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The Marshallian Demand Curve

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# THE JOURNAL OF POLITICAL ECONOMY

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## THE MARSHALLIAN DEMAND CURVE

MILTON FRIEDMAN<sup>1</sup>

ALFRED MARSHALL'S theory of demand strikingly exemplifies his "impatience with rigid definition and an excessive tendency to let the context explain his meaning."<sup>2</sup> The concept of the demand curve as a functional relation between the quantity and the price of a particular commodity is explained repeatedly and explicitly in the *Principles of Economics*: in words in the text, in plane curves in the footnotes, and in symbolic form in the Mathematical Appendix. A complete definition of the demand curve, including, in particular, a statement of the variables that are to be considered the same for all points on the curve and the variables that are to be al-

lowed to vary, is nowhere given explicitly. The reader is left to infer the contents of *ceteris paribus* from general and vague statements, parenthetical remarks, examples that do not purport to be exhaustive, and concise mathematical notes in the Appendix.

In view of the importance of the demand curve in Marshallian analysis, it is natural that other economists should have constructed a rigorous definition to fill the gap that Marshall left. This occurred at an early date, apparently without controversy about the interpretation to be placed on Marshall's comments. The resulting definition of the demand curve is now so much an intrinsic part of current economic theory and is so widely accepted as Marshall's own that the assertion that Marshall himself gave no explicit rigorous definition may shock most readers.

Yet why this particular interpretation evolved and why it gained such unquestioned acceptance are a mystery that requires explanation. The currently accepted interpretation can be read into Marshall only by a liberal—and, I think, strained—reading of his remarks, and its acceptance implicitly convicts him of log-

<sup>1</sup> I am deeply indebted for helpful criticism and suggestions to A. F. Burns, Aaron Director, C. W. Guillebaud, H. Gregg Lewis, A. R. Prest, D. H. Robertson, G. J. Stigler, and, especially, Jacob Viner, to whose penetrating discussion of the demand curve in his course in economic theory I can trace some of the central ideas and even details of this article. The standard comment that none is to be held responsible for the views expressed herein has particular relevance, since most disagreed with my interpretation of Marshall as presented in an earlier and much briefer draft of this article.

<sup>2</sup> C. W. Guillebaud, "The Evolution of Marshall's *Principles of Economics*," *Economic Journal*, LII (December, 1942), 333.

ical inconsistency and mathematical error at the very foundation of his theory of demand. More important, the alternative interpretation of the demand curve that is yielded by a literal reading of his remarks not only leaves his original work on the theory of demand free from both logical inconsistency and mathematical error but also is more useful for the analysis of most economic problems.

Section I presents the two interpretations of the demand curve and compares them in some detail; Section II argues that a demand curve constructed on my interpretation is the more useful for the analysis of practical problems, whatever may be the verdict about its validity as an interpretation of Marshall; Section III demonstrates that my interpretation is consistent with Marshall's monetary theory and with his work on consumer's surplus; and Section IV presents the textual evidence on the validity of my interpretation. Finally, Section V argues that the change that has occurred in the interpretation of the demand curve reflects a corresponding change in the role assigned to economic theory.

#### I. ALTERNATIVE INTERPRETATIONS OF MARSHALL'S DEMAND CURVE

The demand curve of a particular group (which may, as a special case, consist of a single individual) for a particular commodity shows the quantity (strictly speaking, the maximum quantity) of the commodity that will be purchased by the group per unit of time at each price. So far, no question arises; this part of the definition is explicit in Marshall and is common to both alternatives to be discussed. The problem of interpretation relates to the phrase, "other things the same," ordinarily attached to this definition.

In the first place, it should be noted

that "same" in this phrase does not mean "same over time." The points on a demand curve are alternative possibilities, not temporally ordered combinations of quantity and price. "Same" means "same for all points on the demand curve"; the different points are to differ in quantity and price and are not to differ with respect to "other things."<sup>3</sup> In the second place, "all" other things cannot be supposed to be the same without completely emasculating the concept. For example, if (a) total money expenditure on all commodities, (b) the price of every commodity other than the one in question, and (c) the quantity purchased of every other commodity were supposed to be the same, the amount of money spent on the commodity in question would necessarily be the same at all prices, simply as a matter of arithmetic, and the demand curve would have unit elasticity everywhere.<sup>4</sup> Different specifications of the "other things" will yield different demand curves. For example,

<sup>3</sup> Of course, when correlations among statistical time series are regarded as estimates of demand curves, the hypothesis is that "other things" have been approximately constant over time or that appropriate allowance has been made for changes in them. Similarly, when correlations among cross-section data are regarded as estimates of demand curves, the hypothesis is that "other things" are approximately the same for the units distinguished or that appropriate allowance has been made for differences among them. In both cases the problem of estimation should be clearly distinguished from the theoretical construct to be estimated.

<sup>4</sup> Yet Sidney Weintraub not only suggests that Marshall intended to keep *a*, *b*, and *c* simultaneously the same but goes on to say: "Clearly Marshall's assumption means a unit elasticity of demand in the market reviewed and no ramifications elsewhere; that was why he adopted it" ("The Foundations of the Demand Curve," *American Economic Review*, XXXII [September, 1942], 538-52, quotation from n. 12, p. 541). Weintraub even adds the condition of constant tastes and preferences to *a*, *b*, and *c*, speaking of a change in tastes as shifting the demand curve. Obviously, *a*, *b*, and *c* together leave no room for tastes and preferences or, indeed, for anything except simple arithmetic.

one demand curve will be obtained by excluding *b* from the list of "other things"; another, quite different one, by excluding *c*.

a) THE CURRENT INTERPRETATION

The current interpretation of Marshall's demand curve explicitly includes in the list of "other things" (1) tastes and preferences of the group of purchasers considered; (2) their money income, and (3) the price of every other commodity. The quantities of other commodities are explicitly considered as different at different points on the demand curve, and still other variables are ignored.<sup>5</sup>

On this interpretation, it is clear that, while money income is the same for different points on the demand curve, real income is not. At the lower of two prices for the commodity in question, more of some commodities can be purchased without reducing the amounts purchased of other commodities. The lower the price, therefore, the higher the real income.

<sup>5</sup> Explicit definition of the demand curve in this way by followers of Marshall dates back at least to 1894 (see F. Y. Edgeworth, article on "Demand Curves" in *Palgrave's Dictionary of Political Economy*, edited by Henry Higgs [rev. ed.; London: Macmillan & Co., Ltd., 1926]). Edgeworth's article apparently dates from the first edition, which was published in 1894. While Edgeworth does not explicitly attribute this interpretation to Marshall, it is clear from the context that he is talking about a Marshallian demand curve and that he does not regard his statements as inconsistent in any way with Marshall's *Principles*. Though no explicit listing of "other things" is given by J. R. Hicks, *Value and Capital* (Oxford, 1939), the list given above is implicit throughout chaps. i and ii, which are explicitly devoted to elaborating and extending Marshall's analysis of demand. For statements in modern textbooks on advanced economic theory see G. J. Stigler, *The Theory of Price* (New York: Macmillan Co., 1946), pp. 86-99, and Kenneth E. Boulding, *Economic Analysis* (rev. ed., New York: Harper & Bros., 1948), pp. 134-35.

b) AN ALTERNATIVE INTERPRETATION

It seems to me more faithful to both the letter and the spirit of Marshall's writings to include in the list of "other things" (1) tastes and preferences of the group of purchasers considered, (2) their real income, and (3) the price of every closely related commodity.

Two variants of this interpretation can be distinguished, according to the device adopted for keeping real income the same at different points on the demand curve. One variant, which Marshall employed in the text of the *Principles*, is obtained by replacing "(2) their real income" by (2a) their money income and (2b) the "purchasing power of money." Constancy of the "purchasing power of money" for different prices of the commodity in question implies compensating variations in the prices of some or all other commodities. These variations will, indeed, be negligible if the commodity in question accounts for a negligible fraction of total expenditures; but they should not be disregarded, both because empirical considerations must be sharply separated from logical considerations and because the demand curve need not be limited in applicability to such commodities. On this variant, all commodities are, in effect, divided into three groups: (a) the commodity in question, (b) closely related commodities, and (c) all other commodities. The absolute price of each commodity in group *b* is supposed to be the same for different points on the demand curve; only the "average" price, or an index number of prices, is considered for group *c*; and it is to be supposed to rise or fall with a fall or rise in the price of group *a*, so as to keep the "purchasing power of money" the same.

The other variant, which Marshall employed in the Mathematical Appendix of

the *Principles*, is obtained by retaining "(2) their real income" and adding (4) the average price of all other commodities. Constancy of real income for different prices of the commodity in question then implies compensating variations in money income. As the price of the commodity in question rises or falls, money income is to be supposed to rise or fall so as to keep real income the same.

These two variants are essentially equivalent mathematically,<sup>6</sup> but the as-

<sup>6</sup> Let  $x$  and  $y$  be the quantity and price, respectively, of the commodity in question;  $x'$  and  $y'$  the quantity and price of a composite commodity representing all other commodities; and  $m$ , money income. Let

$$x = g(y, y', m, u) \quad (1)$$

be the demand curve for the commodity in question, given a utility function,

$$U = U(x, x', u), \quad (2)$$

where  $u$  is a parameter to allow for changes in taste, and subject to the condition

$$xy + x'y' = m. \quad (3)$$

From eq. (3) and the usual utility analysis, it follows that eq. (1), like eq. (3), is a homogeneous function of degree zero in  $y$ ,  $y'$ , and  $m$ ; i.e., that

$$g(\lambda y, \lambda y', \lambda m, u) = g(y, y', m, u). \quad (4)$$

On the current interpretation, a two-dimensional demand curve is obtained from eq. (1) directly by giving  $y'$  (other prices),  $m$  (income), and  $u$  (tastes) fixed values. A given value of  $y$  then implies a given value of  $x$  from eq. (1), a given value of  $x'$  from eq. (3), and hence a given value of  $U$  (i.e., real income) from eq. (2). The value of  $U$  will vary with  $y$ , being higher, the lower  $y$  is.

On my alternative interpretation,  $u$  and  $U$  are given fixed values and  $x'$  is eliminated from eqs. (2) and (3). This gives a pair of equations,

$$x = g(y, y', m, u_0), \quad (5)$$

$$U_0 = U_0\left(x, \frac{m - xy}{y'}, u_0\right), \quad (6)$$

where the subscript 0 designates fixed values. The two-dimensional variant involving compensating variations in other prices is obtained by eliminating  $y'$  from eqs. (5) and (6) and giving  $m$  a fixed value; the variant involving compensating variations in income, by eliminating  $m$  from eqs. (5) and (6) and giving  $y'$  a fixed value.

sumption of compensating variations in other prices is easier to explain verbally and can be justified as empirically relevant by considerations of monetary theory, which is presumably why Marshall used this variant in his text. On the other hand, the assumption of compensating variations in income is somewhat more convenient mathematically, which is presumably why Marshall used this variant in his *Mathematical Appendix*.

On my interpretation, Marshall's demand curve is identical with one of the constructions introduced by Slutsky in his famous paper on the theory of choice, namely, the reaction of quantity demanded to a "compensated variation of price," i.e., to a variation in price accompanied by a compensating change in money income.<sup>7</sup> Slutsky expressed the compensating change in money income in terms of observable phenomena, taking it as equal to the change in price *times* the quantity demanded at the initial price. Mosak has shown that, in the

The homogeneity of eqs. (5) and (6) in  $y$ ,  $y'$ , and  $m$  means that  $x$  is a function only of ratios among them. Thus eqs. (5) and (6) can be written:

$$x = g(y, y', m, u_0) = g\left(\frac{y}{m}, \frac{y'}{m}, 1, u_0\right) = g\left(\frac{y}{y'}, 1, \frac{m}{y'}, u_0\right), \quad (5')$$

$$U_0 = U_0\left(x, \frac{m - xy}{y'}, u_0\right) = U_0\left(x, \frac{1 - x \frac{y}{m}}{\frac{y'}{m}}, u_0\right) = U_0\left(x, \frac{m}{y'} - x \frac{y}{y'}, u_0\right). \quad (6')$$

The choice of price-compensating variations is equivalent to selecting the forms of these two equations in the next to the last terms of eqs. (5') and (6'); of income-compensating variations, to selecting the forms in the last terms.

<sup>7</sup> Eugenio Slutsky, "Sulla teoria del bilancio del consumatore," *Giornale degli economisti*, LI (1915), 1-26, esp. Sec. §8.

limit, the change in income so computed is identical with the change required to keep the individual on the same level of utility (on the same indifference curve).<sup>8</sup> It follows that a similar statement is valid for compensating changes in other prices. In the limit, the change in other prices required to keep the individual on the same indifference curve when his money income is unchanged but the price of one commodity varies is identical with the change in other prices required to keep unchanged the total cost of the basket of commodities purchased at the initial prices, i.e., to keep unchanged the usual type of cost-of-living index number.

c) COMPARISON OF THE INTERPRETATIONS

The relation between demand curves constructed under the two interpretations is depicted in Figure 1. Curve  $Cc$  represents a demand curve of an individual consumer for a commodity  $X$  drawn on the current interpretation. Money income and the prices of other commodities are supposed the same for all points on it; in consequence, real income is lower at  $C$  than at  $P$ , since, if the individual sought to buy  $OM$  of  $X$  at a price of  $OC$ , he would be forced to curtail his purchases of something else. As the curve is drawn, of course, he buys none of  $X$  at a price of  $OC$ , spending the sum of  $OHPM$  on other commodities that his action at a price of  $OH$  shows him to value less highly than he does  $OM$  units of  $X$ . The ordinate is described as the ratio of the price of  $X$  to the price of other commodities. For the demand curve  $Cc$  this is a question only of the unit of measure,

since other prices are supposed to be the same for all points on it.

From the definition of the demand curve  $Cc$ ,  $OC$  is obviously the maximum price per unit that an individual would be willing to pay for an infinitesimal initial increment of  $X$  when his money income and the prices of other commodities have the values assumed in drawing  $Cc$ . Let us suppose him to purchase this amount at a price of  $OC$ , determine the

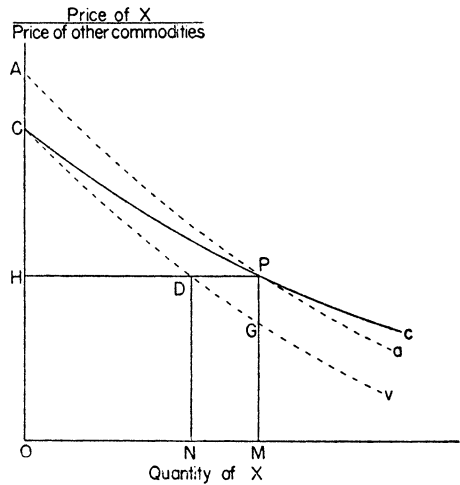


FIG. 1.—Comparison of demand curves constructed under the two interpretations.

maximum price per unit he would be willing to pay for an additional increment, and continue in this fashion, exacting the maximum possible amount for each additional increment. Let these successive maximum prices per unit define the curve  $Cv$ . The consumer obviously has the same real income at each point on  $Cv$  as at  $C$ , since the maximum price has been extracted from him for each successive unit, so that he has gained no utility in the process.

$Cv$  is now a demand curve constructed according to my interpretation of Marshall. If other prices are supposed to be the same, the necessary compensating

<sup>8</sup> Jacob L. Mosak, "On the Interpretation of the Fundamental Equation of Value Theory," in O. Lange, F. McIntyre, and T. O. Yntema, *Studies in Mathematical Economics and Econometrics* (Chicago: University of Chicago Press, 1942), pp. 69-74, esp. n. 5, pp. 73-74, which contains a rigorous proof of this statement by A. Wald.



variations in money income as the price of  $X$  falls are given by triangular areas exemplified by  $HCD$  for a price of  $OH$ :  $OH$  is the maximum price per unit that the individual will give for an additional infinitesimal increment of  $X$  when he has spent  $OCDN$  for  $ON$  of  $X$  out of his initial income of, say,  $m$ ; but his situation is exactly the same if, when the price of  $X$  is  $OH$ , his income is  $(m - HCD)$  and he spends  $OHDN$  on  $X$ ; he has the same amount left to spend on all other commodities, their prices are the same, and he has the same amount of  $X$ ; accordingly, his demand price will be the same, and he will buy  $ON$  of  $X$  at a price of  $OH$  and an income of  $(m - HCD)$ .<sup>9</sup>

If compensating variations in other

<sup>9</sup> In the notation of n. 6, except that  $u$  is omitted for simplicity, the quantities of  $X$  and  $X'$  that will be purchased for any given values of  $y$  and  $y'$  and any given real income,  $U_0$ , are obtained by solving simultaneously:

$$\frac{U_x}{U_{x'}} = \frac{y}{y'}, \quad (1)$$

and

$$U(x, x') = U_0, \quad (2)$$

where  $U_x$  and  $U_{x'}$  stand for the partial derivatives of  $U$  with respect to  $x$  and  $x'$ , respectively, i.e., for the marginal utility of  $X$  and  $X'$ . The solution of these equations gives the demand curve on my interpretation of Marshall, using compensating variations in money income.

$U_0(0, m/y')$  is the utility at  $C$  in the diagram. For any given amount of  $X$  and given value of  $y'$ , the amount of  $X'$  purchased is obtained by solving

$$U(x, x') = U_0 \left( 0, \frac{m}{y'} \right), \quad (3)$$

which is identical with eq. (2). The amount paid for  $X$  (the area under  $Cv$ ) is

$$m - x'y'. \quad (4)$$

The maximum price that will be paid per unit of  $X$  is the derivative of eq. (4), or

$$y = -\frac{dx'}{dx} y' = \frac{U_x}{U_{x'}} y', \quad (5)$$

which is identical with eq. (1). It follows that  $Cv$  is a demand curve constructed on my interpretation of Marshall.

prices rather than in money income are used to keep real income the same, the absolute price of neither  $X$  nor other commodities can be read directly from Figure 1. For each ratio of the price of  $X$  to the price of other commodities, the quantity of  $X$  purchased will be that shown on  $Cv$ . But the prices of other goods will vary along  $Cv$ , rising as the relative price of  $X$  falls, so the absolute price of  $X$  can no longer be obtained by multiplying the ordinate by a single scale factor.

Figure 1 is drawn on the assumption that  $X$  is a "normal" commodity, that is, a commodity the consumption of which is higher, the higher the income. This is the reason  $Cv$  is drawn to the left of  $Cc$ —at every point on  $Cv$  other than  $C$ , real income is less than at the corresponding point on  $Cc$ ; hence less  $X$  would be consumed.

Curve  $Aa$  represents a demand curve on my interpretation of Marshall for a real income the same as at point  $P$  on  $Cc$ ; it is like  $Cv$  but for a higher real income. Real income is higher on  $Aa$  than on  $Cc$  for prices above  $OH$ , lower for prices below  $OH$ , which is the reason  $Aa$  is to the right of  $Cc$  for prices above  $OH$  and to the left of  $Cc$  for prices below  $OH$ .

#### d) WHY TWO INTERPRETATIONS ARE POSSIBLE

The possibility of interpreting Marshall in these two quite different ways arises in part from the vagueness of Marshall's exposition, from his failure to give precise and rigorous definitions. A more fundamental reason, however, is the existence of inconsistency in the third and later editions of the *Principles*. In that edition Marshall introduced the celebrated passage bearing on the Giffen phenomenon. This passage and a related sentence added at the same time to the

Mathematical Appendix fit the current interpretation better than they fit my interpretation. Although these are the only two items that I have been able to find in any edition of the *Principles* of which this is true, they provide some basis for the current interpretation. A hypothesis to explain the introduction of this inconsistency into the *Principles* is offered in Section IVe below.

## II. THE RELATIVE USEFULNESS OF THE TWO INTERPRETATIONS

The relative usefulness of the two interpretations of the demand curve can be evaluated only in terms of some general conception of the role of economic theory. I shall use the conception that underlies Marshall's work, in which the primary emphasis is on positive economic analysis, on the forging of tools that can be used fairly directly in analyzing practical problems. Economic theory was to him an "engine for the discovery of concrete truth."<sup>10</sup> "Man's powers are limited: almost every one of nature's riddles is complex. He breaks it up, studies one bit at a time, and at last combines his partial solutions with a supreme effort of his whole small strength into some sort of an attempt at a solution of the whole riddle."<sup>11</sup> The underlying justification for the central role of the concepts of demand and supply in Marshall's entire structure of analysis is the empirical generalization that an enumeration of the forces affecting demand in any problem and of the forces affecting supply will yield two lists that contain few items in common. Demand and supply are to

him concepts for organizing materials, labels in an "analytical filing box." The "commodity" for which a demand curve is drawn is another label, not a word for a physical or technical entity to be defined once and for all independently of the problem at hand. Marshall writes:

The question where the lines of division between different commodities should be drawn must be settled by convenience of the particular discussion. For some purposes it may be best to regard Chinese and Indian teas, or even Souchong and Pekoe teas, as different commodities; and to have a separate demand schedule for each of them. While for other purposes it may be best to group together commodities as distinct as beef and mutton, or even as tea and coffee, and to have a single list to represent the demand for the two combined.<sup>12</sup>

### a) THE DISTINCTION BETWEEN CLOSELY RELATED AND ALL OTHER COMMODITIES

A demand function containing as separate variables the prices of a rigidly defined and exhaustive list of commodities, all on the same footing, seems largely foreign to this approach. It may be a useful expository device to bring home the mutual interdependence of economic phenomena; it cannot form part of Marshall's "engine for the discovery of concrete truth." The analyst who attacks a concrete problem can take explicit account of only a limited number of factors; he will inevitably separate commodities that are closely related to the one immediately under study from commodities that are more distantly related. He can pay some attention to each closely related commodity. He cannot handle the more distantly related commodities in this way; he will tend either to ignore

<sup>10</sup> Alfred Marshall, "The Present Position of Economics" (1885), reprinted in *Memorials of Alfred Marshall*, ed. A. C. Pigou (London: Macmillan & Co., Ltd., 1925), p. 159.

<sup>11</sup> Alfred Marshall, "Mechanical and Biological Analogies in Economics" (1898), *ibid.*, p. 314.

<sup>12</sup> Marshall, *Principles of Economics* (8th ed.; London: Macmillan & Co., Ltd., 1920), p. 100 n. All subsequent page references to the *Principles*, unless otherwise stated, are to the eighth and final edition.

them or to consider them as a group. The formally more general demand curve will, in actual use, become the kind of demand curve that is yielded by my interpretation of Marshall.

The part of the Marshallian filing box covered by *ceteris paribus* typically includes three quite different kinds of variables, distinguished by their relation to the variable whose adaptation to some change is directly under investigation (e.g., the price of a commodity): (a) variables that are expected both to be materially affected by the variable under study and, in turn, to affect it; (b) variables that are expected to be little, if at all, affected by the variable under study but to materially affect it; (c) the remaining variables, expected neither to affect significantly the variable under study nor to be significantly affected by it.

In demand analysis the prices of closely related commodities are the variables in group *a*. They are put individually into the pound of *ceteris paribus* to pave the way for further analysis. Holding their prices constant is a provisional step. They must inevitably be affected by anything that affects the commodity in question; and this indirect effect can be analyzed most conveniently by first isolating the direct effect, systematically tracing the repercussions of the direct effect on each closely related commodity, and then tracing the subsequent reflex influences on the commodity in question. Indeed, in many ways, the role of the demand curve itself is as much to provide an orderly means of analyzing these indirect effects as to isolate the direct effect on the commodity in question.

The average price of "all other commodities," income and wealth, and tastes and preferences are the variables in group *b*. These variables are likely to be affected only negligibly by factors affect-

ing primarily the commodity in question. On the other hand, any changes in them would have a significant effect on that commodity. They are put into the pound in order to separate problems, to segregate the particular reactions under study. They are put in individually and explicitly because they are so important that account will have to be taken of them in any application of the analysis.

Price changes within the group of "all other commodities" and an indefinitely long list of other variables are contained in group *c*. These variables are to be ignored. They are too numerous and each too unimportant to permit separate account to be taken of them.

In keeping with the spirit of Marshallian analysis this classification of variables is to be taken as illustrative, not definitive. What particular variables are appropriate for each group is to be determined by the problem in hand, the amount of information available, the detail required in results, and the patience and resources of the analyst.

#### b) CONSTANCY OF REAL INCOME

It has just been argued that any actual analysis of a concrete economic problem with the aid of demand curves will inevitably adopt one feature of my interpretation of Marshall—consideration of a residual list of commodities as a single group. For somewhat subtler reasons this is likely to be true also of the second feature of my interpretation of Marshall—holding real income constant along a demand curve. If an analysis, begun with a demand curve constructed on the current interpretation, is carried through and made internally consistent, it will be found that the demand curve has been subjected to shifts that, in effect, result from failure to keep real income constant along the demand curve.

An example will show how this occurs. Let us suppose that the government grants to producers of commodity  $X$  a subsidy of a fixed amount per unit of output, financed by a general income tax, so that money income available for expenditure (i.e., net of tax and gross of subsidy) is unchanged. For simplicity, suppose, first, that no commodities are closely related to  $X$  either as rivals or as complements, so that interrelations in consumption between  $X$  and particular other commodities can be neglected; second, that the tax is paid by individuals in about the same income class and with about the same consumption pattern as those who benefit from the subsidy, so that complications arising from changes in the distribution of income can be neglected; and, third, that there are no idle resources. Let  $DD$  in Figure 2 be a demand curve for commodity  $X$ , and  $SS$  be the initial supply curve for  $X$ , and let the initial position at their intersection, point  $P$ , be a position of full equilibrium. The effect of the subsidy is to lower the supply curve to  $S'S'$ . Since we have ruled out repercussions through consumption relations with other markets and through changes in the level or distribution of money income, it is reasonable to expect that the intersection of this new supply curve and the initial demand curve, point  $P'$ , will itself be a position of full equilibrium, involving a lower price and larger quantity of  $X$ . Yet, if the demand curve is constructed on the current interpretation and if the supply curve is not perfectly inelastic,<sup>13</sup> point  $P'$  is not a position of full equilibrium. This can be

seen most easily by supposing  $DD$  to have unit elasticity, so that the same amount is spent on  $X$  at  $P'$  as at  $P$ . The same amount is then available to spend on all other commodities, and, since their prices are supposed to be the same for all points on  $DD$  under the current interpretation, the same quantity of each of them will be demanded. But then where do the resources come from to produce the extra  $MN$  units of  $X$ ? Obviously, our as-

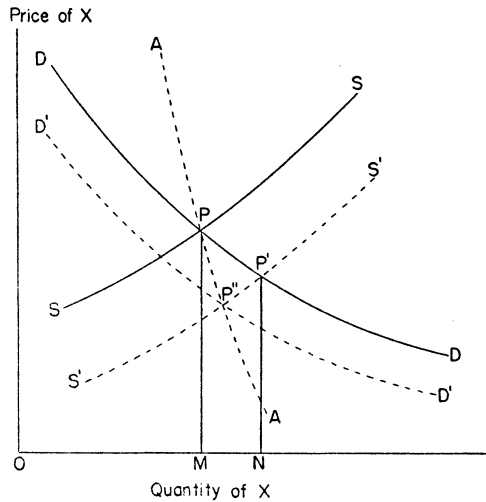


FIG. 2.—Illustrative analysis of effect of subsidy

sumptions are not internally consistent. The additional units of  $X$  can be produced only by bidding resources away from the production of other commodities, in the process raising their prices and reducing the amount of them produced. The final equilibrium position will therefore involve higher prices and lower quantities of other commodities. But, on the current interpretation, this means a shift in the demand curve for  $X$ —say, to  $D'D'$ —and a final equilibrium position of, say,  $P''$ <sup>14</sup>

<sup>13</sup> If it is perfectly inelastic, neither the price nor the quantity of  $X$  is changed, so the new position of equilibrium coincides with the old; but the demand curve will pass through the initial position of equilibrium whether constructed on the current interpretation or on mine; hence the two coincide at the one point on them that is relevant.

<sup>14</sup>  $D'D'$  will not necessarily be to the left of  $DD$  even for a "normal" commodity. The reason is that the ordinate of Fig. 2 measures the absolute price of  $X$ , so that ordinates of the same height on  $DD$

The assumption that the elasticity of  $DD$  is unity is not, of course, essential for this argument. If the elasticity of  $DD$  is less than unity, a larger amount than formerly is available to spend on other commodities; at unchanged prices this means a larger quantity demanded. In consequence, while the additional amount of resources required to produce the increased amount of  $X$  demanded is smaller when  $DD$  is inelastic than when it has unit elasticity, this is counterbalanced by increased pressure for resources to produce other commodities. Similarly, when  $DD$  is elastic, the additional amount of resources required to produce the increased quantity of  $X$  demanded is larger than when  $DD$  has unit elasticity, but some resources are released in the first instance from the production of other commodities.

No such internal inconsistency as that just outlined arises if the demand curve is constructed by keeping real income the same. Curve  $AA$  is such a demand curve. At prices of  $X$  less than  $PM$ , prices of other commodities are supposed to be sufficiently higher than at  $P$  to keep real income the same, which involves the release of just enough resources so that the position of final equilibrium,  $P''$ , lies on the demand curve so constructed—at least for small changes in the price of  $X$ .<sup>15</sup>

and  $D'D'$  represent different ratios of the price of  $X$  to the price of other commodities. If the ordinate measured the ratio of the price of  $X$  to the price of other commodities,  $D'D'$  would always be to the left of  $DD$  for "normal" commodities, always to the right for "inferior" commodities.

<sup>15</sup>Let  $X'$  be a single composite commodity representing all commodities other than  $X$ ;  $x$  and  $x'$ , the quantities of  $X$  and  $X'$ ; and  $y$  and  $y'$ , their prices. Let the subscript 1 refer to values at the initial position of equilibrium,  $P$ ; the subscript 2, to values at the final position,  $P''$ . The condition of constant total expenditures means that

$$x_1y_1 + x'_1y'_1 = x_2y_2 + x'_2y'_2. \quad (1)$$

The fundamental principle illustrated by this example can be put more generally. The reason why a demand curve constructed under the current interpretation fails to give the correct solution even when all disturbing influences can be neglected is that each point on it implicitly refers to a different productive capacity of the community. A reduction in the price of the commodity in question is to be regarded as enabling the community, if it so wishes, to consume more of some commodities—this commodity or others—without consuming less of any commodity. But the particular change in supply whose consequences we sought to analyze—that arising from a subsidy—does not make available any additional resources to the community; any increase in the consumption of the commodity in question must be at the expense of other commodities. The conditions for which the demand curve is drawn are therefore

As was pointed out above (Sec. 1b), in the limit, holding real income constant is equivalent to holding constant the cost of a fixed basket of commodities. Thus, if  $P''$  is considered close to  $P$ ,

$$x_1y_1 + x'_1y'_1 = x_2y_2 + x'_2y'_2. \quad (2)$$

In the neighborhood of  $P$ ,  $y_1$  can be regarded as the cost per unit of producing  $X$ ;  $y'_1$  as the cost per unit of producing  $X'$ . The condition that sufficient resources be released to permit the production of the requisite additional amount of  $X$  is therefore

$$(x_2 - x_1)y_1 = -(x'_2 - x'_1)y'_1, \quad (3)$$

which is equivalent to

$$x_1y_1 + x'_1y'_1 = x_2y_2 + x'_2y'_2. \quad (4)$$

But, in the limit, eqs. (1) and (2) imply eq. (4), as can be seen by subtracting eq. (2) from eq. (1) and replacing  $y_2$  and  $y'_2$  in the result by  $(y_2 - y_1 + y_1)$  and  $(y'_2 - y'_1 + y'_1)$ , respectively.

More generally, constant real income involves keeping a price index unchanged; constant use of resources involves keeping a quantity index unchanged; and, in the limit, a constant price index and constant total expenditures imply a constant quantity index.

Note that  $AA$  need not be steeper than  $DD$  in a graph like Fig. 2. The point in question is that commented on in n. 14.

inconsistent with the conditions postulated on the side of supply. On the other hand, if the demand curve is constructed by keeping "real income" the same, no such inconsistency need arise. True, constant "real income" in the sense of "utility" and constant "real income" in the sense of outputs attainable from a fixed total of resources are different concepts, but they converge and can be treated as the same in the neighborhood of a position of equilibrium.

Of course, not all shifts in supply that it is desired to analyze arise in ways that leave the productive capacity of the community unaltered. Many involve a change in productive capacity—for example, changes in supply arising from improvements in technology or the discovery of previously unknown resources. Even in these cases, however, a demand curve constructed on the current interpretation will not serve. There is no reason to expect the differences in productive capacity implicit in constant money income and constant prices of other goods to bear any consistent relation to the change in productive capacity arising on the side of supply.<sup>16</sup> The better plan, in these cases, is to allow separately and directly for the increase in productive capacity by redrawing the demand curves to correspond to an appropriately higher real income and then to use a demand curve on which all points refer to that higher real income.

The main point under discussion can be put still more generally. The opportunities open to a consumer to satisfy his

wants depend principally on two factors—the total resources at his disposal and the terms on which he can exchange one commodity for another, that is, on his real income and on relative prices. The form of analysis that is now fashionable distinguishes three effects of changes in his opportunities—the income effect arising from changes in his money income; the income effect arising from changes in the price of a commodity, with unchanged money income and prices of other commodities; and the substitution effect arising from a change in the relative price of a commodity, with unchanged real income.

The distinction between the so-called "substitution" and "income" effects of a change in price is a direct consequence of defining the demand curve according to the current interpretation of Marshall. Its basis is the arithmetic truism that at given prices for all commodities but one, a given money income corresponds to a higher real income, the lower the price of the remaining commodity—at a lower price for it, more of some commodities can be purchased without purchasing less of others. In consequence, a decline in the price of a commodity, all other prices constant, has, it is argued, two effects: first, with an unchanged real income, it would stimulate the substitution of that commodity for others—this is the substitution effect; second, if the money income of the consumers is supposed to be unchanged, the increase in their real income as a result of the decline in price causes a further change in the consumption of that commodity as well as of others—this is the income effect.<sup>17</sup>

<sup>16</sup>Note the difference from the previous case of constant productive capacity. As stated above, there is reason to expect constant real income along a demand curve to bear a consistent relation to constant productive capacity in the neighborhood of equilibrium. The reason, in effect, is provided by one of the conditions of equilibrium: the tangency of consumption and production indifference curves,

<sup>17</sup>See Slutsky, *op. cit.*; Henry Schultz, *The Theory and Measurement of Demand* (Chicago: University of Chicago Press, 1938), pp. 40-46; J. R. Hicks and R. G. D. Allen, "A Reconsideration of the Theory of Value," *Economica*, XIV (1934), 52-76 and 196-219; Hicks, *op. cit.*, Part I.

The two different kinds of income effects distinguished in this analysis—one arising from a change in money income, the other from a change in the price of one commodity—are really the same thing, the effect of a change in real income with given relative prices, arising in different ways. It is hard to see any gain from combining the second income effect with the substitution effect; it seems preferable to combine the two income effects and thereby gain a sharp contrast with the substitution effect.

It has often been stated that Marshall “neglected the income effect.”<sup>18</sup> On my interpretation of his demand curve, this statement is invalid. One must then say that Marshall recognized the desirability of separating two quite different effects and constructed his demand curve so that it encompassed solely the effect that he wished to isolate for study, namely, the substitution effect. Instead of neglecting the income effect, he “eliminated” it.

The conclusion to which the argument of this section leads is identical with that reached by Frank H. Knight in a recent article, in which he says:

We have to choose in analysis between holding the prices of all other goods constant and maintaining constant the “real income” of the hypothetical consumer. . . . The treatment of the Slutsky school adopts the assumption that . . . the prices of all other goods (and the consumer’s money income) are constant. Hence, real income must change. Of the two alternatives, this seems to be definitely the wrong choice. . . . The simple and obvious alternative is to draw the demand curves in terms of a change in *relative* prices, i.e., to assume that the value of money is held constant, through compensating changes in the prices of other goods, and not that these other prices are held constant.<sup>19</sup>

<sup>18</sup> Hicks, *op. cit.*, p. 32.

<sup>19</sup> “Realism and Relevance in the Theory of Demand,” *Journal of Political Economy*, LII

### III. THE CONSISTENCY OF THE ALTERNATIVE INTERPRETATION WITH OTHER PARTS OF MARSHALL’S WORK

Marshall’s demand curve is part of a coherent body of thought; it is designed to fit into the rest of his structure of analysis; and it is used extensively in developing and applying this structure. It would take us too far afield to demonstrate in detail that my interpretation of his demand curve is consistent with the rest of his work. However, two special topics call for some explicit consideration: (1) the relation between the demand curve and Marshall’s theory of money, because, in my view, this explains the particular device that he adopted for holding real income constant; and (2) the concept of consumer’s surplus, because this is one of the most important applications of the demand curve and certainly the most controversial and because the passages in the later editions of the *Principles* that are inconsistent with my interpretation were introduced into the discussion of consumer’s surplus.

#### a) THE THEORY OF RELATIVE PRICES AND THE THEORY OF MONEY

Granted that real income is to be held constant along the demand curve, why do so by holding money income and the purchasing power of money constant rather than, for example, by holding prices of other goods constant and permitting compensating variations in money income? What reason is there to treat the prices of all other commodities as moving inversely to the price of the commodity in question?

The answer to these questions is given, I think, by one of Marshall’s basic or-

(December, 1944), 289-318, esp. Sec. III, “The Meaning of a Demand Curve,” pp. 298-301. Quotation from p. 299.

ganizing principles, namely, the separation of the theory of relative prices from monetary theory, the theory of the level of prices. The *Principles* is devoted to the theory of relative prices under given monetary conditions; *Money, Credit, and Commerce* to the analysis of monetary conditions and their effect on the "purchasing power of money." With *given monetary conditions*, is it possible for the prices of all commodities other than the one in question to remain the same, on the average, while the price of this one rises or falls? Will not a rise or fall in the price of the commodity in question set in motion *monetary* forces affecting other prices? A complete answer requires explicit specification of the content of "given monetary conditions" and perhaps also of the source of the initial price change.

Marshall's selection of a constant purchasing power of money as a means of impounding monetary forces is presumably the end-result of a chain of reasoning about the influence of monetary forces, not the direct content that he gave to "given monetary conditions." The beginning of the chain of reasoning may well be his own version of the quantity theory of money. According to this version, "the value of money is a function of its supply on the one hand, and the demand for it, on the other, as measured by 'the average stock of command over commodities which each person cares to keep in a ready form.'"<sup>20</sup> Given monetary conditions would then imply a given stock of money and a given desired "average stock of command over commodities." A decline in one price alone, all other prices remaining the same, is inconsistent with these "givens." It would increase the real value of a

fixed (nominal) stock of money, leave the community with a larger "stock of command over commodities" than previously, and establish an incentive (reflecting "monetary" forces) to increase expenditures and thereby raise prices until the fixed stock of money again represented the same "stock of command over commodities," i.e., until the "purchasing power of money" reached its former level.<sup>21</sup> This argument suggests that not only was constant purchasing power of money a device for separating the theory of relative prices from monetary theory, it was also a bridge between the two. Marshall separated the two theories in his attempt to reduce problems to manageable proportions, but he constructed them in such a way as to make them mutually consistent and thus facilitate ultimate combination.<sup>22</sup>

<sup>21</sup> C. W. Guillebaud has pointed out to me that Marshall typically supposed the desired "stock of command over commodities" to be a given fraction of real income (see *ibid.*) and that the argument in the text might not apply if this fraction were taken as the fundamental given. The monetary effects of a change in one price, other prices given, would then depend on the source of the initial price change. If this involved no change in aggregate real income (e.g., arose from a shift in demand), the argument in the text would remain unchanged. If it did involve a change in aggregate real income (e.g., arose from an invention reducing the cost of producing the commodity in question), no inconsistency need arise, since the desired "stock of command over commodities" would change in proportion to the change in real income. These considerations account for the phrase "perhaps also of the source of the initial price change" at the end of the preceding paragraph of the text.

<sup>22</sup> This interpretation would, of course, be contradicted if Marshall had devised his theory of money after he had substantially completed his theory of relative prices, as might be inferred from the fact that *Money, Credit, and Commerce* was not published until 1923, thirty-three years after the first edition of the *Principles*. But in Marshall's case, the order of publication is a poor guide to the order of construction. Keynes tells us that the essence of his quantity theory of money is contained in a manuscript "written about 1871"; that "by 1871 his progress along" the lines of the material

<sup>20</sup> J. M. Keynes, "Alfred Marshall, 1842-1924," *Memorials*, p. 29.



Marshall was, of course, very much aware of the interaction between real and monetary factors. The 1879 *Economics of Industry* contains an extremely interesting discussion of the trade cycle, part of which Marshall thought sufficiently important to quote at length in 1886 in answering questions circulated by the celebrated Royal Commission on the Depression of Trade and Industry.<sup>23</sup>

Marshall's decision to keep the purchasing power of money the same for different points on a demand curve may not be the device best suited to abstract from monetary factors. It serves, however, to emphasize the necessity of considering explicitly the monetary arrangements under which the forces affecting relative prices are supposed to operate. The best apparatus for tackling problems of relative prices cannot be determined independently of these arrangements and of their mode of operation. Though price theory and monetary theory can be sep-

contained in the *Pure Theory* "was considerably advanced"; that the *Pure Theory* itself was "substantially complete about 1873," though not printed even for private circulation until 1879; that "in 1877 he turned aside to write the *Economics of Industry* with Mrs. Marshall"; and that Mrs. Marshall said "Book III on Demand was largely thought out and written on the roof at Palermo, Nov. 1881—Feb. 1882" (*Memorials*, pp. 28, 21, 23, 39 n.). These dates are extremely suggestive, particularly since the constancy of the purchasing power of money is not explicitly mentioned in the *Pure Theory*, which Marshall was presumably working on at about the same time that he was developing his monetary theory, while it is explicitly mentioned in the 1879 *Economics of Industry*, begun some years later. See also nn. 37 and 38 below.

<sup>23</sup> See Alfred Marshall and Mary Paley Marshall, *Economics of Industry* (London: Macmillan & Co., 1st ed., 1879; 2d ed., 1881), Book III, chap. i, pp. 150-57. This and all later references are to the first edition. "Answers to Questions on the Subject of Currency and Prices Circulated by the Royal Commission on the Depression of Trade and Industry (1886)," *Official Papers by Alfred Marshall* (London: Macmillan & Co., Ltd., 1926), pp. 7-9. See also "Remedies for Fluctuations of General Prices" (1887), *Memorials*, pp. 189-92.

arated, they are not basically independent. From this point of view it is entirely natural that the recent development of alternative monetary theories should have stimulated re-examination of price theory.

#### b) CONSUMER'S SURPLUS

Marshall's discussion of consumer's surplus constitutes one of the most extensive applications that he made of his demand curve and has probably given rise to more controversy and discussion than any other part of his theory. Recently, consumer's surplus has come in for renewed attention, primarily as a result of J. R. Hicks's attempt to rehabilitate and reinterpret the concept.<sup>24</sup> The reason for commenting on it here is not to contribute to the discussion or to evaluate the merits or demerits of the concept but rather to show the relation between Marshall's treatment of consumer's surplus and my interpretation of his demand curve.

Marshall's treatment of consumer's surplus might, offhand, seem inconsistent with my interpretation of his demand curve for either of two different, and almost opposed, reasons. In the first place, consumer's surplus refers to a difference in real income under different situations. But, on my interpretation, all

<sup>24</sup> See Hicks, *op. cit.*, pp. 38-41; "The Rehabilitation of Consumers' Surplus," *Review of Economic Studies*, VIII (February, 1941), 108-16; "Consumers' Surplus and Index Numbers," *ibid.* (summer, 1942), 126-37; "The Four Consumer's Surpluses," *ibid.*, XI (winter, 1943), 31-41. See also A. Henderson, "Consumer's Surplus and the Compensating Variation," *Review of Economic Studies*, VIII (February, 1941), 117-21; Knight, *op. cit.*; Kenneth E. Boulding, "The Concept of Economic Surplus," *American Economic Review*, XXXV (December, 1945), 851-69, reprinted in American Economic Association, *Readings in the Theory of Income Distribution* (Philadelphia: Blakiston Co., 1946), pp. 638-59; E. J. Mishan, "Realism and Relevance in Consumer's Surplus," *Review of Economic Studies*, XV (1947-48), 27-33.

points on the demand curve are to be regarded as corresponding to the same real income. A movement along such a demand curve cannot, therefore, involve a change in consumer's surplus. Does this not eliminate the entire notion of consumer's surplus and make Marshall's entire discussion of it pointless? The answer is clearly "No," the reason being that the two situations compared need not correspond to two points on the *same* demand curve, even though a single demand curve is used to *estimate* the difference in real income between the two situations.

In the second place, Marshall regarded his analysis of consumer's surplus as valid only for commodities that account for a small part of total expenditure. He makes this restriction in order to justify neglecting changes in the marginal utility of money. But, if all points on the demand curve correspond to the same real income, does it not then follow that the marginal utility of money is the same everywhere on the demand curve? And does it not also follow that his estimate of consumer's surplus is exact, so that the assumption that a negligible proportion of expenditures is devoted to the commodity in question becomes unnecessary? Again the answer is "No," and for much the same reason. If the two situations compared differ in real income, the fact that real income is the same along the demand curve becomes something of a vice in using it to measure consumer's surplus. The assumption that a negligible proportion of expenditures is devoted to the commodity in question cannot be dispensed with on my interpretation; indeed, if anything, it is even more necessary than on the current interpretation.

To explain and justify these cryptic answers, it will be necessary to examine

Marshall's definition of consumer's surplus, his suggested estimate of its magnitude, and the relation of this estimate to the correct value under the two alternative interpretations of the demand curve.

Marshall is more explicit and complete in defining consumer's surplus than was his wont, and his definition admits of little ambiguity: "The excess of the price which he would be willing to pay rather than go without the thing, over that which he actually does pay, is the economic measure of this surplus satisfaction. It may be called *consumer's surplus*."<sup>25</sup>

Marshall then proceeds to argue that consumer's surplus can be *estimated* by the famous triangle under the demand curve. As Hicks remarks, this "association of Consumer's Surplus with the curvilinear triangle under the demand curve . . . is not a definition; it is a theorem, true under certain restrictive assumptions, but only true if these assumptions are granted."<sup>26</sup> The confusion of the suggested estimate with the definition is perhaps the chief source of misunderstanding on this exceedingly complex subject.

Figure 1, introduced in Section 1c above to illustrate the relation between demand curves drawn on the current and on my interpretation, can also be used to show the relation between consumer's surplus as defined and estimates of it obtained from demand curves constructed according to the two interpretations. Curve *Cc*, it will be recalled, is a demand curve for the commodity *X* constructed according to the current interpretation. Money income and all other prices are the same for all points on it. *Aa* and *Cv*

<sup>25</sup> *Principles*, p. 124.

<sup>26</sup> "Rehabilitation of Consumers' Surplus," p. 109.

are demand curves constructed according to my interpretation— $Aa$  for a real income the same as at  $P$  on  $Cc$ ;  $Cv$  for a real income the same as at  $C$  on  $Cc$ . At point  $P$  on  $Aa$  and at point  $C$  on  $Cv$ , money income and all other prices are the same as on  $Cc$ . At other points other prices are sufficiently different, or money income is, to compensate for the difference in the price of  $X$  and thereby keep real income the same.

Now consider the consumer's surplus obtained from this commodity when the consumer is at  $P$ .<sup>27</sup> This is *defined* as "the excess of the price which he would be willing to pay rather than go without the thing, over that which he actually does pay." "Price" is here to be interpreted as "total amount" rather than "price per unit."<sup>28</sup> Further, it is clear that the sum he would pay rather than go without is to be determined for circumstances otherwise the same as at  $P$ ; in particular, his money income and the other prices are to be the same as at  $P$ .<sup>29</sup> Now the amount that he actually does pay for  $OM$  of  $X$  is given by the rectangle  $OHPM$  in the figure. By the argument of Section  $Ic$ , the maximum amount that he would be willing to pay for  $OM$  of  $X$  rather than go without any of it is given by the area un-

<sup>27</sup> For simplicity, the discussion is restricted to the consumer's surplus obtained from the entire amount of  $X$  consumed; and to facilitate this, the demand curves have been drawn to cut the price axis.

<sup>28</sup> See Mathematical Note II of the *Principles* (p. 838), in which Marshall defines  $p$  as "the price which he is just willing to pay for an amount  $x$  of the commodity" and then differentiates  $p$  with respect to  $x$  to get the price per unit.

<sup>29</sup> None of the reasons cited earlier for keeping real income the same along the demand curve apply here. The question being asked is purely hypothetical; no other reactions need be allowed for. Further, to keep his real income the same when he has none of  $X$  as when he has  $OM$  of  $X$  would make the entire discussion of consumer's surplus pointless. The whole point of the discussion is to measure the difference in real income between the two situations.

der  $Cv$  between  $O$  and  $M$ , or  $OCDGM$ . The triangular area  $CDH$  minus the triangular area  $DPG$  therefore gives the consumer's surplus. This area is necessarily positive; we know he is willing to pay at least  $OHPM$  for  $OM$  of  $X$ ; hence  $OCDGM$  must be greater than  $OHPM$ .

Marshall's *estimate* of the maximum sum is the area under the demand curve:  $OCPM$  if we use the current interpretation,  $OAPM$  if we use the alternative interpretation. For a "normal" commodity, the case for which the figure is drawn, both are clearly too large. How large the error is depends on the difference between  $Aa$  and  $Cc$ , on the one hand, and  $Cv$ , on the other. Now we have seen (in Sec.  $Ic$ ) that these differences arise entirely from differences in the real income associated with the different curves; if real incomes differ little, so will the curves. Here is where Marshall's assumption about the fraction of expenditures devoted to the commodity enters the picture. If this fraction is small, the differences in real income will tend to be small, and both estimates will approach the correct value.<sup>30</sup> Since the error is

<sup>30</sup> This statement is not rigorous. As the fraction of expenditures devoted to the commodity diminishes, so will aggregate consumer's surplus. It is not enough that the error become small in absolute terms; its ratio to the correct value must become small. This, in general, will occur, as is well known. The chief qualification has to do with the behavior of a demand curve constructed under the current interpretation (e.g.,  $Cc$ ) for small quantities of  $X$ . The crucial question is the difference in real income between  $P$  and  $C$ . Expenditure on the commodity might be a small fraction of total expenditure at  $P$ ; yet, if the demand curve constructed under the current interpretation were extremely inelastic, not near  $C$ . In this case the difference in real income might be large.

This qualification is emphasized by Marshall. For example: "If however an amount  $b$  of the commodity is necessary for existence,  $f(z)$  [*sic*] [ordinate of the demand curve] will be infinite, or at least indefinitely great, for values of  $x$  less than  $b$ . We must therefore take life for granted, and estimate separately the total utility of that part of the supply of

larger for  $Aa$  than for  $Cc$ , it is clear that Marshall's assumption is, if anything, even more necessary on my interpretation of the demand curve than on the current one.<sup>31</sup>

#### IV. TEXTUAL EVIDENCE ON WHAT MARSHALL REALLY MEANT

Marshall's writings on demand bear on three different problems: (1) the defi-

the commodity which is in excess of absolute necessities" (p. 841). See also pp. 133 and 842.  $f(z)$  clearly should be  $f(x)$ , as it is in the first four editions of the *Principles*. See appendix to this paper.

This discussion of the role of the assumption that the commodity absorbs only a small fraction of income throws some light on an issue about which there has been considerable discussion, namely, whether Marshall assumed the marginal utility of money to be roughly constant with respect to a change in price or a change in income. The above analysis suggests that he assumed constancy with respect to a change in income. This is also Hicks's conclusion (*Value and Capital*, p. 40; "Rehabilitation," p. 109). Samuelson denies this and asserts that he assumed constancy with respect to a change in price (see Paul A. Samuelson, "Constancy of the Marginal Utility of Income," in *Studies in Mathematical Economics and Econometrics*, p. 80).

<sup>31</sup> The argument can be easily extended to "inferior" goods. The order of the three curves in Fig. 1 is then reversed; the estimates then become too small, instead of too large; but the error under the alternative interpretation remains larger in absolute value than under the current interpretation.

In the terminology used by Hicks in "The Four Consumers' Surpluses," what I have called the consumer's surplus is what Hicks calls the "quantity-compensating variation." The estimate of consumer's surplus derived from the demand curve constructed under my interpretation (the area  $APH$ ) Hicks calls the "quantity-equivalent variation." The area  $CDH$  in Fig. 1, Hicks calls the "price-compensating variation." Hicks's fourth concept, "price-equivalent variation," is not shown directly in Fig. 1. It is obtained by drawing a horizontal line through  $C$ . Designate by  $E$  the point at which this line cuts  $Aa$ . The "price-equivalent variation" is then equal to the area  $APH$  minus  $AEC$ . These relations can be checked by noting that curve  $mep$  in Hicks's Fig. 3 is  $Aa$  in our Fig. 1; his curve  $PCM$  is  $Cv$  in our Fig. 1. Further, in comparing the two figures, the part of Hicks's diagram for quantities less than  $hN$  should be neglected. That is, his point  $P$  is equivalent to our point  $C$ , his  $p$  to our  $P$ . Our Fig. 1 is also equivalent to Fig. 3B in Boulding, "The Concept of Economic Surplus."

inition of the demand curve—the problem of form; (2) the shape of the demand curve—the problem of content; and (3) the use of the demand curve—the problem of application. In his usual manner Marshall gives precedence to the problem of content and does not explicitly separate his discussion of content from his discussion of form. His definitions are characteristically given parenthetically and implicitly. He went to extreme lengths to present his tools in the context of concrete problems, so that definitions grew out of the uses to be made of them.<sup>32</sup> His discussion of utility and diminishing utility in the chapter of the *Principles* which introduces the concept of a demand curve (Book III, chap. iii, "Gradations of Consumers' Demand") is part of the discussion of content, even though it precedes his definition. It is the means whereby he rationalizes his "one general law of demand:—The greater the amount to be sold, the smaller must be the price at which it is offered in order that it may find purchasers."<sup>33</sup> It is not part of his definition of the demand curve.

Similarly, one of the major applications that Marshall made of the demand curve was his analysis of consumer's surplus. This analysis, too, must be distinguished from his definition of the demand curve. Assumptions made in his

<sup>32</sup> Cf. J. M. Keynes, *Memorials*, esp. pp. 33–38; see also Guillebaud, *op. cit.*

<sup>33</sup> *Principles*, p. 99. Note that on my interpretation this is truly a general law, not subject to the exceptions that have been made in recent literature. It depends for its validity only on (a) the postulate that consumers can be treated as if they behaved consistently and attempted to maximize some function of the quantity of commodities consumed; (b) the observed fact that consumers choose a higher income in preference to a lower, other things the same; and (c) the observed fact that consumers do not spend all their income on one commodity. For proof that a demand curve constructed on my interpretation must slope negatively see Slutsky, *op. cit.*, Sec. 8.

discussion of consumer's surplus cannot, without additional evidence, be supposed to apply equally to other applications of the "demand curve."

a) THE CENTRAL PASSAGES IN THE TEXT  
OF THE "PRINCIPLES"

The central passages in the text of the eighth and final edition of the *Principles* bearing on the other things to be kept the same seem to me to be three: one governing the entire volume, and two essentially parenthetical comments in his discussion of the demand curve:

We may throughout this volume neglect possible changes in the general purchasing power of money. Thus the price of anything will be taken as representative of its exchange value relatively to things in general [p. 62].

The larger the amount of a thing that a person has the less, other things being equal (i.e. *the purchasing power of money, and the amount of money at his command being equal*), will be the price which he will pay for a little more of it: or in other words his marginal demand price for it diminishes [p. 95; italics added].

The demand prices in our list are those at which various quantities of a thing can be sold in a market *during a given time and under given conditions*. If the conditions vary in any respect the prices will probably require to be changed; and this has constantly to be done when the desire for anything is materially altered by a *variation of custom, or by a cheapening of the supply of a rival commodity, or by the invention of a new one* [p. 100; second set of italics added].

For our purposes the critical part of the second quotation is the italicized parenthesis and, of the third, the second set of italicized phrases.

Though these quotations are taken from the eighth edition of the *Principles*, their substantive content is contained in Marshall's earliest published work on the theory of demand. All except the constancy of the purchasing power of money is in *The Pure Theory of (Domestic) Values*, printed for private circulation in

1879<sup>34</sup> but, according to Keynes, "substantially complete about 1873";<sup>35</sup> and the constancy of the purchasing power of money is in his and Mrs. Marshall's *The Economics of Industry*, published in 1879.<sup>36</sup> The actual wording of the first and third quotations can be traced back to the first edition of the *Principles* (1890), of the second quotation, to the second edition (1891).<sup>37</sup>

<sup>34</sup> Reprinted, together with the companion paper, *The Pure Theory of Foreign Trade*, by the London School of Economics and Political Science (1930).

<sup>35</sup> *Memorials*, p. 23.

<sup>36</sup> This work should not be confused with the condensation of the *Principles*, published, under the same title but with Alfred Marshall as sole author, in 1892.

<sup>37</sup> In all editions of the *Principles* the statement corresponding to the first quotation is in a subsection dealing with the meaning of the word "value." In the first (1890), second (1891), and third (1895) editions, the subsection on "value" is at the end of Book I, "Preliminary Survey," chap. i, "Introduction," and contains the statement: "Throughout the earlier stages of our work it will be best to speak of the exchange value of a thing at any place and time as measured by its price, that is, the amount of money for which it will exchange then and there, and to assume that there is no change in the general purchasing power of money" (p. 9, all three editions). In the first edition this assumption is repeated at the beginning of the chapter on "The Law of Demand" (Book III, chap. ii): "The purchasing power of this money may vary from time to time; but in these early stages of our work we assume it to be constant" (1st ed., p. 151). This repetition was eliminated in later editions, apparently in the process of introducing into the second edition the chapter on "Wants in Relation to Activities." In the fourth edition (1898), the subsection on "value" was split, part remaining at the end of Book I, chap. i, the remainder, including the material on the purchasing power of money, being transferred to end of Book II, "Some Fundamental Notions," chap. ii, "Wealth." The wording was changed to essentially its final form; the only difference is that the first sentence is in the passive voice, reading: "Throughout this volume possible changes in the general purchasing power of money will be neglected" (4th ed., p. 130). In the fifth edition (1907), the rest of the subsection on "value" was transferred to the end of Book II, chap. ii, and the quotation revised to its present

b) THE BEARING OF THESE PASSAGES ON  
THE TWO INTERPRETATIONS

The "other things" listed in the three passages cited above are as follows:

1. "Purchasing power of money"
2. "Amount of money at his command"

form; even the page number is the same in the fifth and eighth editions (p. 62).

In both editions of *The Economics of Industry*, subsection 4 in Book II, "Normal Value," chap. i, "Definitions. Law of Demand," contains essentially the same material as the subsection on "value" in the *Principles* referred to in the preceding paragraph, including the following statement: "But while examining the theory of Normal value we shall, for convenience, assume that the purchasing power of money remains unchanged. So that a rise or fall in the price of a thing will always mean a rise or fall in its general purchasing power or exchange value" (pp. 68-69). No corresponding statement appears in *The Pure Theory*.

The italicized parenthesis in the second quotation is identical in the second and all later editions of the *Principles*. The remainder of the quotation was worded as follows in the second edition: "An increase in the amount of a thing that a person has will, other things being equal . . . diminish his Marginal Demand-price for it" (p. 152). In the third edition, the words "marginal" and "demand" were not capitalized, and the hyphen was eliminated after "Demand" (p. 170). In the fourth edition the end of the statement was expanded to read, "diminish the price which he will pay for a little more of it: or in other words diminishes his marginal demand price for it" (pp. 169-70). In the fifth edition the quotation takes its present form, except for the addition of a comma, even the page number being the same as in the eighth edition (p. 95). In all editions from the second on, the indicated quotations are in Book III, chap. iii, the chapter first introducing the demand curve. This chapter is entitled "The Law of Demand" in the second and third editions, "Gradations of Demand" in the fourth, and "Gradations of Consumers' Demand" in the fifth and later editions.

The absence of the statement from the first edition reflects a difference in exposition, not in substance. As noted above, an explicit statement that the purchasing power of money is assumed constant appears in the chapter on "The Law of Demand" in the first edition. In all editions this chapter contains a statement covering the second part of the italicized parenthesis, which is worded as follows in the first edition: "Every increase in his resources increases the price which he is willing to pay for any given pleasure. And in the same way every diminution of his resources increases the

3. "Custom"
4. Price of "a rival commodity" (to avoid "cheapening of the supply of a rival commodity")

marginal utility of money to him, and diminishes the price that he is willing to pay for any pleasure" (p. 156). The only change in this statement in later editions was the substitution of "benefit" for "pleasure" (8th ed., p. 96).

*The Economics of Industry* also contains a statement anticipating the second part of the italicized parenthesis: "The price which he is willing to pay for a thing depends not only on its utility to him but also on *his means*; that is, the amount of money or general purchasing power at his disposal" (p. 70).

In all editions of the *Principles* the statement corresponding to the third quotation is in the final subsection of the chapter first introducing the demand curve (1st ed., Book III, chap. ii; in later editions, Book III, chap. iii). In the first edition it reads: "It must be remembered that the demand schedule gives the prices at which various quantities of a thing can be sold in a market during a given time and under given conditions. If the conditions vary in any respect the figures of the schedule will probably require to be changed. One condition which it is especially important to watch is the price of rival commodities, that is, of commodities which can be used as substitutes for it" (p. 160). A footnote is attached to the word "rival," the first sentence of which reads: "Or to use Jevons' phrase (*Theory of Political Economy*, Ch. IV), commodities that are nearly 'equivalent'" (1st ed., p. 160, n. 2).

The part of the second sentence of the third quotation following the semicolon assumed its final form in the second edition (p. 157), the footnote reference to Jevons being dropped. The rest of the quotation is the same in the second and third editions as in the first and assumes its final form in the fourth (p. 174). The change made in the second sentence from the first to the second edition argues that the list was not intended to be exhaustive, but illustrative. No change in substance is involved (see 1st ed., p. 155). In all editions the quoted statement is followed by the example of tea and coffee to illustrate the necessity of assuming the prices of rival commodities to be known; in the second edition the example of gas and electricity was added, and in the third edition the example of different varieties of tea. The passage itself, the changes in it, and the examples all indicate that Marshall considered the price of "rival" commodities particularly important. The examples, together with the footnote in the first edition, make it clear that he meant "close" rivals.

For a statement in the *Pure Theory* covering the substance of these quotations, except the constancy of the purchasing power of money, see n. 38, below.

5. Range of rival commodities available (to avoid "invention of a new one")<sup>38</sup>

1. *The current interpretation.*—The current interpretation of Marshall's demand curve treats item 2 as referring to the money income of the group of purchasers to whom the demand curve relates, item

<sup>38</sup> The adequacy of this list as a summary of Marshall's views may be checked by comparing it with two others in Marshall's writings. In *The Pure Theory of (Domestic) Values*, he writes: "The periods with which we are concerned . . . are sufficiently long to eliminate . . . casual disturbances. . . . But they are sufficiently short to exclude fundamental changes in the circumstances of demand and in those of supply. On the side of demand for the ware in question it is requisite that the periods should not include (i) any very great change in the prosperity and purchasing power of the community; (ii) any important changes in the fashions which affect the use of the ware; (iii) the invention or the great cheapening of any other ware which comes to be used largely as a substitute for it; (iv) the deficiency of the supply of any ware for which the ware in question may be used as a substitute, whether this deficiency be occasioned by bad harvests, by war, or by the imposition of customs or excise taxes; (v) a sudden large requirement for the commodity, as e.g. for ropes in the breaking out of a maritime war; (vi) the discovery of new means of utilizing the ware, or the opening up of important markets in which it can be sold" (p. 15).

Item i in this list presumably corresponds with 2 in my list; ii corresponds with 3, and iii and iv with 4 and 5, iii excluding a fall in the price of a rival commodity and iv a rise. Items v and the first part of vi would seem to be contained in 3 and largely redundant with ii. The rest of vi is presumably covered by the restriction of the discussion to a demand curve for a particular market.

The other list is in Marshall's discussion in the *Principles* of the difficulties of the statistical study of demand (Book III, chap. iv), where he writes: "Other things seldom are equal in fact over periods of time sufficiently long for the collection of full and trustworthy statistics. . . . To begin with, [a] the purchasing power of money is continually changing. . . . Next come the changes in [b] the general prosperity and in the total purchasing power at the disposal of the community at large. . . . Next come the changes due to [c] the gradual growth of population and wealth. . . . Next, allowance must be made for changes in [d] fashion, and taste and habit, for [e] the opening out of new uses of a commodity, for [f] the discovery or improvement or cheapening of other things that can be applied to the same uses with it" (*Principles*, pp. 109-10;

3 to their tastes and preferences, and item 4 to the price of every other commodity rather than of *rival* commodities alone. It ignores entirely items 1 and 5.

Item 2 is not entirely unambiguous. It might be interpreted as referring to the cash balances of the purchasers or to their wealth instead of, or in addition to, their income. On the whole, the most reasonable course seems to be to interpret it as referring to both income and wealth,<sup>39</sup> particularly since wealth qualifies for the list of "other things" by virtue of its possible importance as a factor affecting consumption. This expansion of the current interpretation does not alter it materially; it merely transfers "wealth" from the category of "other things" implicitly supposed to be the same to the list of things mentioned explicitly.

letters in brackets added). This statement dates from the first edition (pp. 170-71); only trivial editorial changes were made in later editions.

Item *a* in this list corresponds with 1 in my list; *b* with 2; *d* and presumably *e* with 3; and *f* with 4 and 5. Item *c* is presumably in part covered by restriction of the discussion to a demand curve for a particular market; in part it contains an item that may deserve to be added to the list, namely, "wealth." The wording of *f* is ambiguous, since it could refer to substitutes for the good in question, to complements, or to both. The subsequent text and the examples cited make it clear that it refers to substitutes; one example, of petroleum and petroleum lamps, itself ambiguously worded, suggests that it may refer to complements as well.

<sup>39</sup> In the quotations from Book III, chap. iv, in the preceding footnote, "wealth" is mentioned explicitly, though separately from "general prosperity" and "total purchasing power." See also the quotations in the fourth and fifth paragraphs of n. 37. Marshall repeatedly refers to "rich" and "poor" rather than to high- and low-income people (e.g., pp. 19, 95, 98). However, in an illustrative case, a rich man and a poor man are identified by their annual incomes (p. 10). And in Book III, chap. vi, he remarks: "We have throughout this and preceding chapters spoken of the rich, the middle classes and the poor as having respectively large, medium and small incomes—not possessions" (p. 134).

Item 3 requires no discussion, since the only reasonable interpretation of it is that it refers to tastes and preferences.<sup>40</sup>

The important defect of the current interpretation is its treatment of item 4, which is, in turn, responsible for the neglect of items 1 and 5. "Rival commodity" is replaced by, or read to mean, "any other commodity," and hence item 4 is taken to mean that the price of every other commodity is to be supposed the same. For example, Henry Schultz says, as if it were obvious and without citing any statements of Marshall: "Marshall also assumes, in giving definite form to the law of demand for any one commodity, that the prices of all other commodities remain constant."<sup>41</sup> Numerous other statements to the same effect could be cited. It is an amusing commentary on our capacity for self-delusion that the only references to Marshall for support that I have seen are to the page containing the third quotation in Section IVa above—the source of the words quoted in item 4.<sup>42</sup> The first set of italicized

<sup>40</sup> See n. 38, above. In discussing the law of diminishing marginal utility, Marshall says: "We do not suppose time to be allowed for any alteration in the character or tastes of the man himself" (p. 94).

<sup>41</sup> *Op. cit.*, p. 53. Immediately after making this statement he quotes from Edgeworth's article on "Demand Curves" cited in n. 5, above, not as evidence for the validity of his interpretation of Marshall but rather as an indication of the difficulties that it raises.

<sup>42</sup> Joan Robinson states without citation: "Marshall instructs us to draw up a demand schedule on the assumption that the prices of all other things are fixed" (*The Economics of Imperfect Competition* [London: Macmillan & Co., Ltd., 1934], p. 20). Paul Samuelson says, also without citation: "All other prices and income are held constant by *ceteris paribus* assumptions" in the "Marshallian partial equilibrium demand functions" (*Foundations of Economic Analysis* [Cambridge: Harvard University Press, 1947], p. 97). In an unpublished exposition of income and substitution effects prepared for class use about 1939, I stated, also without citation: "There is no question but that it [the Marshallian demand

words in that quotation are the only words on the page even remotely supporting the substitution of "any other" for rival. The specific examples that follow the quotation—tea and coffee, gas and electric lighting, different varieties of tea, beef and mutton—make it clear that Marshall was using the word "rival" in a narrow sense and not in that broad sense in which it may be said that all commodities are "rivals" for the consumer's income.<sup>43</sup> Whatever the merits of the current interpretation, it cannot be found explicitly in Marshall.

The interpretation of item 4 as referring to all other commodities makes item 5 unnecessary and contradicts item 1. Item 5 is unnecessary because the introduction of a new commodity is equivalent to a decline in its price from infinity to a finite amount; hence is ruled out if the price of every other commodity is to be unchanged. Item 1 is contradicted because, if all other prices are unchanged, the purchasing power of money will be lower, the higher the price of the commodity in question. The purchasing power of money cannot, therefore, be the same for all points on the demand curve.

The redundancy of item 5 on this interpretation of item 4 is unimportant; this item is in a list that is illustrative rather than exhaustive, and there is no reason why Marshall should have scrupulously

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curve] was not intended to be . . . interpreted" as "showing the effect of *compensated* variations in price." Similar statements, all citing p. 100 of the *Principles* as authority, are made by Robert Triffin, *Monopolistic Competition and General Equilibrium Theory* (Cambridge: Harvard University Press, 1940), p. 44; Ruby Turner Norris, *The Theory of Consumer's Demand* (New Haven: Yale University Press, 1941), p. 82; and Weintraub, *op. cit.*, p. 539.

<sup>43</sup> If any doubt remains, it is removed by the footnote in the first edition attached to the word "rival" referring to Jevons' phrase "commodities that are nearly 'equivalent'" (see n. 37, above).



pulously avoided overlapping. The logical inconsistency between items 1 and 4 cannot, however, be dismissed so lightly. Retention of the current interpretation requires either that item 1 be eliminated, on the grounds that the quotations on which it is based are exceptional and peripheral, or that Marshall be convicted of logical inconsistency on a fundamental point in his theory of demand.<sup>44</sup> Item 1 cannot, I think, be eliminated. The constancy of the purchasing power of money is clearly fundamental in Marshall's thought, probably more fundamental than any other item on our list.<sup>45</sup>

One excuse for retaining the current interpretation of Marshall, despite the logical inconsistency that it introduces, is to suppose that Marshall intended to restrict the use of his demand curve to commodities that account for only a small fraction of total expenditures. A change in the price of such a commodity would have only a small effect on the purchasing power of money, and it could be argued that Marshall neglected it as a "second-order effect." On this

<sup>44</sup> The extent to which the current interpretation dominates economic thought could not be more strikingly illustrated than by the fact that so acute an economic theorist as J. R. Hicks can write: "No doubt it [the constancy of the marginal utility of money] was . . . associated in his [Marshall's] mind with the assumption of a constant value of money (constant prices of other consumers' goods than the one, or sometimes ones, in question)" ("The Rehabilitation of Consumers' Surplus," p. 109). Hicks here treats constancy of all other prices as an alternative statement of item 1, when, in fact, it is logically inconsistent with item 1.

<sup>45</sup> See nn. 37 and 38, above. Note also that constancy of the purchasing power of money was a standard assumption of economic theory long before Marshall's day. It was made by Ricardo in his price theory, and Marshall refers to Cournot's discussion of the reasons for making this assumption (see Marshall, *Principles*, pp. ix, 62; Augustin Cournot, *Researches into the Mathematical Principles of the Theory of Wealth* [1838], Nathaniel Bacon translation [New York: Macmillan Co., 1897], p. 26).

rationalization, item 1 becomes redundant, but, in the limit, not logically inconsistent with an item 4 taken to refer to all other commodities.

I do not believe that Marshall intended to restrict the use of the demand curve to commodities accounting for only a small fraction of total expenditure. He speaks of a demand curve for wheat (p. 106), for housework (p. 107), and for other commodities that he cannot have regarded as unimportant. He first explicitly introduces the restriction to unimportant commodities in connection with his discussion of consumer's surplus, which comes well after the initial discussion of the demand curve—in the eighth edition, three chapters later; and the restriction is repeated at most points at which the argument depends on it. At one point the restriction is said to be "generally," not universally, justifiable. This evidence may not be conclusive but it certainly establishes a strong presumption that Marshall did not intend the restriction to carry over to all uses of the demand curve.<sup>46</sup>

<sup>46</sup> In connection with the discussion of consumer's surplus and the assumption of a constant marginal utility of money implicit in that discussion, Marshall says: "The assumption . . . underlies our whole reasoning, that the expenditure on any one thing . . . is only a small part of his whole expenditure" (p. 842). The first sentence of the paragraph from which this quotation is taken explicitly limits it to "the discussion of consumers' surplus" (p. 842). The quotation is followed by a cross-reference to the part of Marshall's famous analysis of the process by which equilibrium is reached in a corn-market in which he discusses "the latent assumption, that the dealers' willingness to spend money is nearly constant throughout" (p. 334). "This assumption," he says, "is justifiable with regard to most of the market dealings with which we are practically concerned. When a person buys anything for his own consumption, he generally spends on it a small part of his total resources" (p. 335).

Nowhere in Book III, chap. iii, does Marshall explicitly restrict his discussion to unimportant commodities. The one statement in that chapter that might be regarded as so restricting the dis-

It should be noted that Marshall's explicit introduction of the restriction to unimportant commodities has no bearing on the relative validity of the two interpretations of his demand curve. The restriction is necessary on either of the two interpretations at each point at which Marshall explicitly makes it. So the restriction cannot be regarded as called for by the inconsistency of items 1 and 4 on the current interpretation of 4.

2. *The alternative interpretation.*—My interpretation of the Marshallian demand curve resolves almost all the difficulties that plague the current interpretation, since it accepts at face value the five "other things" listed at the beginning of Section IV*b*. Marshall's words can be taken to mean what they say without uncomfortable stretching, and there is no logical inconsistency in the constancy of both item 1, the purchasing power of money, and item 4, the prices of rival commodities. Item 5, the range of rival commodities available, is still redundant, since, if "rival" has the same meaning in 4 and 5, the invention of a new rival commodity means a change in its price from infinity to a finite value.

My interpretation explains also the precise wording of the second quotation

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cussion is the statement on p. 95 that "the marginal utility of money to him is a fixed quantity." But the context argues and Note II in the Mathematical Appendix demonstrates that this is merely a verbal statement of an identity (if income is unchanged, so is marginal utility of money), and thus is not really relevant to the issue. In the eighth edition, Note II is referred to only at the end of the subsection following the paragraph containing the passage quoted. However, in the first edition, the corresponding note (Note III) is referred to at the end of the paragraph containing the passage quoted, and hence clearly covers it (pp. 155-56, 737-38).

The above quotations are essentially unchanged from the first edition on. The restriction to unimportant commodities is, however, mentioned neither in Marshall and Marshall, *Economics of Industry*, nor in the *Pure Theory*.

in Section IV*a*, which reads, in part: "The larger the amount of a thing that a person has the less . . . will be the price which he will pay for a little more of it." This is a curious form of phrasing on the current interpretation. Why emphasize the amount of a thing that a person has and the marginal expenditure that he can be induced to make rather than the amount he purchases and the average price he pays? On my interpretation, this phrasing follows directly from the argument of Section I*c* above (and Note II of Marshall's Mathematical Appendix), according to which a demand curve constructed on my interpretation can be viewed as showing the maximum price per unit that a person can be induced to pay for successive increments of the commodity.

One minor puzzle remains on my interpretation. Why does Marshall restrict his attention to "rival" commodities? Why not to "closely related" commodities, whether rivals or complements? His use of the word "rivals" in discussing the demand curve is apparently not a mere verbal accident. He uses the word repeatedly; almost all his examples deal with the effect of, or through, substitutes. I have no very good answer to this puzzle; the only one that seems at all persuasive is that he thought the concept of "joint demand" and the associated analytical apparatus better suited to problems involving complementary goods.<sup>47</sup>

My interpretation follows so directly

<sup>47</sup> In Note VII of the Mathematical Appendix, Marshall qualifies a suggested formula for combining consumer's surplus from different commodities by saying: "if we could find a plan for grouping together in one common demand curve all those things which satisfy the same wants, and are rivals; and also for every group of things of which the services are complementary (see Book V, chap. vi) . . ." (p. 842). Book V, chap. vi, contains the discussion of joint demand. The qualification quoted appears first in the third edition.

from Marshall's words that further defense of it would be unnecessary were it not for the unquestioned dominance of the current interpretation in the economic thinking and writing of the past half-century. This circumstance explains the presentation of additional textual evidence bearing on the validity of the alternative interpretation.

c) COUNTEREVIDENCE FROM THE TEXT OF  
THE "PRINCIPLES"

I have been able to find only one passage in the text of the eighth edition of the *Principles* that is in any way inconsistent with my interpretation of Marshall. This is the celebrated passage, adverted to above, which deals with the so-called "Giffen phenomenon" and which was first introduced in the third edition:

For instance, as Sir R. Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the *poorer labouring families* and raises so much the marginal utility of money to them, that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and bread being still the cheapest food which they can get and will take, they consume more, and not less of it [p. 132; italics added].

This passage clearly offsets an income effect against a substitution effect, whereas, on my interpretation of Marshall, real income is the same at all points on the demand curve, so there is no "income effect" (see Sec. IIb, above). The passage is thus in the spirit of the current interpretation. Yet the words I have italicized indicate that it does not necessarily contradict my interpretation of Marshall. The purchasing power of money and the real income of the community at large may remain constant; yet the real income of a particular group in the community that has a special consumption pattern may be adversely

affected by the rise in the price of a particular commodity.<sup>48</sup>

d) THE EVIDENCE OF THE MATHEMATICAL  
APPENDIX

The Mathematical Appendix to the *Principles* confirms and extends the evidence already presented from the text of the *Principles* and from Marshall's other writings. Note II (III in the first edition) explicitly derives a relation between price and quantity demanded that is identical with a demand curve on my interpretation, in which real income is kept constant by compensating variations in money income. Indeed, my derivation of such a demand curve in Section Ic above is a verbal paraphrase of Marshall's mathematics. Marshall does not explicitly say that the relation he derives is a demand curve, but Note II is attached to his initial discussion of the demand curve (Book III, chap. iii, in the eighth edition) and is given as the authority for statements made about the demand curve; hence there can be no doubt that it presents the pure theory of his demand curve.

In all editions of the *Principles* Note VI, attached to Marshall's discussion of consumer's surplus, contains a sentence that is definitely wrong on the current interpretation of his demand curve but correct on my interpretation.

Finally, a sentence added to Note VI in the third edition, referred to in the text of the *Principles* in connection with the material added on the Giffen phenomenon, contains an implicit mathematical proposition that is correct on the current interpretation but incorrect on my interpretation. The mathematical point in question is considerably more

<sup>48</sup> See Marshall's explicit discussion of, and emphasis on, this possibility in "Remedies for Fluctuations of General Prices" (1887), *Memorials*, p. 207.

subtle than those referred to in the two preceding paragraphs, so it cannot be given the same weight.

These two notes are examined in some detail in the appendix to this paper, to which the reader is referred for proof of the above statements.

e) A SYNTHESIS OF THE EVIDENCE

There are two differences between the current interpretation of Marshall's demand curve and my interpretation: (1) On the current interpretation, account is taken of the price of each other commodity individually; on my interpretation, only of the average price of all commodities other than the one in question and its close rivals. (2) On the current interpretation, real income varies along the demand curve with the price of the good in question; on my interpretation, real income is constant along the demand curve.

On the first, and less important, point, it is mathematically convenient to consider each other price separately, and this procedure might well have recommended itself to the writer of mathematical Notes XIV and XXI. On the other hand, it is impossible to consider each price separately in a practical analysis; so the use of an average price would clearly have recommended itself to the writer of the text of the *Principles* and is entirely in the spirit of Marshall's explicit methodological statements (see Sec. IIa, above). Marshall does not discuss this point explicitly; hence the textual evidence is all indirect.

On the second and basic point of difference the evidence leaves little room for doubt: Marshall's theory of demand, in the form in which it is presented in the first edition of the *Principles*, is explicitly based on constancy of real income along the demand curve. This interpre-

tation not only is consistent with both the letter and the spirit of the entire text of the first edition of the *Principles* but is almost conclusively established by the evidence cited above from two notes in the Mathematical Appendix of the first edition. In his determined effort to be persuasive and to make his work accessible to educated laymen, Marshall might well have been vague in his verbal presentation, though even there it seems unlikely that he would have been logically inconsistent. It is hardly credible that he would have been not merely vague but downright wrong on simple mathematical points stated in mathematical language, especially since the mathematical points in question could hardly even have arisen if he had been explicitly using the current interpretation of the demand curve.

I am inclined to believe, however, that by the time Marshall made the revisions incorporated in the third edition of the *Principles*—presumably between 1891, when the second edition appeared, and 1895, when the third edition appeared—he had himself been influenced by the current interpretation, probably without realizing that it was different from his own. This conjecture is based primarily on the two passages cited above as inconsistent with my interpretation: the passage dealing with the Giffen phenomenon and the last sentence of Note VI of the Mathematical Appendix. Both were added in the third edition—and these are the only passages I have been able to find in any edition of the *Principles* that fit the current interpretation better than they fit my interpretation. Further, both show some evidence of confusion about the fine points of his theory of demand (see last paragraph of appendix to this paper).

The hypothesis that Marshall did not

recognize the contradiction between the current interpretation and his earlier work would hardly be tenable if the lapse of time between the work incorporated in the first and the third editions of the *Principles* were as short as between their publication. But, as already noted, this is not the case. The essence of both his theory of demand and his analysis of consumer's surplus is contained in the *Pure Theory of Domestic Values*, which, though not printed until 1879, "must have been substantially complete about 1873."<sup>49</sup> The one important point in the theory of demand that is not in the *Pure Theory*—explicit mention of constancy of the purchasing power of money—is in the 1879 *Economics of Industry*. The only important addition in the *Principles* is the concept of "elasticity of demand"; and even this concept, which is not relevant to the present problem, was worked out in 1881–82.<sup>50</sup> No important substantive changes were made in the theory of demand in successive editions of the *Principles*, though the exposition was amplified and rearranged, the wording changed in detail, and some examples modified. The only important change of substance introduced into the discussion of consumer's surplus (in the third edition) was in connection with a point that has no bearing on the present issue.<sup>51</sup>

<sup>49</sup> Keynes, *Memorials*, p. 23.

<sup>50</sup> *Ibid.*, p. 39, n. 3.

<sup>51</sup> This change does not reflect favorably on Marshall's willingness to admit error. The first edition states: "Subject to these corrections then we may regard the aggregate of the money measures of the total utility of wealth as a fair measure of that part of the happiness which is dependent on wealth" (pp. 179–80), the corrections referred to being for "differences in the wealth of different purchasers" (p. 178) and "elements of collective wealth which are apt to be overlooked" (p. 179). A footnote to the first quotation refers to mathematical Note VII, in which he says, subject to the same two qualifications: "if  $a_1, a_2, a_3 \dots$  be the amounts consumed of the several commodities of which

Marshall himself writes: "My main position as to the theory of value and distribution was practically completed in the years 1867 to 1870. . . . By this time [from the context, 1874] I had practically completed the whole of the substance of my Mathematical Appendix."<sup>52</sup> Thus Marshall appears to have completed his fundamental work on the theory of demand in the early 1870's and to have made no important substantive changes thereafter. The third edition appeared some twenty or more years later—an ample lapse of time for the precise details of an essentially mathematical analysis to have become vague and their difference from a superficially similar set of details to pass unnoticed. This seems especially plausible in view of the acceptance of the current interpretation by others and the absence of controversy about it.

$b_1, b_2, b_3 \dots$  are necessary for existence, if  $y = f_1(x)$ ,  $y = f_2(x)$ ,  $y = f_3(x) \dots$  be the equations to their demand curves . . . , then the total utility of his wealth, subsistence being taken for granted, is represented by

$$\sum \int_b^a f(x) dx''$$

(1st ed., p. 741).

The eighth edition does not contain the first statement. Instead, the text contains an explicit warning against adding consumer's surpluses for different commodities, and a footnote says: "Some ambiguous phrases in earlier editions appear to have suggested to some readers the opposite opinion" (p. 131). Note VII in the Mathematical Appendix was modified by replacing "his wealth" by "income" and, of more importance, "is represented" by "might be represented" and by adding after the formula the significant qualification, "if we could find a plan for grouping together in one common demand curve all those things which satisfy the same wants, and are rivals; and also for every group of things of which the services are complementary. . . . But we cannot do this; and therefore the formula remains a mere general expression, having no practical application" (p. 842). As noted, these changes date from the third edition.

<sup>52</sup> Letter to J. B. Clark, *Memorials*, p. 416.

Further circumstantial evidence that Marshall did not recognize the contradiction between the current interpretation and his earlier work is provided by the apparent absence of any explicit discussion of the question in the writings of either Marshall or the more prominent of his students or even of any comments that could reasonably be interpreted as implying recognition of the existence of alternative interpretations of the demand curve. Yet, as noted earlier (n. 5), the current interpretation is explicitly given by Edgeworth as early as 1894 in an article on "Demand Curves" in *Palgrave's Dictionary of Political Economy* that Marshall must be presumed to have read. Though the assumption of constant prices of commodities other than the one in question is not explicitly attributed to Marshall, most of the article is based on Marshall; and there is no suggestion that this assumption does not apply to Marshall's demand curve. Further, Walras' definition of the demand curve, which presumably influenced Edgeworth, is identical with the current interpretation of Marshall's demand curve, and Marshall refers to Walras several times in the first edition of the *Principles*, though it seems clear that Marshall developed his theory of demand independently of Walras.<sup>53</sup> So Marshall must have been exposed to a definition of the demand curve corresponding to the current interpretation at a time when he was still making substantial revisions in the *Principles*. If he had recognized that this interpretation was incorrect, would he not have taken the opportunity to clarify his statements in later editions?

<sup>53</sup> *Principles* (1st ed.), pp. xi, xii, 425; Keynes, *Memorials*, pp. 19-24; Marshall's letter to J. B. Clark, *ibid.*, pp. 416-18.

#### V. ALTERNATIVE CONCEPTIONS OF ECONOMIC THEORY

There remains the mystery how the current interpretation of Marshall's demand curve gained such unquestioned dominance at so early a date and retained it so long, not only as an interpretation of Marshall, but also as "the" definition of "the" demand curve.

One obvious explanation is that mathematical economists were more likely than others to state explicitly and precisely their assumptions about the behavior of other prices; that mathematical economists were likely to be familiar with Walras' independent definition and to take it as a point of departure; and that, in any event, the current interpretation is mathematically more convenient. Other economists, it could be argued, followed the lead of the mathematical economists, and thus the current interpretation was taken for granted and accepted without question.

This explanation seems to me a significant part of the answer; however, I do not believe that it is the entire answer. If, as I have argued above, my interpretation of Marshall is more useful for most practical problems, why has its use been so rarely proposed; why has there been no general feeling of dissatisfaction with the current interpretation? There must, it would seem, be something about the role that has been assigned to economic theory that has made the current interpretation acceptable.

I am inclined to believe that this is, in fact, the case; that, by slow and gradual steps, the role assigned to economic theory has altered in the course of time until today we assign a substantially different role to theory than Marshall did. We curtsy to Marshall, but we walk with Walras.

The distinction commonly drawn between Marshall and Walras is that Marshall dealt with "partial equilibrium," Walras with "general equilibrium." This distinction is, I believe, false and unimportant. Marshall and Walras alike dealt with general equilibrium; partial equilibrium analysis as usually conceived is but a special kind of general equilibrium analysis—unless, indeed, partial equilibrium analysis is taken to mean erroneous general equilibrium analysis. Marshall wrote to J. B. Clark in 1908: "My whole life has been and will be given to presenting in realistic form as much as I can of my Note XXI."<sup>54</sup> Note XXI, essentially unchanged from the first edition of the *Principles* to the last, presents a system of equations of general equilibrium. It ends with the sentence: "Thus, however complex the problem may become, we can see that it is theoretically determinate, because the number of unknowns is always exactly equal to the number of equations which we obtain."<sup>55</sup> The explanation given above why Marshall might have decided to hold the purchasing power of money constant was entirely in terms of constructing the demand curve so that it would be consistent with general equilibrium in those parts of the system not under direct study.

The important distinction between the conceptions of economic theory implicit in Marshall and Walras lies in the purpose for which the theory is constructed and used. To Marshall—to repeat an expression quoted earlier—economic theory is "an engine for the discovery of concrete truth." The "economic organon" introduces "systematic and organized methods of reasoning." Marshall wrote:

<sup>54</sup> *Memorials*, p. 417.

<sup>55</sup> *Principles*, p. 856. This note was numbered XX in the first edition.

Facts by themselves are silent. . . . The most reckless and treacherous of all theorists is he who professes to let facts and figures speak for themselves, who keeps in the background the part he has played, perhaps unconsciously, in selecting and grouping them, and in suggesting the argument *post hoc ergo propter hoc*. . . . The economist . . . must be suspicious of any direct light that the past is said to throw on problems of the present. He must stand fast by the more laborious plan of interrogating facts in order to learn the manner of action of causes singly and in combination, applying this knowledge to build up the organon of economic theory, and then making use of the aid of the organon in dealing with the economic side of social problems.<sup>56</sup>

Economic theory, in this view, has two intermingled roles: to provide "systematic and organized methods of reasoning" about economic problems; to provide a body of substantive hypotheses, based on factual evidence, about the "manner of action of causes." In both roles the test of the theory is its value in explaining facts, in predicting the consequences of changes in the economic environment. Abstractness, generality, mathematical elegance—these are all secondary, themselves to be judged by the test of application. The counting of equations and unknowns is a check on the completeness of reasoning, the beginning of analysis, not an end in itself.

Doubtless, most modern economic theorists would accept these general statements of the objectives of economic theory. But our work belies our professions. Abstractness, generality, and mathematical elegance have in some measure become ends in themselves, criteria by which to judge economic theory. Facts are to be described, not explained. Theory is to be tested by the accuracy of its "assumptions" as photographic descrip-

<sup>56</sup> The quotations are all taken from Marshall, "The Present Position of Economics" (1885), *Memorials*, pp. 159, 161, 164, 166, 168, 171.

tions of reality, not by the correctness of the predictions that can be derived from it. From this viewpoint the current interpretation of the demand curve is clearly the better. It is more general and elegant to include the price of every commodity in the universe in the demand function rather than the average price of a residual group. Any price may affect any other, so a demand equation including every price is a more accurate photographic description. Of course, it cannot be used in discovering "concrete truth"; it contains no empirical generalization that is capable of being contradicted—but these are Marshallian objections. From the "Walrasian" viewpoint, to take one other example from recent developments in economic theory, it is a gain to eliminate the concept of an "industry," to take the individual firm as the unit of analysis, to treat each firm as a monopoly, to confine all analysis to either the economics of the individual firm or to a general equilibrium analysis of the economy as a whole.<sup>57</sup> From the Marshallian viewpoint this logical terminus of monopolistic competition analysis is a blind alley. Its categories are rigid, determined not by the problem at hand but by mathematical considerations. It yields no predictions, summarizes no empirical generalizations, provides no useful framework of analysis.

Of course, it would be an overstatement to characterize all modern economic theory as "Walrasian" in this sense. For example, Keynes's theory of employment, whatever its merits or demerits on other grounds, is Marshallian in method. It is a general equilibrium theory containing important empirical content and constructed to facilitate meaningful prediction. On the other hand, much recent work based on

Keynes's theory of employment is Walrasian.<sup>58</sup>

## VI. CONCLUSION

Modern economic theory typically defines the demand curve as showing the relation between the quantity of a commodity demanded and its price for given tastes, money income, and prices of other commodities. This definition has also been uniformly accepted as a correct interpretation of the demand curve defined and used by Alfred Marshall in his *Principles of Economics*. Rarely has the view been expressed that a different definition would be preferable.

Despite its unquestioned acceptance for over half a century, this interpretation of Marshall is, in my view, wrong. Marshall's early writings, the text of the *Principles*, and, even more definitely, the Mathematical Appendix provide almost conclusive proof that Marshall's demand curve differs in two respects from the one commonly used and attributed to him: first, commodities other than the one in question and its close rivals are treated as a group rather than individually, and only their average price is explicitly taken into account; second, and far more important, real income is considered the same at all points on the demand curve, whereas constant money income and other prices imply a higher real income, the lower the price of the commodity in question. Two variants of Marshall's demand curve can be distinguished: one, employed in the text of the *Principles*, uses variations in the prices of other commodities to compensate for variations in the price of the commodity in question and thereby keeps the purchasing power of money constant; the other, employed in the Mathematical

<sup>58</sup> O. Lange, *Price Flexibility and Employment* (Bloomington, Ind.: Principia Press, 1944), is perhaps as good an example as any.

<sup>57</sup> See Triffin, *op. cit.*, pp. 188-89.



Appendix, uses variations in money income to compensate for variations in the price of the commodity in question.

The only textual evidence that conflicts with this interpretation is a passage in the text and a related sentence in the Mathematical Appendix that were added to the third edition of the *Principles*. The inconsistency of these with the rest of the *Principles* can be explained by the hypothesis that Marshall himself was after a point influenced by the current interpretation of the demand curve without recognizing its inconsistency with his earlier work. Some circumstantial evidence also supports this hypothesis.

The alternative interpretation of the demand curve not only is faithful to both the letter and the spirit of Marshall's work but also is more useful for the analysis of concrete problems than is the demand curve commonly employed. The acceptance of a less useful definition seems to me to be a consequence of a changed conception of the role of theory in economic analysis. The current interpretation of the demand curve is Walrasian; and so is current economic theory in general.

#### APPENDIX ON TWO NOTES IN THE MATHEMATICAL APPENDIX TO THE *PRINCIPLES*

##### I. NOTE II OF THE EIGHTH EDITION

This note is numbered III in the first edition of the *Principles*, II in the rest. In the first edition the relevant parts are worded as follows (pp. 737-38):

"If  $m$  is the amount of money or general purchasing power at a person's disposal at any time, and  $\mu$  represents its total utility to him, then  $d\mu/dm$  represents the marginal utility of money to him.

"If  $p$  is the price which he is just willing to pay for an amount  $x$  of the commodity which gives him a total pleasure  $u$ , then

$$\frac{d\mu}{dm} \Delta p = \Delta u ; \quad \text{and} \quad \frac{d\mu}{dm} \frac{dp}{dx} = \frac{du}{dx} \dots$$

"Every increase in his means diminishes the marginal utility of money to him; . . .

"Therefore,  $du/dx$ , the marginal utility to him of an amount  $x$  of a commodity remaining unchanged, an increase in his means . . . increases  $dp/dx$ , that is, the rate at which he is willing to pay for further supplies of it. Treating  $u$  as a variable, that is to say, allowing for possible variations in the person's liking for the commodity in question, we may regard  $dp/dx$  as a function of  $m$ ,  $u$ , and  $x$  . . ."

The wording in the eighth edition is identical except that "marginal utility of money" is replaced by "marginal degree of utility of money" and that " $du/dx$ " and the words "Treating  $u$  . . . in question" are omitted from the last paragraph quoted (pp. 838-39). The changes were first made in the third edition.

In the second sentence of this note, the word "price" is to be interpreted as "total amount" not as "price per unit." This is clear from the context and is demonstrated by the equation that follows and the designation of  $dp/dx$  as "the rate at which he is willing to pay for further supplies of it." The words "just willing" in the second sentence and the equations that follow demonstrate that  $p$  is the maximum amount he can pay for an amount  $x$  and have the same utility as if he had none of the commodity. Thus Marshall is describing a process like that outlined in Section Ic of this paper, whereby the maximum possible amount is extracted from the individual for each successive increment of the commodity, the individual retaining the same "real income," that is, remaining on the same indifference curve, throughout the process.

The last sentence quoted shows that  $u$  is to be regarded as a parameter to allow for changes in tastes. The rest of that sentence simply describes a function like that obtained by eliminating  $y'$  from equations (5) and (6) of note 6 of this paper. The parameter  $m$  in Marshall's function takes the place of  $U_0$  in our footnote, since  $dp/dx$  is still to be regarded as the price per unit paid for an additional increment of the commodity rather than as the price per unit at which any amount can be purchased. In consequence, no explicit statement is needed as yet about the compensating variations in income that are implicit in Marshall's analysis.

The word "demand" does not appear in this note. But the note is attached to the chapter in the *Principles* in which Marshall first introduces the demand curve (Book III, chap. ii, in

the first edition; Book III, chap. iii, in later editions) and is cited as proof of statements about the demand curve; hence there can be no doubt that the "function" mentioned in the last sentence quoted is the counterpart of Marshall's demand curve.

I have been able to construct no interpretation of this note that would render it consistent with the current interpretation of Marshall's demand curve.

## 2. NOTE VI

This note has the same number in all editions. In the first edition the relevant parts are worded as follows (p. 740):

"If  $y$  be the price at which an amount  $x$  of a commodity can find purchasers in a given market, and  $y = f(x)$  be the equation to the demand-curve, then the total utility of the commodity is measured by  $\int_0^a f(x) dx$ , where  $a$  is the amount consumed.

"If however an amount  $b$  of the commodity is necessary for existence,  $f(x)$  will be infinite, or at least indefinitely great, for values of  $x$  less than  $b$ . We must therefore take life for granted, and estimate separately the total utility of that part of the supply of the commodity which is in excess of absolute necessities: it is of course  $\int_b^a f(x) dx \dots$

"It should be noted that, in the discussion of Consumers' Rent, we assume that the marginal utility of money to the individual purchaser is the same throughout. . . ."

Only trivial changes were made in these sentences in subsequent editions: a typographical error in the fifth edition, which remained uncorrected thereafter, substituted  $f(z)$  for  $f(x)$  in the second sentence; and "consumers' surplus" replaced "Consumers' Rent." In the third edition the following sentence was added at the end of the note:

"If, for any reason it be desirable to take account of the influence which his expenditure on tea exerts on the value of money to him, it is only necessary to multiply  $f(x)$ , within the integral given above by that function of  $xf(x)$  (i.e. of the amount which he has already spent on tea) which represents the marginal utility to him of money when his stock of it has been diminished by that amount" (3d ed., p. 795). The only subsequent changes were the addition of a comma after "reason" and the deletion of the comma before "within" (8th ed., p. 842).

In its final form Note VI seems internally inconsistent: the second sentence is wrong on the current interpretation of Marshall's demand curve, correct on my interpretation; the final sentence, added in the third edition, seems correct on the current interpretation, wrong on my interpretation.

### a) THE SECOND SENTENCE

The second sentence is wrong on the current interpretation, which holds money income and other prices constant along the demand curve, since the ordinate of the demand curve for any quantity  $x$  cannot then exceed money income divided by  $x$ , and this is not "indefinitely great" for a fixed value of  $x$ —say,  $x_0$ —whether  $x_0$  is greater or less than  $b$ . True,  $f(x)$  might approach infinity as  $x$  approaches zero, but this is not what Marshall says; he says it is "indefinitely great, for values of  $x$  less than  $b$ ," i.e., for any particular value of  $x$  less than  $b$ —say,  $x_0 = 0.99b$ .

On the variant of my interpretation involving compensating variations in money income—the variant that the note numbered II in the eighth edition leads me to believe Marshall used in the Mathematical Appendix—this sentence is entirely valid. As  $x$  declines from a value larger than  $b$ , the compensating variation in money income required to keep the individual's real income the same becomes larger and larger, approaching infinity as  $x$  approaches  $b$ , the minimum amount necessary for existence. This permits the ordinate of the demand curve likewise to approach infinity as  $x$  approaches  $b$ . On the variant involving compensating variations in other prices—the one Marshall used in the text—the definition of the demand curve breaks down for values of  $x$  less than  $b$ : for a finite price of the commodity in question, sufficiently high so that the given money income could purchase only a quantity  $x$  less than  $b$ , there will exist no set of non-negative prices for the remaining commodities that will keep the purchasing power of money constant in the sense of enabling the same money income to provide the same level of utility; money income and real income cannot both be held constant and at the same time all prices be kept nonnegative. This sentence can therefore be defended as valid on either variant of my interpretation.

One possible ground for dismissing this sentence as evidence against the current inter-

pretation is that the so-called "error" on that interpretation is of my own making, arising from a too subtle and too literal reading of the note. Marshall, it could be argued, was using "demand curve" to mean "utility curve" and  $f(x)$  to mean "marginal utility," and therefore he did not consider whether the sentence would be valid if  $f(x)$  were to be interpreted literally as the ordinate of the demand curve. A note that Marshall published in 1893 on "Consumer's Surplus" could be cited as evidence for this contention. In this note he quotes part of Note VI as follows: "If, however, an amount  $b$  of the commodity is necessary for existence, [the utility of the first element]  $a$  will be infinite."<sup>59</sup> The bracketed expression that Marshall substituted for  $f(x)$  would support the notion that he was using "demand curve" and "utility curve" interchangeably.

I do not myself accept this argument; it seems to me to do much less than justice to Marshall. In the first place, I am inclined to give little weight to an incidental, explanatory, phrase inserted by Marshall as late as 1892 or 1893, some twenty years after the fundamental analysis incorporated in Note VI had been completed. I have noted above and shall presently cite evidence that Marshall may have been somewhat confused about the fine points of his own theory of demand by the early 1890's. In the second place, and more important, Marshall clearly distinguishes in the earlier notes in the Mathematical Appendix between a utility curve and a demand curve, repeatedly using the word "utility," and in the first sentence of Note VI says that "the total utility of the commodity is *measured* by  $\int_0^a f(x) dx$ " (1st ed., p. 740; italics added). If he had been using  $f(x)$  to stand for marginal utility, the words I have italicized could have been omitted. Finally, Note VI, like most of the rest of the Mathematical Appendix, summarizes a subtle, closely reasoned, and by means obvious, mathematical argument, in which, so far as I know, few errors have ever been found. Is it credible that it would have been worded as loosely and carelessly as the argument being criticized

<sup>59</sup>"Consumer's Surplus," *Annals of the American Academy of Political and Social Science*, III (March, 1893), 618-21 (brackets in original). This note is a reply to some comments by Simon Patten. The letter  $a$  after the brackets which appears in the *Annals* note does not appear in the *Principles*, and I can explain it only as a typographical error.

requires; or that, if at one stage it had been, Marshall would have failed to see the simple mathematical error implicit in a literal reading of his words on the current interpretation of the demand curve? It seems to me far more credible that he meant what he said and that the correctness of what he said on my interpretation of his demand curve is strong evidence for that interpretation.

#### b) THE FINAL SENTENCE

The explanation that follows of the final sentence added to Note VI in the third edition, though not completely satisfactory, is reasonably so, and I have been able to construct no other even remotely satisfactory explanation.

Let  $U$  be the utility function of the "individual purchaser" and  $U_x$  the marginal utility of  $x$  units of tea to him, i.e., the partial derivative of  $U$  with respect to  $x$ . Now the increase in utility attributable to having  $a$  rather than  $b$  units of tea—consumer's surplus in utility units—is given by

$$\int_b^a U_x dx, \quad (1)$$

where the integral is computed for constant quantities of other commodities equal to the amounts consumed when  $a$  units of tea are consumed and other conditions are those corresponding to the demand curve  $y = f(x)$ .

At every point along the demand curve,

$$U_x = ny = n(x) f(x), \quad (2)$$

where  $n$  is the marginal utility of money—*itself*, of course, a function of  $x$  along the demand curve. Integrating both sides of equation (2) gives

$$\int_b^a U_x dx = \int_b^a n(x) f(x) dx. \quad (3)$$

The left-hand side of equation (3) is symbolically identical with equation (1); yet there is an important difference between them. In equation (1),  $U_x$  is computed, holding the quantities of other commodities constant as  $x$  varies; in equation (3),  $U_x$  is computed, holding constant whatever is held constant along the demand curve (money income and other prices on the current interpretation; real income on my interpretation). In general, quantities of other commodities vary along the demand curve (on either interpretation), and  $U_x$  may depend on the quantities of other commodities, so the

$U_x$  in equation (3) may be numerically different from the  $U_x$  in equation (1) for a value of  $x$  other than  $a$ . This difficulty disappears if  $U_x$  is supposed to be independent of the quantities of other commodities—an assumption that Marshall pretty clearly makes as a general rule (e.g., see Nn. I and II of the Mathematical Appendix). On this assumption, then, the right-hand side of equation (3) measures consumer's surplus in utility units.

It is at this point that difficulties of interpretation arise; for the right-hand side of equation (3) is obtained by multiplying " $f(x)$  within the integral given above by that function of"  $x$  "which represents the marginal utility . . . of money." Why does Marshall say "that function of  $xf(x)$ " rather than of  $x$  alone? And is it valid to make this substitution? One can argue that to each value of  $x$  there corresponds a value of  $f(x)$  and hence of  $xf(x)$ , so that the two forms of statement are equivalent: Marshall has simply made the transformation  $z = xf(x)$  and converted  $n(x)$  into  $n(z)$ . This argument is not, however, rigorous. In general,  $x$  will not be a single-valued function of  $z$ ; hence to any given value of  $z$  there may correspond more than one value of  $x$  and hence more than one value of  $n$ . The two forms of statement are equivalent if and only if  $n$  is a single-valued function of  $z$ , i.e., if  $n(x)$  is the same for all values of  $x$  for which  $xf(x)$  is the same.

Given independence between the marginal utility of tea and the quantity of other commodities, this condition is always satisfied on the current interpretation of the demand curve but not on the alternative interpretation. Let  $x'$  stand for the quantity of a composite commodity representing all commodities other than tea,  $y'$  for its price, and  $U_{x'}$ , for its marginal utility. At each point on the demand curve,

$$\frac{U_x}{y} = \frac{U_{x'}}{y'} = n.$$

On the current interpretation of the demand curve, money income and the prices of other commodities are the same for all points on the demand curve. It follows that, for all values of  $x$  that yield the same value of  $xf(x)$ , the same amount will be spent on other commodities; so  $x'$  is the same (since  $y'$  is by definition); so  $U_{x'}$  is (since, on the assumption of independence,  $U_{x'}$  depends only on  $x'$ ); and so  $n$  is. Marshall's use of  $xf(x)$  instead of  $x$  is thus valid on the current interpretation of the demand curve.

On my interpretation, either money income varies along the demand curve, so as to keep real income constant, or other prices do; hence the preceding argument is no longer valid. That the two forms of statement are no longer always equivalent can be shown by a counterexample. If other prices are held constant and compensating variations of income are used to keep real income constant,  $U = \sqrt{x} + \sqrt{x'}$  is a utility function that gives different values of  $n$  for different values of  $x$  yielding the same value of  $xf(x)$ . If money income is held constant and compensating variations of other prices are used to keep real income constant,  $U = 3 + x - \frac{1}{10}x^2 + \sqrt{x'}$  is such a utility function. Hence Marshall's use of  $xf(x)$  instead of  $x$  is invalid on either variant of the alternative interpretation.

This explanation leaves a number of Marshall's verbal statements wrong or ambiguous, whichever interpretation of the demand curve is accepted. (1) The parenthetical explanation of the meaning of  $xf(x)$  seems wrong—why the word "already"? If one is thinking of going through the process of extracting as much as possible from the consumer for each successive unit of tea and is supposing the maximum price that he will pay for successive units to be given by the demand curve, then  $\int_b^x f(x)dx$  and not  $xf(x)$  is the amount he has "already spent on tea." If one is thinking of the amount spent on tea at a given price for tea, then  $xf(x)$  is the amount spent when the price is  $f(x)$ , not the amount "already spent." The explanation offered above accepts the latter rendering of the parenthesis, i.e., supposes the word "already" omitted. (2) The last clause—"when his stock of money has been diminished by that amount"—is ambiguous. To make it consistent with the explanation offered above, one must add "and tea is unavailable, so that the balance is spent solely on other commodities at the prices assumed in drawing the demand curve for tea." The reference to "stock of money" suggests that Marshall was supposing money income constant and so, independently of the rest of the quotation, would tend to rule out compensating variations in money income. It should be noted that there are no such ambiguities in the original version of Note VI, either in the parts quoted above or in the parts not quoted.