articles on mathematical statistics. As a result he was able, in *Metretike* (Edgeworth, 1887), to list his own papers alphabetically, using every letter in the alphabet!

Subsequent generations of historians of economic thought have given Edgeworth very little attention. The secondary literature on Edgeworth is minute compared with that on Marshall, and most texts do not even devote a separate chapter to Edgeworth's contributions to the subject. Much of this difference can be explained by the approaches and objectives of the two writers, as will be seen below, but care must be taken in making such comparisons. The small professional group to whom Edgeworth directed his many articles, and which was able to appreciate the technicalities involved, has grown very significantly since the Second World War. Many theorists now acknowledge his importance, even though probably very few have ever read a line of *Mathematical Psychics* (Edgeworth, 1881). By contrast, many of Marshall's insights have been lost or deliberately discarded by those too impatient to search for the analytical depths beneath the smooth surface of Marshall's literary style. The perspective continues to alter while receding beyond the relatively short period of a hundred years.

Personal relationship

Keynes (1972, p. 255) has suggested that Marshall's review of Edgeworth's *Mathematical Psychics* in the *Academy* in 1881 led to the friendship between the two economists. But contact between Marshall and Edgeworth seems to have begun some time in late 1879 or early 1880, when Edgeworth sent a copy of his *New and Old Methods of Ethics*. Marshall replied: 'I have now read nearly all the book you sent us, and I am extremely delighted by many things in it. There seems to be a very close agreement between us as to the promise of mathematics in the sciences that relate to man's actions' (letter of 8 February 1880). In the same letter Marshall expressed a desire for more 'dynamical' analysis. The passage quoted above is rather unusual in that it is the only example of the explicit encouragement by Marshall of Edgeworth's mathematical economics. He later became critical of Edgeworth's concentration on theory; indeed, only a few weeks later Marshall stressed in another letter that he intended 'never to use analysis when I can use geometry' (letter of 28 March 1880). However, Edgeworth himself fully recognized Marshall's ambivalence concerning the use of

mathematics, and characterized him as 'bearing under the garb of literature the armour of mathematics' (in Pigou, 1925, p. 66).

Edgeworth's movement towards economics is in fact closely connected with his association with Jevons, which also began in 1879. Jevons was responsible for bringing Marshall's *Economics of Industry* and the *Pure Theory of Foreign Trade and Domestic Values* to Edgeworth's attention. Edgeworth later wrote: 'Eagerly studying these writings I discerned a new power of mathematical reasoning not only in the Papers bristling with curves and symbols, but also in certain portions of the seemingly simple textbook' (in Pigou, 1925, p. 66).

Even though Edgeworth was only three years younger, he continued to look up to Marshall as the more 'senior' economist. Despite their different natures, there seems little doubt that the word 'friendship' is most appropriate to describe their relationship, even though Edgeworth reserved his most sympathetic remarks for Jevons and Sidgwick. Guillebaud, for example, reports that Marshall had 'a strong personal liking' for Edgeworth (1969, p. 7), though Marshall characterized Edgeworth's approach as follows: "'Let us assume" says Edgeworth "that we have two elephants suspended from the end of a rope. Give the elephants a push and then, disregarding the weight of the elephants, work out what happens to the rope"' (quoted by Guillebaud, 1969, p. 7). Mary Paley Marshall recalls simply that 'Professor Edgeworth was also a frequent visitor and kept us in touch with Oxford economics' (1947, p. 45).

It is relevant that Edgeworth is one of the few people to refer to Marshall's humour. While others, quoted for example by Guillebaud (1969), mention a total lack of humour, Edgeworth wrote that many (discarded) letters 'would not only have been of scientific interest, but also would have preserved touches of his peculiar humour' (in Pigou, 1925, p. 69). MacGregor (1942) also discusses Marshall's humour, though it does seem (from this distance) as if it was rather awkward, if not actually embarrassing, such as a propensity to laugh alone at his own jokes during lectures. Marshall ended a letter to Edgeworth, of 7 February 1893, with the rather laboured 'yours obligatedestly & (Mount) Everestly'. By contrast, Edgeworth allowed a rather sharp and dry sense of humour to colour his writings, giving light relief along the brisk ascent to the rarefied atmosphere in which he thrived.

While Edgeworth was an admirer of Marshall, he was no sycophant. In return for the occasional mild criticism Edgeworth received very strong rebukes from Marshall. Two examples are discussed below. Perhaps these episodes explain Marshall's comment, quoted by Keynes (1972, p. 265), that 'Francis is a charming fellow, but you must be careful with Ysidro'. Keynes also remarks that 'there can seldom have been a couple whose conversational methods were less suited to one another than Francis Edgeworth and Alfred Marshall' (1972, p. 255).

It may be worth mentioning at this point that the close relationship

between Marshall and Edgeworth can perhaps be said to have had its negative side, concerning the international transmission of ideas. Edgeworth regularly solicited Marshall's advice about the running of the *Economic Journal*, but received the full force of Marshall's wrath after he allowed Cunningham's attack on the *Principles* to be published, without consultation. Edgeworth then refused to publish Cunningham's rejoinder to Marshall's reply (see Coats, 1967, p. 712, n. 10). Despite Edgeworth's wide international sympathies and knowledge of foreign literature, the *Economic Journal* published very little by major overseas writers. Robbins (1970, p. 196) suggested that the refusal to publish a piece by Barone, much to Walras's annoyance, was due to a fear on Edgeworth's part of offending Marshall. This 'protection' of Marshall was also noted by Schumpeter (1954, pp. 831, n. 4; 839, n. 4). It is relevant that the only translation that Edgeworth encouraged was of Pierson's *Principles* (1902); Mary Paley Marshall recalled that 'we were very fond of Professor Pierson and his wife, who stayed with us several times' (1947, p. 45).

II

APPROACHES TO ECONOMICS

Some comparisons

The two major figures of British economics present some interesting contrasts, although it is important to keep a sense of proportion in making comparisons. Marshall, after moving to economics, concentrated exclusively on the subject and indeed it may be said that he devoted his life to it. Guillebaud mentioned that 'he disapproved profoundly if he thought he saw any indication of my having wider interests in life than the only one by which he himself was activated' (1969, p. 5). But Edgeworth, who certainly devoted himself wholeheartedly to academic work, devoted at least as much energy to mathematical statistics as to economics.

Marshall was motivated, to use Viner's terms (1958, p. 252), by a greater element of 'feeling' or 'warmth', and he often said that the primary problem facing economists was that of poverty. By contrast, Edgeworth's interests were largely intellectual. Keynes (1972) compared the two by saying that Marshall was concerned with developing *maxims*, whereas Edgeworth was concerned with developing *theroems*. Marshall's analogies were mainly biological, whereas Edgeworth's were mainly mechanical and often reflected his interest in physics.

A contrast is sometimes detected in their attitudes to theory. Marshall often appears to be negative about theory, as in his comment to Edgeworth's that 'economic theory is, in my opinion, as mischievous an imposter when it claims to be economics *proper* as is mere crude unanalysed

history. Six of ye one 1/2 dozen of ye other' (in Pigou, 1925, p. 437). But it must be recognized that Edgeworth, the archetypal theorist, was at least as cautious as Marshall in its use. Indeed, Edgeworth mainly saw theory as having the negative value of indicating what cannot be achieved: 'Reason is here no guide, but still a guard.'

Their attitudes towards mathematics are also of interest. Marshall, the highly trained mathematician, began his economic studies by 'translating' J. S. Mill into mathematics and working on Cournot's *Recherches*, but he later 'had what almost amounted to an obsession for hiding his tools away' (Pigou, in Pigou and Robertson, 1931, p. 3). But Edgeworth, the self-taught though more creative mathematician, 'gloried in his tools'. It has been suggested by Keynes (1972) and Viner (1958) that Marshall's attitude towards mathematics and diagrams reflects a puritanical reaction against the things that gave him most pleasure.

The desire to conceal his working methods is also associated with Marshall's desire to reach a wide audience. While actively seeking to establish economics as a profession he attempted to write for the widest possible readership. Edgeworth, by contrast, wrote exclusively for a small group within the nascent profession, believing that economics was much too difficult for the majority and expressing some contempt for the audience which Marshall attempted to reach (see 1925, III, p. 64). Edgeworth unfortunately never followed the wise advice of Jevons and Marshall, reviewers of *Mathematical Psychics*, to improve the clarity of his writing, although his style became more restrained in later years. But while Marshall's polished style did not allow a single spark of humour to light his pages, Edgeworth's patient reader is regularly rewarded with examples of his sharp wit. It seems rather churlish to criticize these authors from such a distance, however. Marshall's gifts were ideally suited to the polished treatise, while Edgeworth's were suited to the more spontaneous article, and all subsequent generations of economists have cause to be grateful for their different contributions.

Their approaches to teaching may also be contrasted. Whereas Marshall devoted much energy to his pupils and to the establishment of the Cambridge economics tripos (see Groenewegen, 1988), Edgeworth took no interest in the development of the subject at Oxford. Rather, his 'administrative' energies were devoted to the Royal Economic Society, the Royal Statistical Society and the British Association (of which he was elected a Fellow in 1903).

Philosophical background

Whatever their differences, Marshall and Edgeworth both 'arrived' at economics from mathematics and moral philosophy. They both shared a background in the active philosophical debates of the last quarter of the nineteenth century, a background that makes the training of modern

graduates seem very narrow. Whitaker's comment on Marshall, that 'the broadness of Marshall's concerns illustrates by contrast the narrowness of ours' (1977, p. 197) applies equally well to Edgeworth. Both were influenced by Spencer's application of evolutionary doctrines to moral philosophy. They were both affected in their approach to economic analysis by their philosophical, especially Utilitarian, background. But only Edgeworth could be accurately described as a thoroughgoing Utilitarian. Marshall was much more sympathetic to the Idealists, whereas appreciation of Edgeworth's Utilitarianism is fundamental to an understanding of his work, and he was strongly critical of anything approaching Idealism.² Edgeworth's own comment on Marshall is of interest; he suggested that 'Marshall observed, what according to Tacitus is most difficult, moderation in philosophy. He had not Bentham's rabid antipathy to other people's formula. He had a good word for T. H. Green' (in Pigou, 1925, p. 71).

It can be argued, however, that Schumpeter exaggerated the position when he said that 'it was one of the many merits of Marshall's treatment of utility that he deplored and renounced the alliance with utilitarianism' (1954, p. 1056). But in reading Schumpeter on this question it must be remembered that he had a strong abhorrence of Utilitarianism, and even went so far as to claim that Utilitarianism was not basic to Edgeworth's economics. (This is considerably stronger than the simple result, known to Edgeworth, that demand functions are invariant with respect to monotonic transformations of the utility function.) It is perhaps more accurate to say, with O'Brien (1988, n. 117), that his later writings involved a 'renunciation of a thoroughgoing utilitarianism'.

As part of this 'renunciation', Guillebaud (1961, II, p. 20) reports that by the third edition of the *Principles* 'Marshall was becoming sensitive to contemporary criticisms of utilitarianism' and where he previously had 'pleasure and pain', he deleted 'pain' and in most cases substituted an alternative word (such as satisfaction or benefit) for 'pleasure'. But when discussing Marshall's deletions, Guillebaud stressed that he often suppressed sensitive parts simply because he felt he could not afford the space to develop the argument adequately. Indeed, Marshall himself commented on his deletion of material on the 'balancing of motives', by saying in a letter to Edgeworth that 'I found it was habitually misunderstood, especially by Ethicists: they *would* take such phrases as utilitarian manifestos. So I set myself to cut out short sentences on a big subject' (in Pigou, 1925, p. 437).³

² Edgeworth's Utilitarianism is discussed at length in Creedy (1986).

³ On the question of interpersonal comparisons, Marshall was quite explicit: 'It is useless to say that various gains and losses are incommensurable, and cannot be weighed against one another. For they must be, and in fact they are, weighed against one another before any deliberate decision is or can be reached on any issue' (in Pigou, 1925, p. 302).

III

WAGE BARGAINING

Unions and wage bargaining

The stimulus provided by Marshall's *Pure Theory of Foreign Trade and Domestic Values* and the book (with Mary Paley Marshall) on *Economics of Industry* has been mentioned above. The attraction to Edgeworth of the analytical apparatus is readily apparent, but a very important factor is the relevance to wage bargaining and the possibility of indeterminacy in bargaining between a trade union and a firm. This context increased the relevance of Edgeworth's analysis of barter with small numbers; the problem of indeterminacy had also been raised directly by the criticisms of Jevons's *Theory of Political Economy* in the *Saturday Review* (Black, 1981 pp. 152–7).

Marshall had earlier hinted at the application of international trade barter analysis to wage bargaining, in his 1876 paper on J. S. Mill's theory of value (see Pigou, 1925, pp. 132–3). It is also of interest that Marshall delayed the formal publication of his trade diagrams for fifty years, and felt unhappy about the implicit assumption that the demand for imports in general has similar characteristics to that for a single commodity (see Pigou, 1925, pp. 449–50). Nevertheless, he always thought that the diagrams could be applied to wage bargaining. For example, he made the point quite clearly in a letter to Edgeworth of 1891 (see Whitaker, 1975, II, p. 112).

The relevance of the 'seemingly simple textbook' to wage bargaining can also be traced without difficulty. When discussing Thornton's comments on the difference between Dutch and English auctions, Edgeworth (1881, p. 48, n. 1) referred not to the original source but to Marshall and Marshall (1879, p. 200, n. 1). This reference by the Marshalls appears, significantly, in their chapter on the influence of trade unions on wages. On the same page, they suggest that, 'If then the labourers enter into local trade combinations, and refuse to sell their labour except at a reserve price, it is quite possible that they may increase their share of the Wages-and-Profits fund, and raise wages at the expense of profits.'⁴

It is therefore perhaps no coincidence that Edgeworth placed considerable stress on the indeterminacy that arises in wage bargaining, and saw clearly that the growth of unions would increase the need for peaceful

⁴ Edgeworth's copy of the *Economics of Industry* is now held by the University of Kansas Library, as part of the magnificent collection assembled by R. S. Howey. Reference to this was made by Howey (1960, p. 239, n. 29). The word 'indeterminate' is written in the margin on p. 210, where the effects of unions on wages is discussed; but it cannot be claimed with certainty that it is Edgeworth's handwriting, and the book was in the circulating section of the Kansas Library for nearly thirty years before being placed in the Special Collections' closed stack. I am very grateful to James D. Neeley for sending photocopies of relevant pages and further information. Edgeworth also referred to Marshall and Marshall (1879) in the *Psychics* (1881, pp. 136–7).

arbitration. Or as Edgeworth stressed, rather less prosaically, 'The whole creation groans and yearns, desiderating a principle of arbitration, an end of strifes' (1881, p. 51). It is in this area also that Edgeworth took his utilitarianism further than Marshall. Whereas the latter was prepared to arbitrate on the basis of what the corresponding competitive wage would have been, Edgeworth argued that this could be improved upon by choosing the point that maximizes the total utility of the contracting parties.

Edgeworth's argument that indeterminacy was likely to be widespread also took him away from Marshall's position, and was the basic source of the controversy discussed in the next section. But it is of interest that Edgeworth argued that his analysis of 'contract' provided 'a sort of unification likely to be distasteful to those excellent persons who are always dividing the One into the Many, but do not appear very ready to subsume the Many under the One' (1881, p. 146). Here Edgeworth is seen to be using the Platonic motto for *Industry and Trade* thirty-eight years before Marshall, although the latter used it in a letter to Edgeworth of 1909 (see Pigou, 1925, p. 442).

Despite Edgeworth's argument that the same basic analysis of 'contract' can be applied to many contexts, he did not actually develop the special feature of wage bargaining at any length.⁵ It was implicitly assumed that the axes of the 'Edgeworth box' would measure the wage bill and the level of employment, but the special characteristics of the two sets of indifference curves were not explored. What Edgeworth did make clear, however, was the result that although settlements would generally be expected to be on the contract curve, if bargaining is over wages *only*, then settlements will be along the demand curve for labour (or what is essentially the employer's offer curve of wages for employment). Thus:

the property of *indeterminateness, plurality of final settlements*, will abide. Only the final settlements will now be by way of demand-curve, not contract-curve. If, for instance, powerful trade unions did not seek to fix the *quid pro quo*, the *amounts* of labour exchanged ... but only the *rate of exchange*, it being left to each capitalist to purchase as much labour as he might demand at that rate, there would still be that sort of *indeterminateness favourable to unionists* above described. (1881, p. 48; see also pp. 137–8)

Later developments

The subject was later examined in a little-known book by Pigou (1905), who may be said to have taken a 'Marshallian' position that bargaining will only concern the rate of wages and consequently 'settlement between the parties

⁵ It is however very surprising that, with respect to Edgeworth's stress on the indeterminacy of wages in union/employer bargaining, Hicks (1932, p. 26, n. 2) wrote, 'Edgeworth did not himself imagine that his proposition was very important in practice.'

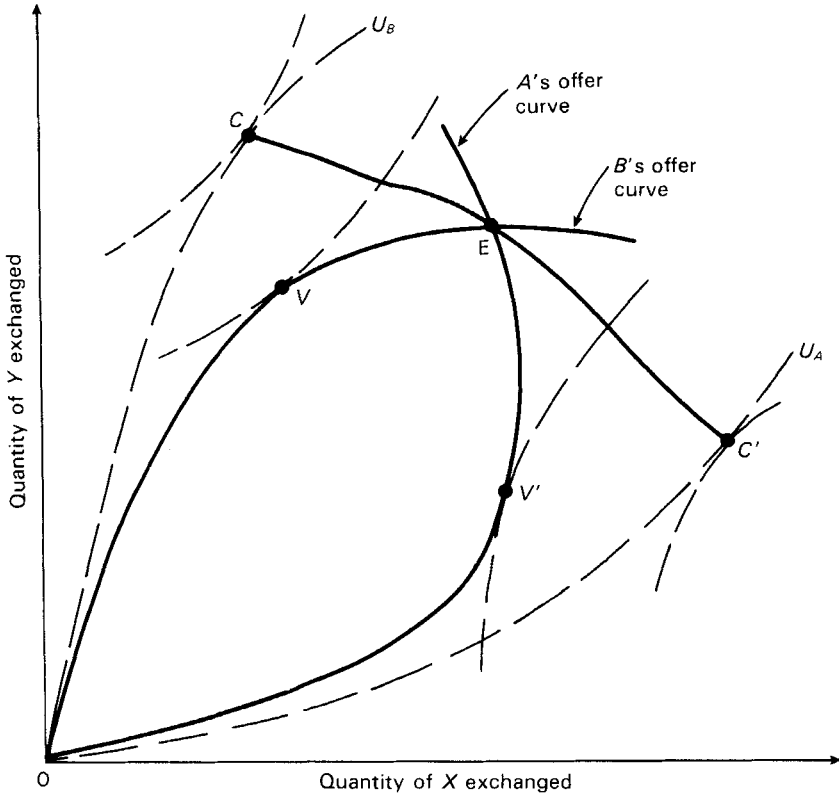


Figure 1

is determined by way of demand and not by way of contract curve' (1905, p. 210). The only relevant position on the contract curve is the point of intersection of the two offer curves; Pigou implicitly ruled out multiple equilibria.

The typical 'bargaining' framework in the context of exchange of two goods between individuals or groups was later described by Bowley (1924, p. 8), who achieved something that hardly seems possible; his book was even more terse than *Mathematical Psychics*! Consider Figure 1, which shows the position of 'A-type' traders holding stocks of good X, dealing with 'B-type' traders who hold good Y. The competitive equilibrium is the point E at the intersection of the offer curves. If the A-type individuals form themselves into a monopoly and are therefore able to set the price, the best they can reach is point V on the offer curve of the Bs. Similarly V' represents the best position that a monopoly of the Bs could reach. The points V and V' correspond, of course, to points reached by the imposition of 'optimum' tariffs in the international trade context. Bowley stated that 'the double curve [VEV'] is called the bargaining locus' (1924, p. 8), but

argued that E will be reached with 'equal bargaining strength'. Bowley did not, however, develop the analysis further and in particular did not examine the special circumstances of bargains between a union and employers.⁶

The view taken by Edgeworth was, of course, that bargaining over only wages is inefficient, so that the contract curve is really the bargaining locus, with employment and wages subject to negotiation. Except for a brief comparison of Marshall and Edgeworth on wage bargaining by Hicks (1930), which did not really take the analysis further, the most important subsequent development seems to be that of Dunlop (1944, although some of this book had been published earlier in articles). Dunlop devoted much attention to the nature of the union's utility function and made explicit what the previous writers mentioned above had left implicit, that the vertical axis (in a diagram such as Figure 1) represent the total wage bill, while the length of the other axis measures employment. A ray drawn from the origin therefore represents a 'price line' whose slope equals the wage. (A represents the union and B is a firm or group of firms). The 'indifference curves' of the firm must represent iso-profit lines, which therefore depend on the role of labour in production and the nature of the market in which the good is sold.

The next contribution is rather curious. Dunlop (1944, p. viii) mentioned that several chapters had been read by Leontief. Two years later Leontief (1946, reprinted 1966) published a brief paper which in virtually all its essential arguments did not go beyond the position reached by Edgeworth in 1881. Furthermore, Leontief did not refer to a single book or article. The main thrust of his argument was that efficient bargains require both wages and employment to be subject to negotiation, so that employment guarantees (or 'take it or leave it' contracts) are preferred. Nevertheless, more recent writers on union bargaining have emphasized Leontief's 'original' contribution, to the neglect of earlier literature; see, for example, Hall and Lilien (1979, p. 868) and Oswald (1985, p. 169).

The most original suggestion made by Leontief (1966, p. 111) was that if employment exceeds the size of the union, efficient contracts are in fact on the demand curve for labour. Several years later Fellner (1949) extended the analysis by producing diagrams which have the wage rate, rather than the wage bill, on the vertical axis. He compared situations in which unions are prepared to trade employment for wage gains, where indifference curves are downward sloping in the usual way, with those where unions are not concerned with the level of employment and therefore have horizontal indifference curves. In the latter case, Fellner (1949, p. 276) pointed out that the demand curve for labour is the contract curve and that 'neither the firm nor the union would gain from an employment guarantee'. This ties in with Leontief's point, made at the beginning of this paragraph, since indif-

⁶This situation corresponds more closely to that of bilateral monopoly (a monopolist in a goods market buying inputs from a monopolistic supplier), which Bowley examined in more detail in response to Wicksell's criticisms.

ference curves will anyway become horizontal after all the union's members are employed, even if they are downward-sloping for lower employment levels.

These contributions did not form the basis of a continuous literature, and in fact the next thirty years saw few economic analyses along the lines originated by Edgeworth and Marshall. The next important step, marking the beginning of a large resurgence of interest in these issues, was actually made in ignorance of the details of the earlier literature. Precisely one hundred years after the appearance of *Mathematical Psychics*, an important analysis was published by McDonald and Solow (1981). Considering the union's indifference curves and a firm's iso-profit curves in detail, they produced the contract curve of efficient bargains between limits specified by zero profits and the opportunity cost of working. McDonald and Solow examined the Nash bargaining equilibrium, producing a wage equal to the average of the marginal and average product of labour, rather than the 'utilitarian arrangement' proposed by Edgeworth, or the equivalent of the competitive equilibrium proposed by Marshall. In considering the long delay involved, it would perhaps be unfair to attribute the slowness in extending the theory solely to Marshall's desire to conceal his methods and Edgeworth's terseness in communicating his results.

IV

DEMAND THEORY

Multiple equilibria

Edgeworth's argument in *Mathematical Psychics* was that with few transactors in the market, there would be an infinite number of final settlements along the contract curve. He then argued that with a sufficient number of competitors, the range of indeterminacy would shrink and there would be '*settlement of the whole field at a single point in the contract-curve*' (1881, p. 41). It is interesting that in his statements of the result of introducing more competitors, Edgeworth did not discuss explicitly the possibility that there may be more than one point in the contract curve. Yet he was perfectly well aware of Marshall's and Walras's demonstrations of multiple equilibria in the context of markets with price-taking behaviour. Indeed, he mentioned their 'doubtless independent' results no less than seven times in *Mathematical Psychics* (1881, pp. 5, 26, 38, 46, 105, 125, 147). It can therefore be assumed that Edgeworth, eager to move from the 'economical' to his 'utilitarian' calculus, simply did not bother to linger over the further details.

The correspondence between Marshall's treatment, using what are now called offer curves, and Edgeworth's contract curve analysis, is shown in Figure 2 for the case where there are three equilibria. It is assumed that

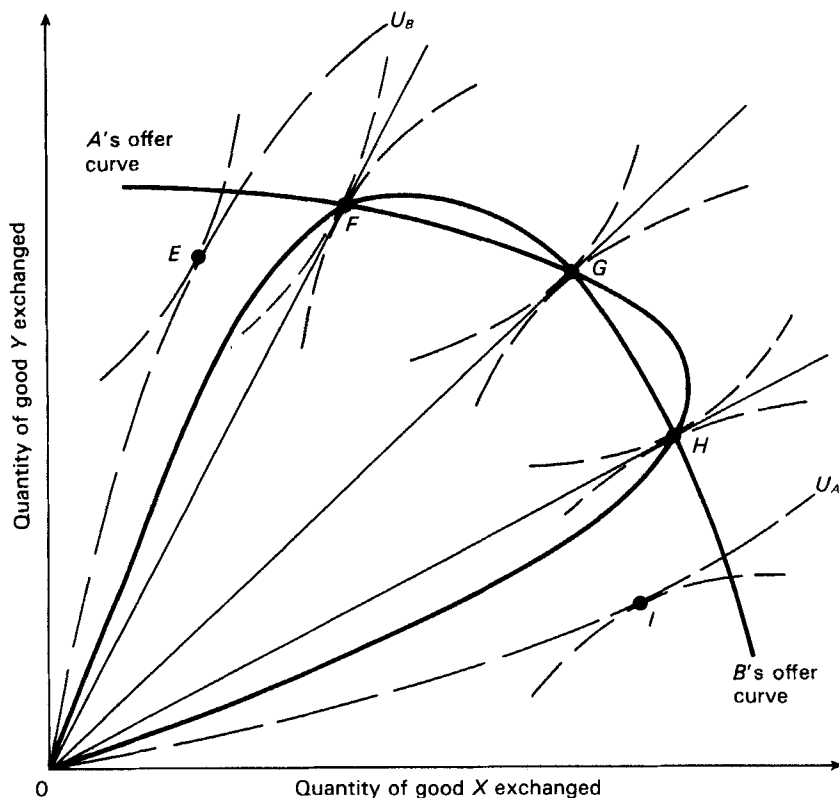


Figure 2

there are many individuals of 'type A' and 'type B', with equal numbers of each. The initial endowment position is assumed to be the origin, O , and the indifference curves that can be reached before trade by individuals of type A and B are shown as U_A and U_B respectively. Using the indifference curve approach of Edgeworth, the competitive equilibria occur at points F , G , and H . At each point the common tangent of the indifference curves coincides with a price line going through the origin, whose slope is equal to the relative price of good X to that of good Y. The contract curve, the locus of points of tangency of indifference curves, runs through the points E , F , G , H and I , although it is not shown in the Figure. Following Marshall's use of offer curves, shown as the heavy lines in Figure 2, the three points of intersection are also F , G and H .

The relationship between the two approaches was explored by Edgeworth, who showed that the offer curve can be derived as the locus of points of tangency between price lines and indifference curves. Edgeworth's own exchange diagram (1881, p.28) showing a single competitive equilibrium, was very difficult for readers to follow as the only indifference

curves shown fully were the pre-trade curves running through the initial endowment point. He also drew the offer curves, but did not define them until ten pages later, and it was only in an appendix where a less terse explanation appeared.

Suppose that, starting at the equilibrium point G , there is a decrease in the price of good X relative to that of good Y , so that the price line, going through the origin, has a lower slope than OG . This leads to an increase in the offer of good X by the A s, because of their inelastic demand for good Y . The price change can be seen to lead to an excess supply of good X , along with a corresponding excess demand for good Y . Hence the relative price of good X would be expected to fall further, and the 'middle' point G is unstable.

Indeterminacy in exchange

It has been seen that Edgeworth placed great stress in *Mathematical Psychics* on what he believed to be the widespread existence of indeterminacy in exchange, leading to the need for a principle of arbitration. He was then able to show that the utilitarian principle of maximizing the total utility of the parties would place them on the contract curve and would be an acceptable principle to individuals who regard all allocations as equally probable; this was his 'neo-contractarian' justification of utilitarianism. Indeterminacy, as Edgeworth argued, means that the contract curve, rather than demand or offer curves, is the appropriate analytical device for the analysis of exchange.

In his review, Marshall (1881, reproduced in Whitaker, 1975, I, p. 267) took a rather different position, arguing that 'the greater part of economic theory can be dealt with most easily' by the use of supply and demand curves. This was of course also the position taken by Marshall throughout his *Principles*. Not surprisingly, Edgeworth subsequently criticized Marshall for not dealing sufficiently with the problem of indeterminacy. On first seeing the review by Edgeworth (in Italian), Marshall did not fully recognize the criticism. He wrote to Edgeworth saying that 'it would never do for me to substitute your argument for mine—since it is so put as to be of little use for my purpose' (1961, II, p. 793), adding that he intended to refer to the contract curve in his Appendix F (on barter). He also mentioned the relevance of his foreign trade diagrams to wage determination, though ignoring the indeterminacy that was so important to Edgeworth.

It seems that the full force of Edgeworth's criticism did not become clear to Marshall until he asked Arthur Berry, a mathematician with a good knowledge of Italian, to look at Edgeworth's paper. After discussing the article with Berry, Marshall sent a long and deeply felt reply to Edgeworth. A sample of this letter is as follows:

I now throw myself on your kind and generous forbearance, and ask you

to listen without anger ... What I want to say is that I do not think you at all appreciate the deadly and enduring injury that *A* does to *B*, if he reads rapidly a piece of hard argument on which *B* has spent an immense amount of work; and then believing that argument to be wrong, writes an article full of the most polite phrases, in which a caricature of that argument is held up to the most refined, but deadly scorn. I fancy you think that the polite phrases diminish the mischief ... Their effect ... is that of a white flag under which the ship approaches close to another and rams or torpedoes it ... There! I feel so much better. I am like a person who has held his mouth full of air under water for a minute. (1961, II. pp. 796–8)⁷

The essential analytical point behind Marshall's indignation was that he believed he had avoided the problem of indeterminacy by assuming that the utility function is additive and that the marginal utility of one good is constant. Marshall's assumptions can be examined in Edgeworth's framework as follows. Individuals *A* and *B* have initial endowments, of goods *X* and *Y*, equal to *a* and *b* respectively. Person *A* exchanges an amount *x* of *X* for *y* of *Y* from person *B*. Their utilities after exchange takes place, U_A and U_B , are given by:

$$U_A = U_A(a - x, y) \quad (1)$$

$$U_B = U_B(x, b - y). \quad (2)$$

The equation of the contract curve, the locus of points of tangency between sets of indifference curves, is given by:

$$\frac{\partial U_A / \partial x}{\partial U_A / \partial y} = \frac{\partial U_B / \partial x}{\partial U_B / \partial y}. \quad (3)$$

For additive utility function and constant marginal utility of good *Y*, (1) and (2) may be written:

$$U_A = U_1(a - x) + \alpha y$$

and

$$U_B = U_2(x) + \beta(b - y). \quad (4)$$

Hence differentiation and substitution into (3) gives

$$\frac{\partial U_1 / \partial x}{\partial U_2 / \partial x} = -\frac{\alpha}{\beta}. \quad (5)$$

Equation (5) shows that $-\alpha/\beta$ is a function of *x* only, so it implies that the

⁷ For a further example of Marshall's ability to write rather strong letters, see his treatment of Sidgwick in Sidgwick and Sidgwick (1906, pp. 394–5), which was perhaps even more harsh than his letter to Edgeworth. This is despite the fact that Marshall at one time described Sidgwick as his 'spiritual father and mother'.

contract curve is a straight line parallel to the y axis. For example, suppose that $U_1(x) = (a - x)^\gamma$ and $U_2(x) = x^\sigma$. Substituting the appropriate partial derivatives into (5) gives:

$$x^{\sigma-1}(a-x)^{1-\gamma} = \gamma\beta/\alpha\sigma \quad (6)$$

which can be solved for x . In the simple case where $\sigma = \gamma$, and writing the right-hand side of (6) as k , it can be seen that x can be solved explicitly as:

$$x = a\{1 + k^{1/(1-\sigma)}\}^{-1}. \quad (6a)$$

It is true that the value of x is determinate in this special case, but there still remains a range of values of y . Edgeworth himself made this point in his original review, reprinted in (1925, II, p. 37, n. 1). Marshall had supposed that Y represented money, so that while the quantity demanded of good X is determinate, the price is not.

It is an indication of the nature of the relationship between the two economists that Edgeworth did not press his point, even though he was strictly correct in his argument that Marshall's special case did not remove indeterminacy. While conceding to Marshall he nevertheless refused to accept any of the points made by Berry. Against the lesser adversary, Edgeworth defended himself 'with all modesty, as one who has already burnt his fingers and fears the fire of controversy' (quoted by Guillebaud, 1961, p. 798). When introducing the translation of his Italian review for his *Papers* (1925, II, p. 313) Edgeworth suggested that 'the term "determinate" is used by Marshall in a somewhat different sense from that which I have adopted. Apropos, it may be remarked that there is a certain indeterminateness about the use of the term "determinate" by economists.' He was therefore able to have the last word, after Marshall had died.

Bowley was clearly familiar with this debate when writing the *Groundwork* (1924), though he did not allude to any of it. He went through the algebra very quickly and showed that 'the equation of the contract curve... only involves x and represents therefore a line (or conceivably lines) parallel to OY ' (1924, p. 13). Bowley's point in parentheses is a useful qualification, since the equation may not necessarily have a unique root. Rather surprisingly, when Samuelson (1942, p. 91) briefly mentioned the debate between Edgeworth and Marshall on this point, he incorrectly stated that indeterminacy regarding the quantity of the good, x , rather than its price, would remain.

It is of interest to compare this general context of exchange with the wage and employment bargaining framework discussed earlier. If the members of the union are assumed to be risk neutral, such that the marginal utility of the wage rate is constant, then it can be shown that the contract curve (in a diagram with the wage rate on the vertical axis and employment on the horizontal axis) is also vertical. The implication here is that the level of employment is determinate, but that the wage rate is indeterminate.

Giffen goods

A later, though milder, disagreement took place over the case of so-called Giffen goods. Edgeworth was not concerned with the theoretical *possibility* of an upward-sloping demand curve for an individual, familiar to every student from the indifference curve analysis of income and substitution effects. Rather, in the course of a review of a book by Rea, he argued that an upward-sloping demand curve was highly *improbable*. Thus: 'Even the milder statement that the elasticity of demand for wheat *may* be positive, though I know it is countenanced by high authority, appears to me so contrary to *a priori* probability as to require very strong evidence' (1909, p. 104). The 'high authority' alluded to was Marshall, who made the suggestion in the *Principles* (1961, p. 132). Marshall wasted no time in responding by letter, and wrote: 'I have just noticed your review of Rae ... I don't want to argue. But the hint that a rather rash and random guess has been made by those who suggest that a (moderate) rise in the price of wheat might increase its consumption in England (not generally) provokes me to say that the matter has not been taken quite at random' (in Pigou, 1925, p. 438). The following day he sent another, much longer, letter to Edgeworth concerning both the supply and demand for wheat. The Giffen good has, of course, provoked an enormous literature that continues to grow, and has taken a life of its own in textbooks, with one text even expressing uncertainty about the very existence of Giffen himself.⁸ This literature may be quite out of proportion to the importance of the subject, but an argument used by Marshall in his second letter seems to have escaped close attention. This argument is examined here not only for its intrinsic interest but for the light it casts on Marshall's desire to justify his earlier argument.

In his second letter to Edgeworth, dated 22 May 1909, Marshall gave an example involving the choice of transport in order to minimize travel time, subject to a fixed budget. The relevant part of his letter is as follows:

I am even more perplexed by what you say about elasticity of demand ... I object to the phrase negative elasticity ... it suggests a paradox. And I submit that there is no paradox at all. Take a parallel case. I believe that people in Holland travel by canal boat instead of railway sometimes on account of its cheapness. Suppose a man was in a hurry to make a journey of 150 kilos. He had two florins for it, and no more. The fare by boat was one cent a kilo, by third class train two cents. So he decided to go 100 kilos by boat and fifty by train: total cost two florins. On arriving at the boat he found the charge had been raised to $1\frac{1}{4}$ cents per kilo.

⁸ In a textbook in microeconomics, Call and Holahan (1980, p. 86) write, 'The supposed Giffen good ... has created more interest among historical economists trying to identify Mr Giffen, the unverified discoverer of the phenomenon, than among research economists in the possibility of hoping ever to observe an upward-sloping demand curve.' I am grateful to Frank Stephen for bringing this gem to my attention.

'Oh: then I will travel $133\frac{1}{3}$ kilos (or as near as may be) by boat, I can't afford more than $16\frac{2}{3}$ kilos by train.' Why not? Where is the paradox? (Pigou, 1925, p. 441)

It is intuitively obvious that if the person were to travel only 100 kilometres by boat at the higher price, he would not be able to pay for 50 kilometres by rail, and so would be unable to complete the required distance. He would therefore have to travel further by the slower method. Using Marshall's figures it is clear that if the price of boat travel were to increase to one and one third cents, the traveller would have no choice but to go the whole distance by boat. Having set up a particular situation in which the distance and the budget are fixed, Marshall's point could easily have been made without using numerical examples. It is therefore instructive to examine the basic model that may have been used by Marshall in producing his illustration. At first sight it may perhaps seem that in general the choice of travel method would be influenced by the relative speeds of the two types, but the following analysis shows why this could safely be ignored by Marshall.

Suppose that the cost of train and boat travel per unit of distance is c_b and c_t respectively, and that their speeds are s_b and s_t . It is required to travel a distance L , using a fixed budget of B . The problem is to determine the distance travelled by boat, L_b , in order to minimize travel time; the remaining distance $L - L_b$ is of course covered by rail. The traveller's problem is thus to minimize travel time, given by:

$$\frac{L_b}{s_b} + \frac{L - L_b}{s_t} \quad (7)$$

subject to the constraint that

$$B = c_b L_b + c_t (L - L_b). \quad (8)$$

Form the Lagrangean, \mathcal{R} where

$$\mathcal{R} = \frac{L_b}{s_b} + \frac{L - L_b}{s_t} + \lambda [B - \{c_b L_b + c_t (L - L_b)\}]. \quad (9)$$

Differentiation of (9) with respect to the Lagrangean multiplier, λ , obviously gives the budget constraint in (8). Differentiation with respect to L_b gives:

$$\frac{\partial \mathcal{R}}{\partial L_b} = \frac{1}{s_b} - \frac{1}{s_t} + \lambda(c_t - c_b) = 0. \quad (10)$$

The first order condition (10) therefore gives λ as:

$$\lambda = \frac{1/s_b - 1/s_t}{c_b - c_t}. \quad (11)$$

Now multiply (11) by L_b to get:

$$L_b \left(\frac{1}{s_b} - \frac{1}{s_t} \right) = \lambda (c_b L_b - c_t L_b). \quad (12)$$

But (8) can be rearranged to get:

$$c_b L_b - c_t L_b = B - c_t L. \quad (13)$$

Substitute (11) and (13) into (12) to solve for L_b as:

$$L_b = \frac{c_t L - B}{c_t - c_b}. \quad (14)$$

This problem is of course only interesting where the cost of covering the whole distance by the fastest and most expensive mode of travel exceeds the budget. Hence both numerator and denominator of (14) are positive. It is easily confirmed that Marshall's numerical results are obtained by substituting directly into equation (14). Notice that Marshall chose his numbers so that the numerator of (14) is equal to unity. Furthermore the speeds s_t and s_b do not appear in (14).

From (14) it is easily seen that:

$$\frac{\partial L_b}{\partial c_b} = \frac{c_t L - B}{(c_t - c_b)^2} > 0 \quad (15)$$

and an increase in c_b increases L_b .

A diagrammatic analysis

It is useful to examine the diagrammatic form of Marshall's model, which indicates a simply alternative method of solution. Consider Figure 3, in which the distances travelled by boat and train are shown respectively on the ordinate and the abscissa. The 'distance constraint' is shown as the straight line LL , which is negatively inclined at an angle of 45° . In order to travel the necessary total distance, all feasible combinations must be along this line. The budget constraint is shown as the line AC in Figure 3, and has a negative slope of c_t/c_b . The assumption that the journey could not be completed by the fastest method is reflected by the fact that the distance OC is less than OL . Similarly OA exceeds OL , so that it is not necessary to go the whole distance by boat.

The feasible region for this problem is therefore the shaded area in the diagram. Consider now the objective function. It is possible, given (7), to define a set of 'equal time' curves, along each of which the total time for the journey remains fixed as the combination of modes varies. Each curve is in fact a straight line, negatively sloped at an angle of s_b/s_t . So long as rail transport is faster than that by canal, each 'equal time' curve is flatter than the distance constraint and, as long as trains are more expensive than boats, the latter constraint is flatter than the budget line. The choice will thus be at point E , along the lowest 'equal time' line TT . The 'south-east' point of the feasible region will always be chosen irrespective of the ratio of s_b to s_t , so long as the latter is less than unity.

The effect of an increase in the cost of boat travel, c_b , can therefore easily

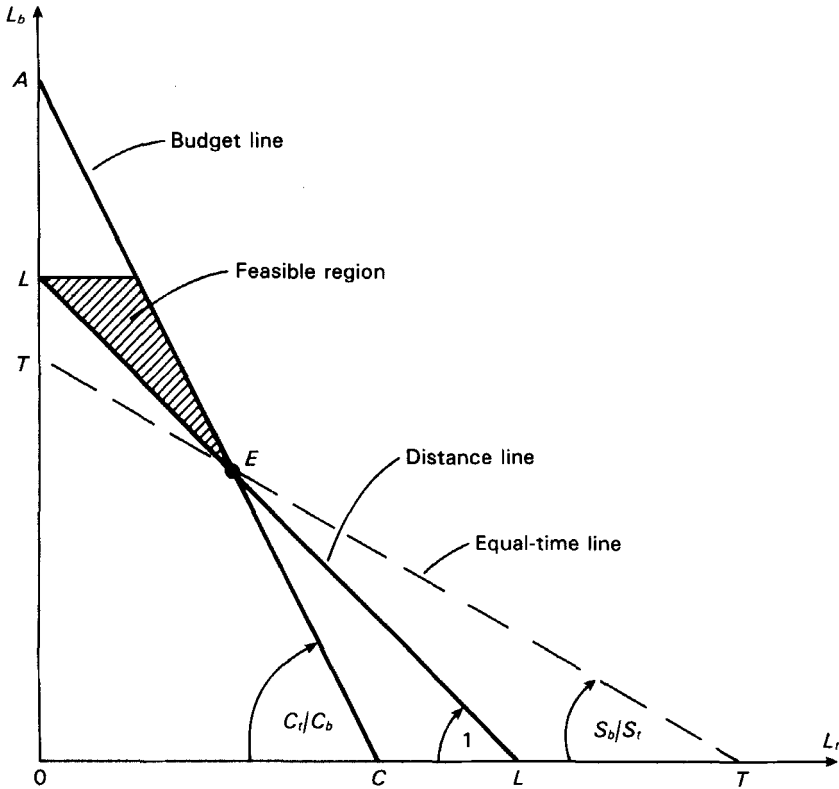


Figure 3

be seen from the figure. The increase in c_b causes the budget line to become flatter, pivoting about the point C . The minimum time feasible choice thus moves up the fixed distance constraint LL ; the demand for boat travel increases and that for rail travel falls.

The diagrammatic approach indicates a simple method of solution. The point E is the point of intersection between the budget line and the distance constraint. These two lines have the following equations:

$$L_b = L - L_t \quad (16)$$

$$L_b = \frac{B}{c_b} - \frac{c_t}{c_b} L_t \quad (17)$$

Use (16) to solve for L_t , substitute this into (17) to get an equation involving L_b , the two costs and the values of the budget and the total distance to be covered. Solving this resulting equation for L_b gives the same result as equation (14). There is of course no way of knowing which approach was used by Marshall.

There is in fact another, even more straightforward, way of approaching this problem. Since the objective is to minimize the total journey time, this

is achieved by maximizing the time spent on the fastest mode of transport. But this can only be maximized subject to the constraint that sufficient money is left to complete the journey on the cheaper, and slower, mode. Hence the individual may be regarded as maximizing L_t subject to the constraint that:

$$B - c_t L_t = c_b(1 - L_t). \quad (18)$$

Substitute for $L_t = L - L_b$ in (18), and collect terms in L_b to get:

$$B - c_t L = L_b(c_b - c_t) \quad (19)$$

from which (14) is easily obtained. This approach obviously does not yield as much insight into the nature of the model as that taken above. But there would seem to be a good chance that Marshall used such a direct attack on his problem, especially as he did not (unlike Edgeworth) use the Lagrange multiplier technique in his published work.

It is of interest that Marshall could have made his basic point quite strongly using only intuition for this example of choice of travel method, yet he nevertheless devoted some energy to solving the model explicitly in order to give numerical illustrations to Edgeworth. Marshall simultaneously concealed his own mathematics and criticized Edgeworth for an excessive use of mathematics. This ambivalence characterizes much of Marshall's work, as has often been noted. In order to make his point, Marshall was also quite prepared to use a highly artificial example, in which individuals can instantaneously transfer between boat and train at any point in the journey.

It may be added that Marshall was not alone in his use of a rather artificial example to illustrate the Giffen good phenomenon. Bowley (1924, p. 52) referred to an example of a utility function given by Johnson (1913, p. 500). He then gave his own illustration of an individual wishing to spend a fixed sum in order to maximize the area of land purchased, but with a constraint on the length of the frontage required. Wicksell (1934, pp. 60–2) gave an example of an individual exchanging wheat for rye in order to achieve 'maximum nourishment', up to a specified limit or target. As the price of rye in terms of wheat increases, the individual must sell more wheat to meet the target, up to the point where the price is so high that even by selling all his wheat, the dietary aim is not achieved. Neither Wicksell nor Bowley made any reference to Marshall or Giffen, and it is perhaps not surprising that their discussions have been neglected in the extensive literature on Giffen goods.

V

CONCLUSIONS

Marshall and Edgeworth, the two acknowledged British academic leaders

of their generation, present a number of interesting comparisons. While there are several sharp contrasts between their personalities and approaches to economics, their professional relationship was very close, with Edgeworth content to remain, if not in the shadow, in the penumbra of Marshall's dominance. Section II of this paper has examined some general features of their work in economics, features which are all too easy to distort with the passage of time. Sections III and IV then went on to consider in more detail several economic issues which show the two economists taking slightly different points of view, although it has been seen that Marshall provided an important stimulus to Edgeworth's early work. On the question of wage bargaining, it has been seen that Edgeworth's contribution, following Marshall, has been largely neglected in the more recent 'explosion' of work. It has not been possible, in the space available, to explore other common interests such as international trade, but there seems little doubt that the close study of their work can still provide useful insights for the modern theorist.

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