

The Course of Marshall's Theorizing about Demand

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Introduction

One way of reading Marshall on demand, especially the teasingly elusive book 3 of the *Principles of Economics*, is to draw on modern theory for conceptions of what he must have been getting at.¹ But Marshall's work is too unlike modern consumer theory for the result to be anything but disappointing. There is another way to acquire a sense of the possible by starting with Marshall's early ideas and watching them develop. A longer perspective can add much to such accounts as those of John K. Whitaker (1975), R. S. Howey (1960), or George J. Stigler (1950), which concentrate on a single period in Marshall's career.

The central theme in Marshall's work on demand is the interplay of a person's pleasure from acquiring a thing, the monetary valuation of this pleasure, the price of the thing, and the loss of pleasure from paying this price. These concerns converge on the notion of consumers' rent or surplus. This was more than an application of the theory. It was its *raison d'être*.

The passage introducing consumers' rent, which appears in all editions of the *Principles* from 1890 to 1920, literally embodies the history of

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1. Some of these readings are discussed below.

Marshall's theorizing. It starts by following the *Essay on Value* from around 1870, with its echoes of John Stuart Mill's *Principles of Political Economy*, and then switches to the *Pure Theory of Domestic Values* of 1879.²

The price which a person pays for a thing can never exceed, and seldom comes up to that which he would be willing to pay rather than go without it: so that the gratification which he gets from its purchase generally exceeds that which he gives up in paying away its price; and he thus derives from the purchase a surplus of pleasure. The excess of the price which he would be willing to pay rather than go without it, over that which he actually does pay, is the economic measure of this surplus pleasure. . . . (1890, 175; 1920, 124n).³

Continuity is not constancy. There was no primeval theory that was elaborated or just more fully revealed over the years.⁴ New lines of thought came on as old ones atrophied. Doctrines rose, for example, the larger the means, the smaller the loss of pleasure in paying the price, and doctrines fell, for example, the greater the availability of substitutes, the less the gratification from buying a thing.

Marshall's work impinged on the interests of contemporaries and successors, but his preoccupations were not widely shared. The result has been a history of misunderstanding through misassimilation. When Léon Walras greeted Marshall's acceptance of Stanley Jevons's doctrine of proportionality of final utilities to prices, he read into *The Economics of Industry* something that was not there.⁵ It was in the *Principles*, but it was not Marshall's "main argument" as Hicks (1939, 11–12) thought. The allocation of expenditure among goods was not a primary problem for Marshall. In some early work, he went beyond Jevons but he did not develop the analysis or publish the results.⁶

2. For the *Essay on Value*, see section 1 and for the *Pure Theory of Domestic Values*, see section 3.

3. In references to the *Principles*, for example, (1890, 175; 1920, 124n), the first location has the exact quote. The 1920 location has Marshall's final formulation of the passage. The 1920 reference also gives access to the notes in Guillebaud's 1961 edition, which has the same pagination as the 1920 edition.

4. Before *Early Writings* was published, the main source of information was Marshall's reminiscences. Friedman and others naturally accepted Marshall's claim to have "practically completed the whole of the substance of my Mathematical Appendix by 1874" (Friedman 1949, 488).

5. See section 2 below for the exchange between Marshall and Walras.

6. See section 2 below. In the *Principles* the paper is boiled down to a paragraph, and the

In sections 1 through 3 below I treat Marshall's early work. This phase ended in 1879 with the presentation of consumers' rent in association with final utility. Section 4 turns to the *Principles* and looks at the overall design of its treatment of demand. Sections 5 through 7 examine the three core chapters of book 3—the only chapters in the first edition. There were big changes in later editions.⁷ Most of the rearrangement and expansion occurred when the ragged and congested first edition was replaced by the most fluent and uninhibited of all the versions. However, the most significant changes came in the third edition when the bolder applications of an essentially single good analysis were smothered. Section 8 discusses these changes. A final section comments on Marshall's theorizing in relation to "Marshallian demand theory."

1. Value in Use and Demand⁸

Marshall's first publication was in 1872. In a letter to J. B. Clark dated 1908 (reproduced in Pigou 1925a, 416), he recalled "my main position as to the theory of value and distribution was practically completed in the years 1867 to 1870." Recreating his early views involves much guesswork. The surviving manuscripts give an incomplete record of his thinking and a person's recollections are usually in some part rationalization.

The position represented by the *Essay on Value*, the most finished of the relevant manuscripts, indeed anticipates his mature position in certain respects. Unlike the "modern consumer," Marshall's buyer is not allocating expenditure between commodities but buying, or not buying, a particular one: "A buyer is a person who wishes to obtain some particular commodity in exchange for command over commodities in general. The amount of this command [called the price] which he is willing to give in order to obtain it represents the value in use of it to him" (Whitaker 1975, 1:128).⁹ The commodity is exchanged for money which indicates

allocation analysis to a sentence; "it is commonly argued . . . a tax on expenditure is *prima facie* the best tax" (1890, 454; 1920, 475).

7. Book 3 occupied 36 pages in the first edition, 50 in the second, 55 in the third, 55 in the eighth. Other books also have relevant material.

8. For the *Essay*, see Bharadwaj 1978; Hicks 1976; Whitaker's editorial commentary (1:119–25); for general background, Whitaker 1:3–113.

9. References to Whitaker's edition of *Early Writings* will appear as (W, 1:128) for page 128 of volume 1.

“command over commodities in general.” The price is a “real price” not a “nominal price.”¹⁰

The *Essay* bases price determination on demand and supply curves. The demand curve represents the “price at which buyers can be found” for varying levels of quantity to be sold. Identifying the origins of this quantity-into-price formulation of demand, which reverses the order given by Mill and Cournot, illustrates the difficulty of interpreting the available record. The formulation harmonizes with the analogy underlying consumers’ rent and a utility theory developed in the direction of marginal demand price.¹¹ But the entire package appears only later. The quantity-into-price formulation may have been chosen on other grounds; thus the case of fixed supply can be dispatched in a sentence without considering with Mill (1848, 446) how equilibrium is reached.

The most striking change from Mill’s treatment—after the use of diagrams—is the salience given the notion of “value in use” in relation to both buying and selling. Some of what Marshall says about value in use is a paraphrase of Mill. But Marshall brings value in use into effective connection with demand, supply, and exchange value. Mill discussed it only to clear the way for his real concern—exchange value.

Marshall takes a proposition from Mill’s *Principles*:

Value in use . . . is the extreme limit of value in exchange. The exchange value of a thing . . . may fall short to any amount of its value in use; but that it can ever exceed the value in use implies a contradiction; it supposes that persons will give, to possess a thing, more than the utmost value which they themselves put upon it as a means of gratifying their inclinations (1848, 437)

and reformulates it as a proposition about a point on the demand curve. “It is obvious from the definition that the ‘value in use’ of the commodities to the buyers who between them take off the amount OM_2 cannot be less, though for many of them it may be much greater, than M_2P_2 [the price associated with the amount OM_2]” (W, 1:130).

Here “value in use” seems to be a price; the definition is less clear: “The value in use of a thing to a person is the value of the things which

10. Marshall uses “real price” for “command over commodities in general” and “nominal price” for command over money (Whitaker 1975, 1:128). Mill uses the terms (exchange) “value” and “price” to mark the same distinction (437–40). Neither wished to become entangled with changes in the value of money.

11. Gordon 1982 has a good account of this harmonization.

must be given him in order that he may be induced to give it up, or which he will give rather than not obtain it: in other words it is the amount of pleasure he would derive from obtaining it or of pain he would undergo from losing it" (W, 1: 125). Four quantities are presented as equivalent. The first pair seem to be prices, the seller's compensation price for parting with the object and the buyer's reservation price. The reservation price most closely corresponds to Mill's usage. The second pair are explicitly amounts of pleasure, not sums of money. Marshall is not yet making the distinction he later emphasizes between pleasure and its money measure (see below, section 8). He never made much of the distinction between gaining and losing except in the context of gambling.

Marshall recognizes that for any price there are marginal purchasers: "The price P_2M_2 represents the 'value in use' to those buyers who are the last induced to buy the commodity when an amount OM_2 per annum is sold, and who would not buy if the price were higher" (W, 1:143). There is a range of more or less eager purchasers each buying a unit of the commodity.¹² Demand curves slope downward because there are more buyers at lower prices. Perhaps this was too obvious to be worth elaboration.

Traders decide whether to trade now or to wait by comparing value in exchange with value in use. Marshall begins a detailed analysis of the seller's case: "How are we to estimate the value in use to the seller? . . . In general terms he will accept an offer if he thinks that if he refuses this he will not be able to get a better one" (W, 1:133). The seller's "estimate" is the mathematical expectation of the future price discounted to the present. The buyer's estimate is not worked out in detail: Similar considerations enter into the question of determining the "value in use to the buyer. Though as he is in general somewhat pressed for time, they are in general simpler in this case" (W, 1:133). Marshall does not identify the "simpler considerations" that determine the prospects of a better offer. However, when discussing Smith's paradox of value, he stressed the importance of the "terms on which, if [the buyer] refuses this thing, another like it will probably be offered to him" (W, 1:125). The buyer speculates about alternative options. By contrast, the formal analysis of the *Principles* weighs the need for the good now against need for money now.

12. Whitaker (1:45, 122) and Krishna Bharadwaj 1978 make this point.

The *Essay* taps what Marshall later calls “the full power which is latent, if not patent, in Mill’s work” (1876, 119). In his later writings, the latent power in Mill’s account of “value in use” is harnessed to consumers’ rent. This notion does not appear in the *Essay*—it does not appear in publishable form before 1879. Yet Marshall’s annotations to Cournot (in W, 2:246) and his mathematical notebooks contain applications of rent analysis to market demand that seem to be earlier than the *Essay*. The terminology is different from that of the *Essay*; being notes on a text, this may not be surprising. Terse descriptions of triangles under demand curves “representing satisfaction of new consumers” (an annotation to Cournot in W, 2:246) or “total loss” to those who will not pay a toll (from a mathematical notebook in W, 2:279) are little help in understanding what was behind such calculations.

Yet it is natural to see the otherwise inexplicable overelaboration of value in use in the *Essay* as backing for this analysis.¹³ Cournot (1838)—whom Marshall read around 1868—had not only used the market demand curve for explaining price determination but also for measuring the gains or losses associated with price changes. Mill’s value in use gave Marshall a tool for tackling a welfare issue that Cournot noticed in 1838 but thought untreatable: “we have not considered the loss experienced by that class of home consumers who stop buying the dearer commodity, and who thus make a use less to their liking of a part of their incomes. This loss, as has been explained, is not capable of measurement” (1963, 130). For Cournot the loss belongs to the region of “moral causes capable of neither enumeration nor measurement” (39). At this time, Marshall was teaching Bentham, who took a different view of the extent—or existence—of this region.¹⁴

Some features of his mature position are present in the *Essay on Value*. The analysis is a partial equilibrium analysis of price determination. Value in use is presented in association with the demand curve and seems designed to complement consumers’ rent analysis. This was new—in neither Mill nor Cournot—and it lasted. His views on what determines a good’s value in use for its buyer were new but did not last; the views that replaced them owed much to Jevons.

13. As Whitaker points out, in the case of purchasing a single unit of a commodity, the consumers’ rent is the difference between the value in use and the value in exchange (W, 1:45).

14. I owe this suggestion to a referee.

2. Value in Use and Final Utility

Marshall's first publication was an unwelcoming review of Jevons's *Theory of Political Economy*. His recollection to Clark continues, "when Jevons' book appeared I knew at once how far I agreed with him and how far I did not" (in Pigou 1925a, 416).¹⁵ He may have remembered feeling so, but he was still reacting to Jevons's ideas in 1890: adopting some and finding reasons for rejecting others.

At this distance, it takes an effort to recreate the difficulties faced by the first readers of the *Theory*—including Marshall. Its account of buying and selling and the transactors' choices is peculiar. Jevons's traders usually exchange goods for other goods, not for money. When the analysis is extended to include production, the traders become producers as well. Marshall never took the exchange model seriously and never acknowledged the extension to production (1871, 181–3) with its derivation of the law of cost of production.¹⁶

Jevons's "final utility" combines utility and finality. Marshall differed from Jevons on utility, which he identified with his own value in use. Arguing against Jevons that difficulty of attainment is "concealed" in utility, Marshall insisted, "The utility of a commodity is in part 'prospective,' that is, dependent on the benefit which will at a future time accrue from its possession: and this depends partly upon the difficulty that there might be in obtaining something before that time to supply its place" (1872, 93). In Marshall's framework, a commodity's utility reflects everything that makes it a good buy—except its price. In Jevons's framework, and in Hicks's, the utility of a commodity does not reflect circumstances relating to the future availability of other commodities.

Marshall was more sympathetic to finality. He reports Jevons accurately and approvingly but denies Jevons's originality—"we meet with old friends in new dresses" (1872, 95).

Thus it is a familiar truth that the total utility of any commodity is not proportional to "its final degree of utility." . . . But Professor Jevons has made this the leading idea of the costume in which he has displayed a large number of economic facts. In estimating . . . the benefit of foreign trade, we must pay attention to the total utility of what we

15. Recent commentators on Jevons agree that Marshall's understanding of Jevons was far from perfect (see, for example, Black 1970, 24).

16. Jevons's analysis was closer to that of Mill—and of Marshall—on international values than to these authors on domestic values (see Aldrich 1992).

obtain by it, as much as to its final utility, which alone is indicated by the rate of exchange. (1872, 95)

Some of Jevons's ideas connect so readily with Marshall's that they may have been "old friends" even though no documents establish this. Later in 1883, Marshall told Walras he had developed the ideas independently, "I cannot be said to have accepted Mr Jevons' doctrine of 'final utility.' For I had taught it publicly in lectures at Cambridge before his book appeared. I had indeed used another name viz.: 'terminal value in use.'"¹⁷ Perhaps. Yet "terminal value in use" may have referred to price not utility, for "value in use" in the *Essay* is equivocal. "Amounts of pleasure" appear there but not as material for the calculus. For the individual buyer there is no continuously varying quantity of good nor continuously varying utility.

Marshall also wrote, "following the lead of Cournot I had anticipated all the central points of Jevons' book and had in many respects gone beyond him" (Jaffé 1965, 1:794). Yet, Marshall and Walras had different maps; for Walras and Jevons, proportionality of prices to marginal utilities was central, "beyond" were extensions to production. Multi-good analysis was not central for Marshall: on one occasion he applied Jevons's analysis of the allocation of labor between different tasks to the consideration of an income tax¹⁸ (Jevons 1871, 178–80). For Marshall, the most central of "central points" seems to have been terminal value in use; "beyond" were consumers' rent and its applications to taxation and monopoly.

3. Price, Utility, and Means

In 1879 Marshall published an elementary textbook, *The Economics of Industry*, with his wife Mary Paley Marshall.¹⁹ He also allowed some chapters of his foreign trade manuscript dealing with consumers' rent to be circulated. The book and the set of chapters known as the *Pure Theory of Domestic Values* were his first publications to deal extensively

17. Marshall sent Walras a copy of the *Economics of Industry*; and this letter was a reply to Walras's acknowledgment. See Jaffé 1965 for the letters (1:792, 794) and for another Marshall letter in similar vein (2:162).

18. The "Abstract Theory of a General Uniform Tax" (W 2:289–302) presents Jevons's analysis diagrammatically and extends it to treat the effects of the imposition of an income tax. Whitaker dates the manuscript at around 1873.

19. For this section, see, in addition to Whitaker, Howey 1960, chapter 10.

with demand. Written in different idioms for different audiences, they achieved a synthesis of Mill, Cournot, and Jevons.²⁰ Behind them, of course, was the *Essay on Value*.

In his review, Marshall had described the difference between Jevons's theory and Ricardo's as "mainly a matter of form" (93). In "Mr Mill's Theory of Value," he writes, "much of what Professor Jevons says about 'final utility' is contained implicitly at least in Mill's account: but he has brought out with excellent distinctness many vital points connected with this notion" (1876, 128). The *Economics of Industry*, which purports to be an introduction to Mill, gave Marshall a chance to make the implicit explicit. Its treatment of demand is condensed into just over three pages. In framing his definitions, Marshall takes the extra pleasure formulation from the four in the *Essay*: "The value in use or utility of a thing to a person is the amount of pleasure or satisfaction which he derives from possessing it" (68).

The downward sloping market demand curve (68) introduces what Jevons calls the "great principle of the ultimate decrease of the final degree of utility":

the larger the stock which sellers determine to sell, the lower will be the price at which it can be got rid of. . . . These facts shew how the Utility of anything to a man, its power of satisfying his wants, depends partly upon the quantity of things of the same kind he has already. The more he has of it the less will be the utility of more of it to him. (1879, 63)

"Shewing" presupposes a certain relation between the principle and the downward sloping market demand curve; the latter holds because of the former. No fuller demonstration is offered until the *Principles* (see section 5 below). "Measurement by price" replaces the *Essay*'s "representation by price:"

[We] may suppose that he is able to get flannel at one shilling a yard, and that at this price he buys twenty yards. This shews that the utility to him of the twentieth yard is not less than the satisfaction he could get by spending the shilling in other ways but that the utility of a twenty-first yard would be less than this satisfaction. In other words a

20. O'Brien 1990 gives a general account of Marshall's relationship to his classical inheritance. There is no evidence of influence from other directions—from Jules Dupuit, William Whewell, or Fleeming Jenkin.

shilling just measures the utility of the twentieth yard, the final yard which he buys. To use Mr Jevons's happy phrase, the Final Utility of a yard of flannel to him is measured by one shilling. (68)

The discussion culminates in a "law of demand," both the downward sloping market demand curve (this time the price into quantity formulation) and the measurement of final utility by price: "The amount of a commodity which finds purchasers in a market in a given time depends on the price at which it is offered for sale; and varies so that the amount demanded is increased by a fall in price and diminished by a rise in price. Its price measures its Final Utility to each purchaser, that is, the value in use to him of that portion of it which it is only just worth his while to buy" (71).

Two aspects of the treatment are noteworthy: one looks back to the *Essay* and the other forward to the *Principles*. Though Marshall follows Jevons on utility—it is the "power of satisfying wants"—he retains his conception of the buyer's situation in which the availability of substitutes is prominent. "The utility of a commodity to any one depends on the amount of it he has at any time, and the opportunity he has or expects to have of getting it, or other things that will serve as substitutes for it" (70). This conception was not given any analytical development in Marshall's later work,²¹ but the next sentence introduced a consideration that was. Means is juxtaposed with utility as a determinant of the price the consumer is prepared to pay.

But further, the price which he is willing to pay for a thing depends not only on its utility to him but also on his means; that is, the amount of money or general purchasing power at his disposal. A greater utility will be required to buy it if he is poor than if he is rich. . . . The clerk with £100 a year will walk into business in a much heavier rain than the clerk with £300 a year; for a sixpenny omnibus fare measures a greater utility to the poorer man than to the richer. (70)

In ascribing to Mill the view that the demand of a person is "dependent upon his means and the value in use to him of the commodity" Marshall was more ingenious than accurate but there was something there (1876, 128). When Mill discussed the extent of the price rise when supply is less

21. It did not disappear completely; Marshall added this sentence in the third edition of the *Principles*: "But we must recollect that the desire for anything is much dependent on the difficulty of getting substitutes for it" (1895, 209; 1920, 1:133; see section 8 below).

than effectual demand he argued that a rise in price short of the deficiency of one-third “may place the article beyond the means, or beyond the inclinations, of purchasers to the full amount” (1848, 447).²² Marshall's version was that the

price [of pictures by Raphael] depends on the desire that people have for such pictures, and the means at their disposal for purchasing them. According to the Law of Demand the price of a commodity ‘measures its Final utility to each purchaser’. . . . The greater the number of pictures, the less will be the competition among purchasers for them, the less will be the price which measures their value in use to those who are only just induced to purchase them. In this exceptional case in which the supply is fixed, the price is determined solely by the utility of the thing; Demand is the sole regulator of value. (*Economics of Industry*, Marshall and Marshall 1879, 93)

However, Jevons was the important influence. He transformed the issue of means in relation to demand; he noted the difference in utility of a penny to a poor and to a rich family (1871, 133–34) and discussed the variation of utility in the mathematicians' analysis of gambling (154–55).

Consumers' rent has its first public outing in the *Domestic Values* with an application to the burden of indirect taxes.²³ The style here is closer to the *Essay* than to Mill. The supporting diagrams are given and also an account of how the necessary information is to be obtained.²⁴ “Now that which a person would be just willing to pay for any satisfaction rather than go without it, is . . . the ‘economic measure’ of the satisfaction to him” (W, 2:213). The concepts of “value in use” in the *Essay* are now distinguished: “satisfaction” versus its “economic measure.”

22. Compare this to *The Wealth of Nations*: “Among competitors of equal wealth and luxury the same deficiency will generally occasion a more or less eager competition, according as the acquisition of the commodity happens to be of more or less importance to them” (1:74); and to Cournot: the form of the total demand function depends “on the kind of utility of the article, on the nature of the services it can render or the enjoyments it can procure, on the habits and customs of the people, on the average wealth, and on the scale on which wealth is distributed” (1838, 38).

23. The name “consumers' rent” first appears in *Domestic Values* (W, 2:216) with the analogy to landlords' rent. The apostrophe appears after the letter “s” even when there is only one consumer.

24. The (market) demand curve is defined in the same way as in the *Essay on Value* but, as in the *Economics of Industry*, there is an explicit proposition governing its shape—it is “throughout negatively inclined” (W, 2:190–91).

The economic measure of the whole is the sum of the measures of the parts:

Take the case of a man who, if the price of coals were £10 a ton, would just be induced to buy one ton annually; who would just be induced to buy two tons if the price were £7, three tons . . . £5, four tons . . . £3, five tons . . . £2, six tons . . . £1.10s., and who, the price being actually £1, does purchase seven tons. We have to investigate the consumers' rent which he derives from his power of purchasing coal at £1 a ton. . . .

[The] satisfaction derived from, or the 'value in use' to him of, a single ton a year, is economically measured by £10. Therefore his power of purchasing one ton of coals for £1 gives him . . . a consumers' rent of £9.

[If] the price were £7 a ton, he would just be induced to purchase a second ton; so that the value in use to him of a second ton is measured by £7. . . . Thus the whole consumers' rent which he derives from the power of purchasing coal at £1 a ton is $9 + 6 + 4 + 2 + 1 + 1/2$, i.e. £22 1/2 . . .

The economic measure of the total value in use, or, as Mr Jevons says, of the 'total utility of the coal' is the sum of the prices that he would be just willing to give for each successive ton: i.e. $£10 + 7 + 5 + 3 + 2 + 11/2 + 1$, i.e. 29.10s. He has to pay for them seven times the value in exchange or market-price of a ton of coal. This value in exchange is of course equal to the measure of the value in use to him of the last ton of coal which he purchases, or in Mr Jevons' phrase, to the measure of the final utility of a ton of coal to him. (W, 2:213)

Marshall might have added that Jevons (1871, 140–41) had already suggested the same technique for measuring total utility, a technique based explicitly on the constancy of the marginal utility of money. Others also missed the identification of consumers' rent analysis with the money measure of Jevons's total utility. Fleeming Jenkin, in advancing his own consumers' rent analysis, thought that "utility, as [Jevons] defines it, admits of no practical measurement" ([1871–72] 1931, 109).

Marshall does not place any limitations on the validity of the procedure in the case of the individual: he does not mention special assump-

tions, relating to the utility of money, as he does later.²⁵ He mentions an obstacle—no data on individuals—but for the body of consumers this is not a problem:

the statistics of trade will generally enable us to draw the Demand curve of the commodity for the whole market: that is, will enable us to estimate the total amount of the commodity which could be sold at a given price to the whole body of consumers. And by this means we are enabled to find the economic measure of the value in use of the commodity to the several members of the community. (W, 2:214)

Yet there is a problem for the community, one that gave him much trouble: “The measure of human satisfaction thus obtained is indeed a rough measure. For . . . it is necessary, as a first approximation, to treat a pleasure that is worth one shilling to one man as equivalent to a pleasure that is worth a shilling to any other man” (215). Allowance must be made for the fact that a “satisfaction which a rich man values at a shilling is slight in comparison with one for which a poor man will be willing to pay a shilling” (215). *How*, Marshall does not say.

Marshall had now produced reasonably extended accounts of all the ideas present in his earlier manuscripts relating to demand. He had also brought together the contributions of other students—even if he divined affinities rather than made tight connections. He could—and would—make more use of Jevons, in particular of his notion of the marginal utility of money. Marshall had begun to recognize the significance of differences in means for his analysis. If there was an obvious priority, it was to improve upon the “first approximation” underlying the calculation of consumers’ rent.

4. The Principles of Economics

We now turn to the *Principles of Economics* of 1890.²⁶ The *Economics of Industry* was an introduction to Mill and *Domestic Values* a specialized account of welfare analysis. The *Principles* was a treatise that aimed to “present a modern version of old doctrines with the aid of the new

25. John Hicks describes the association of the surplus with the demand curve triangle as a “theorem, true under certain restrictive assumptions” (1941, 109) (for further stations on the road to theoremhood see below).

26. In the years between 1879 and 1890, Marshall touched on the topic of demand occasionally. See, for example, 1881; 1885a; 1885b.

work, and with reference to the new problems, of our own age" (1890, v; 1920, v). Realizing the aim took more than theory: plenty of factual material and also reflections on such matters as "wisdom in the pursuit and use of wealth" (1890, 181; 1920, 136). The reader is not allowed to forget that there is a higher domain than the one the economist studies.²⁷

The *Principles* was no *Value and Capital*, a specialized "inquiry into fundamental principles of economic theory," but it did have new theoretical arguments. These are treated in detail in later sections. Here I make some general points about the analysis. Marshall (1890, 147–49) raises three important issues relating to demand. The first is the role of demand in determining value. The others are connected with consumers' rent; they are the measurement of demand, with its promise of throwing light on "difficult questions of great importance to public well-being," and the question "how far the exchange value [of a commodity] represents accurately the addition which it makes to happiness and well-being."²⁸ None of the issues were new. The role of demand in determining value was basically unchanged from the *Essay*. The other matters had been treated in *Domestic Values*.

The utility analysis was now much more elaborate. Two developments involved the relationship between price, means, and utility. People with different means apply different utility scales to price but one scale would usually be valid for a person for the course of trading.²⁹ Marshall saw this second proposition, the constancy of the marginal utility of money, as underlying consumers' rent. He also gave serious attention to Bernoulli's suggestion of a logarithmic relation between utility and income.

27. Regarding the "utility of wealth," he wrote, "This is a wide subject on which economic science has very little to say, but that little is important" (1890, 175). In the second edition, he added a cautionary remark: "The higher study of Consumption must come after, and not, before the main body of economic analysis; and though it may have its beginning within the proper domain of economics, it cannot find its conclusions there, but must extend far beyond" (1891, 148; 1920, 90).

28. Marshall enlarges on his hopes for a statistical study of consumers' rent in section 8 of his chapter on the "Theory of Monopolies" (1890, 468; 1920, 489).

29. Whitaker interprets the passage from the "Abstract of a General Scheme of Taxation" as showing that Marshall already realized the significance for his analysis of the constancy of the marginal utility of money around 1873 (W, 2:286–88). Yet the paper does not mention the concept. In a note dated October 1881, Marshall mentions the changing final utility of money in the same labor market context as in the *Principles* (W, 2:333). He refers to non-extant papers for amplification. He had not used the concept in his review of Edgeworth's *Mathematical Psychics* earlier in the year (W, 2:267).

Of the utility theorists mentioned in *Principles*, Jevons is the only one who matters; his results are cited and his views criticized. The argument also drew on his work in uncredited ways and not just for its "form" as Whitaker states (W, 1:103). The portrayal of the consumer follows Jevons's case of a person paying money for a good (1871, 111–13, 124–27, 140–41). The person has a utility function for the good and a utility function for money. Jevons treats only the case of an "inconsiderable item of expense" (1871, 112). He thought the analysis of possible value for deriving laws of utility from market data, though it could only apply to a few goods. Marshall took this case far more seriously. At the same time, he stressed that the final degree of utility of money varies between persons as income varies. Marshall counted Jevons's neglect of this point a great error, invalidating the notion of a trading body.³⁰ But Marshall found a casualty nearer home: consumers' rent is an exact measure of "human satisfaction" only when consumers are identical.

As he went Jevonian, Marshall distanced himself from the man, not least with his "modern version of old doctrines." He found false perspectives as well as analytical errors. In the chapter, "Wants in Relation to Activities," added in the second edition, Marshall proclaims, "It is not true therefore that 'the Theory of Consumption is the scientific basis of economics'" (1891, 147; 1920, 90) as Jevons had thought (1871, 49).³¹ Jevons's "delight in stating his case strongly has led him to a conclusion which . . . does mischief by implying that the older economists were more strongly at fault than they really were" (Marshall 1891, 147; 1920, 90).

Their technical work on consumption had a different orientation. For Jevons (1879, 21), the main purpose of utility theory was to provide basic laws from which the laws of market phenomena could be deduced: to achieve the "subordination of the more complicated phenomena of the market to the simple laws of sentience," as Edgeworth described it (1881,

30. Marshall writes, "A trading body is not a 'person,' it gives up things which represent equal purchasing power to all of its members, but very different utilities" (1890, 534; 1920, 818). In 1889, he wrote to J. N. Keynes, "I hold that Jevons's great error was that of applying to utility propositions that are only true of price. It was here that he thought himself most profound: and it is because I think he was wrong in this one point in which he differed from his predecessors Von Thünen and Cournot that I consider his claims to greatness do not to any considerable extent rest on his *Theory of Political Economy*" (1961, 2:260).

31. Despite a long history of discussion from Parsons 1931 onwards, it is much harder to identify a positive message in this chapter.

108).³² Marshall's interest was centered more on welfare. This is not to say that he took utility theory any less seriously; behind his interest in consumers' rent was a belief that this sum of money measured *something*. Figuring out *what* demanded close consideration of utility. Of the three issues relating to demand, the third was the most important, "how far the exchange value [of a commodity] represents accurately the addition which it makes to happiness and well-being."

In the *Principles*, Marshall paid closer (but to modern eyes still inadequate) attention to the properties of money as a "measuring rod." The possibility of measuring motives by money gave economics an advantage over other social sciences.³³ Measurability by money was at the heart of consumers' rent and of Marshall's demand theory. All editions have basically the same discussion of how desirability is measured by price. In the first edition, it is presented with the law of demand; in the second with consumers' rent; and finally, it comes to rest in the general discussion of the study of economics at the start of the book. Each deployment has its own logic.

Marshall explains the role of money and makes an assumption about its value.

Nearly all actions of life are governed, at least in part, by desires the force of which can be measured by the sacrifice which people are willing to make in order to secure their gratification. . . . In our world [this sacrifice] has nearly always consisted of the transfer of some definite material thing which has been agreed upon as the common medium of exchange, and is called "money." The purchasing power of this money may vary from time to time; but in these early stages of our work we assume it to be constant. (1890, 151)

The same assumption was made by Mill. There is no adequate discussion of the "purchasing power of money" and its "constancy," in the *Principles* or elsewhere.³⁴ Marshall evidently thought the formula justified treating

32. Edgeworth was commenting on his derivation of the offer curve from utility maximization (1881, 108).

33. See (1890, 78; 1920, 14) and his praise of Smith—his "chief work was to indicate the manner in which value measures human motive" (1885a, 157).

34. In the *Economics of Industry*, Marshall follows Mill (1848, 37–440) and distinguishes the exchange value of a thing, or its power of purchasing other goods, from its price, or power of purchasing money. Like Mill, he overlooks the fact that a change in price necessarily alters the purchasing power of money when he writes, "we shall, for convenience, assume that the purchasing power of money remains unchanged. So that a rise of price of a thing will always

money, which is not wanted for its own sake, as an object of utility like tea or nuts. He was no more precise about "income" or "plentifulness of money."

The measuring rod of money is not even fixed for an individual. Marshall explains why a "shilling may measure a greater pleasure at one time than at another": money may be "more plentiful with him" or his "sensitivity to pleasure may be different at different times" (1890, 151; 1920, 18). However the real issue concerned differences between persons.

In groups, differences in the sensibility of individuals average out:

It is not at all safe to say that two men with the same income derive equal pleasure from its use. . . . Nevertheless, if we take averages sufficiently broad to cause the personal peculiarities of individuals to counterbalance one another, the money which people of equal incomes will give to obtain a pleasure or avoid a pain is an extremely accurate measure of the pleasure or pain. (1890, 152; 1920, 18)

Advancing beyond the "first approximation" of treating a pound's worth of pleasure as the same for everyone was trickier. Marshall makes two points: as a matter of fact, the approximation works well but, if it did not, Bernoulli's suggestion about the form of the dependence of happiness on income could be exploited. Both figure in the following passage.

And in fact it happens that by far the greater number of the events with which economics deals affect in about equal proportions all the different classes of society; so that if the money measures of the happiness caused by the two events are equal, there is not in general any very great difference between the amounts of happiness in the two cases. If however it should appear that the class affected in the one case is on the average, say, ten times as rich as in the other, then we shall probably not be far wrong in supposing that the increment of happiness measured by a given sum of money in the one case, is so far as its direct results go, about one-tenth as great as in the other. (1890, 152; 1920, 20)

Marshall kept Bernoulli's law, but after the first edition he did not mention

mean a rise or fall of its . . . exchange value" (1879, 68–69). Marshall's conception seems very remote from Hicks's (1939, 33) interpretation of "money" as a composite commodity.

that it could be used for comparing changes in collective happiness. The neutrality of economic events made it unnecessary.

The sections that follow focus on innovations in the *Principles*. The demand schedule for a single good remained central. The buyer spends on the good or keeps his money. Marshall's work remains in a good (not goods!) versus money framework. He added a theory about the effects on demand of income and liking, factors in which people differ. The question posed by Milton Friedman, *what* is being held constant as price and quantity vary (1949, 463), has a different character when there is no theory of other goods. Changes in the prices of other goods are treated as disturbing causes in the application of the theory, not as an integral part of it.

Another framework is latent in the *Principles*, its full power made patent by others. Marshall treats "the choice between different uses of the same thing" where the "same thing" may be money (1890, 156; 1920, 117–19). These few sentences grow to half a (short) chapter in the second edition. From the first, Jevons's "doctrine of proportionality" had been present. Yet nothing comes of this.³⁵ After the *Essay*, Marshall had emphasized the significance for the buyer's decision of the availability of substitutes, but there was no formal development. We see below how he recognized some of the difficulties arising from the existence of other goods. But recognizing that a feather does not fall in the air as it does in a vacuum is not the sum of aeronautics.

5. "The Law of Demand"

"The law of demand"³⁶ of the *Economics of Industry* covered two points: the market demand curve slopes downward and the individual's final utility is measured by price. The chapter in the *Principles* of the same name proceeds from an account of utility to the conclusion about market demand schedules, "There is then one law and one law only which is common to all demand schedules, viz. that the greater the amount to be sold the smaller the price at which it will find purchasers" (1890, 159; 1920, 99). From the second edition "the law of demand" is the margin entry beside this proposition.³⁷

35. Marshall does not link up this discussion of consumption with the law or principle of substitution in production (1890, 401; 1920, 341).

36. In the fourth edition the title was changed to "Gradations of Demand" and in the fifth to "Gradation of Consumers' Demand."

37. In the first edition, the margin entry was "the law of diminution of marginal demand-price" (1890, 159).

The law applies to an aggregate not an individual. There is no individual law of demand for things “the need for which on the part of any individual is inconstant, fitful, and irregular” (1891, 155–56; 1920, 98). Marshall's theorizing is not concerned with wedding cakes but with items like tea. There is an individual law of demand, “every variation in its price is likely to affect the amount [a person] will buy” (1891, 155–56; 1920, 98). From the first, tea was the paradigm commodity in this part of the *Principles* but tea did not replace coal in the rent analysis until the third edition. The change was less in the interest of a tidy exposition than because changes in the marginal utility of money could be safely neglected for “something of which the price is measured in pence rather than in pounds” as he put it to Leslie Stephen.³⁸

Marshall uses new terminology and formal apparatus to express the principles; the pleasure from a good has a monetary value derived from balancing the need for the good against the need for money and the more of a commodity a person has, the less pleasure is derived from (and hence smaller value put on) further supplies of it. The richer a person is, the greater the value put on any given pleasure; such private valuations are revealed by the way a person's purchases change as price changes.

Marshall's arguments for these principles are hard to grasp because the premises are not fully specified and the conclusions are not clearly expressed. An embarrassment of theorems can be proved with assumptions made somewhere in the *Principles*, and this chapter is often taken to include a derivation of some kind of demand curve and a proof that its slope is negative.³⁹ It does neither, on any reasonable interpretation of derivation or proof.

I base my account on the second and later editions rather than on the very congested first edition. In the second edition “marginal demand price” was made central to the exposition.⁴⁰ This notion, which descends from the price variant of the *Essay*'s “value in use,” figures in two lines

38. In this letter of March 1891, Marshall was replying to Stephen's criticism of consumers' rent. Their correspondence is in the British Library of Economics and Political Science.

39. Hicks (1939, 27) and Stigler (1950, 141) state that Marshall deduced the downward slope of the demand curve from the law of diminishing marginal utility and the condition that the marginal utility of money is constant. Peter Dooley (1983, 27) states that Marshall was the first to derive a demand curve from utility analysis.

40. The term “Marginal demand price” is hardly used in the first edition, though Marshall may have meant to use it more. He writes, rather fancifully, of “phrases used by . . . economists [before Jevons], such as the ‘the price which consumers are only just willing to pay,’ phrases which in this treatise are condensed into ‘marginal demand price’” (1890, 634; 1920, 818).

of argument, one in the text and one in the mathematical notes. The note was already fixed in the first edition. It has more of an independent life than the usual mathematical gloss on a proposition in the text. The main lines of the text were fixed in the second edition, but there were significant revisions up to the fourth edition. These seem to represent, not changes of mind, but attempts to improve the original formulation. They were not entirely successful.

The text's account of how the pleasure from a good can be given a monetary valuation appears in the course of a "translation" into price terms of "the law of diminishing utility." Utility is "pleasure affording power." The law of satiable wants or law of diminishing utility states that "the TOTAL UTILITY . . . of a commodity to a person increases with every increment in his stock of it, but does not increase as fast as his stock increases" (1891, 150; 1920, 93).

The first paraphrase is, "the Marginal Utility of a commodity to any one diminishes with every increase in the amount of it he already has." The ground is shifting because marginal utility is defined in the context of a person choosing to "incur the outlay" needed to acquire some of the commodity, "the increment which he is only just induced to acquire . . . may be called its MARGINAL INCREMENT; because he is on the margin of doubt whether it is worth his while to incur the outlay required to obtain it. And the pleasure giving power, or Utility, of that increment may be called the MARGINAL UTILITY of the commodity to him" (1891, 151; 1920, 93).

Marshall has a very light touch with the implied transaction; he does not say anything about the terms on which the person can acquire the good. When the cognate term "marginal demand price" is introduced, the person is buying tea at a given price.⁴¹ When the price is 2 shillings per pound, 10 pounds are bought annually: "the difference between the happiness which he gets from buying 9 lbs and 10 lbs is just enough for him to be willing to pay 2s. for it. . . . 2s. ["which measures the utility of the tea which lies at the margin . . . of his purchases"] is his MARGINAL DEMAND-PRICE" (1891, 152; 1920, 94). His "marginal increment" is the tenth pound, "marginal utility" the utility derived from it and "marginal demand price" the price paid for it.⁴²

41. The formulation echoes the discussion of flannel and paintings in the *Economics of Industry* and coal in *Domestic Values*.

42. Marginal demand-price is the "price which measures their [Raphael's] value in use to those who are only just induced to purchase them" in the *Economics of Industry*.

The final translation is: "an increase in the amount of a thing that a person has will, other things being equal (i.e. the purchasing power of money, and the amount of money at his command being equal) diminish his Marginal Demand-price for it" (1891, 153; 1920, 95).

Two other generalizations about valuation are given in the text. One is a marginal demand price version of Jevons's result on proportionality of final utilities to prices: "At one and the same time, a person's material resources being unchanged, the marginal utility of money to him is a fixed quantity, so that the prices which he is just willing to pay for two commodities are to one another in the same ratio as the utility of those two commodities" (1891, 152; 1920, 95).

The other proposition compares individuals with different incomes, or the same individual at different income levels.⁴³ The utility of money is now on stage; it had not been when the law of diminishing marginal utility was discussed. The treatment of valuation in relation to utility and means, clearly descended from the *Economics of Industry*, reproduces the old example of the two clerks with the gloss: "the richer a man becomes, the less is the marginal utility of money to him; every increase in his resources increases the price which he is prepared to pay for any given pleasure. And in the same way every diminution of his resources increases the marginal utility of money to him, and diminishes the price that he is willing to pay for any pleasure" (1891, 153; 1920, 96).

The demand schedule is defined as in earlier writings. The price into quantity and the quantity into price formulations are used interchangeably. How much of a thing a person "would be willing to purchase at each of the prices at which it is likely to be offered" or equally "the prices which he is willing to pay . . . for different amounts of it" (1891, 153; 1920, 96) is the question.

In note 2, functions $u(x)$ and $\mu(m)$ describe how a person's utility from a commodity varies with the quantity of the commodity and how utility from money varies with the "amount of money or general purchasing power at a person's disposal at any time" (1890, 737; 1891, 749; 1920, 838). Total utility is evidently the sum of u and μ . Both d^2u/dx^2 and $d^2\mu/dm^2$ are negative.

The argument turns on the construction of dp/dx , marginal demand price:

43. Patinkin says this and the associated appendix material treat the "income effect" (1963, 106). But "income effect" is short for "income effect of a price change"; Marshall is treating the income effect of an income change!

If p is the price which he is just willing to pay for an amount x of the commodity which gives him a total pleasure u , then

$$\frac{d\mu}{dm} \Delta p = \Delta u; \quad \text{and} \quad \frac{d\mu}{dm} \frac{dp}{dx} = \frac{du}{dx}$$

Therefore, du/dx , the marginal utility to him of an amount x of a commodity remaining unchanged, an increase in his means increases $du/dx \div d\mu/dm$; i.e. it increases dp/dx , that is, the rate at which he is willing to pay for further supplies of it. Treating u , as variable, that is to say, allowing for possible variations in the person's liking for the commodity in question, we may regard dp/dx as a function of m , u , and x ; and then we have $d^2p/dmdx$ always positive. Of course $d^2p/dudx$ is always positive. (1890, 737; 1891, 749; 1920, 838)

Friedman (1949, 492) argues that "the price which he is just willing to pay for an amount x " is the total price or expenditure that he will pay for x rather than go without it completely.⁴⁴ Marshall's argument can be written more explicitly if we introduce some extra notation. Suppose the person has an amount of money, m_0 . Define e (expenditure equals Marshall's "price") implicitly by

$$u(x) + \mu(m_0 - e) = \mu(m_0)$$

so that

$$\frac{de}{dx} = \frac{u'(x)}{\mu(m_0 - e)}.$$

This construction of marginal demand price fits the phrasing of the note but not the text's treatment of the tea buyer nor its account of the demand schedule. To fit these, it would be more natural to take as the base point not $(0, m_0)$ but $(x^*, m_0 - px^*)$, the buyer's optimal response to the given price, p ; that is, x^* is the value of x for which $u(x) + \mu(m_0 - px)$ is a maximum.

Proceeding as before,

$$u(x^* + x) + \mu(m_0 - px^* - e) = u(x^*) + \mu(m_0 - px^*),$$

which implies that

$$\frac{de}{dx} = \frac{u'(x^* + x)}{\mu(m_0 - px^* - e)}.$$

44. See also John Whitaker's very neat presentation of Marshall's theory (1986, 354).

When the derivative is evaluated at $x = e = 0$, the right hand side equals p from the first order condition for a maximum. This construction rationalizes Marshall's identifying the marginal demand price and the ruling price.

The proposition that marginal demand price dp/dx (our de/dx , or one of them!) decreases as x increases is not derived in the note. Two demonstrations more or less fit the words of the text, though neither gives the strongest result possible. One is to argue that $du/dx \div d\mu/dm$ is decreased by an increase in x when m remains unchanged. This fits the statement of the law with its qualification "the purchasing power of money, and the amount of money at his command being equal." This line of argument fits in with the "translation" of the law of satiable wants, for that initial law relates to the effects of varying the person's endowment of the good. It does not fit in with the person buying the good. Another way of arguing is to assume that $d\mu/dm$ is constant as m varies. This would explain Marshall's light touch with the means of payment. In later parts of the *Principles*, he took this assumption of constancy of the marginal utility of money very seriously.

Marginal demand price is a peculiarly Marshallian notion without precedent in Jevons. Nor did Jevons give conclusions about the effects of variation in means and in liking on demand. This difference was reflected in Marshall's great complaint that Jevons neglected the heterogeneity in trading bodies. Marshall did re-derive Jevons's result on proportionality, though perhaps Marshall's is better described as a companion result on marginal demand price. It is important to note that this prime "allocation" result is obtained (1891, 749; 1920, 838) by applying the argument for a single good seriatim to the two goods. An allocation theorist might say that Marshall assumed an additive utility function over the goods, but that was not how Marshall argued.

The outlines of the chapter are clear. The details are incoherent. Coherence can be imposed. Friedman rewrites the chapter around the note, even redefining the demand curve. Patinkin rewrites the note (1963, 86). Or perhaps masters of the "Cambridge didactic style" say one thing and have another in mind.⁴⁵ Such impositions suppose that Marshall under-

45. According to Lawrence Fouraker, "Instead of leading the reader through the intricate analytical processes that their own minds had recently traversed, [Marshall and Keynes] would provide a short cut, in the form of an assumption whose purpose was to eliminate consideration of the difficult problem they had faced and solved" (1958, 66).

stood the issues that absorbed economists of Hicks's generation but that his skill at explaining was not equal to his understanding. This seems very unlikely.

6. "Elasticity of Demand"⁴⁶

The chapter "Elasticity of Demand" introduces and applies the concept of the elasticity of demand, considering some goods in detail. And it discusses the measurement of demand schedules. There is also a less prominent issue, the treatment of cross-section variation in income, which will concern us.

When introducing elasticity, Marshall glides between desire and price, in accordance with the "translation" theme.

This diminution [in a person's desire for a commodity with every increase in his supply] may be slow or rapid. If it is slow the price that he will give for the commodity will not fall much in consequence of a considerable increase in his supply of it: . . . his willingness to purchase the thing stretches itself out a great deal under the action of a small inducement. . . . [The] elasticity of his demand, we may say is great. (1890, 162; 1920, 102)

Marshall uses the elasticity of demand to organize his ideas on the shape of the demand curve.⁴⁷ There is a typical shape for the demand curve for a given income class, or "general law of variation of the elasticity of demand". "The elasticity of their demand is small when the price of a thing is *very high* relatively to their means and again when it is *very low*: while the elasticity is much greater for prices intermediate between what we may call the *high level* and the *low level*" (1890, 163; 1920, 103). There was always a law with this name in the *Principles*, but in the third edition it was rewritten to say that elasticity increases with price.⁴⁸

46. In the second and third editions the title was "Law of Demand continued. Elasticity of Demand." Subsequently it was "The Elasticity of Wants."

47. Elasticity first appeared in "The Graphic Method in Statistics" (1885), which is not such an odd place. The geometrical method for determining the rate of growth of a curve—in the case of statistics a smoothed time series plot—is related to that for determining the elasticity of a curve. Then the graphic method is a preliminary to induction; the essay finishes with, "I believe that inductions with regard to the elasticity of demand, and deductions based on them, have a great part to play in economic science" (187).

48. The change was in line with the argument of Johnson and Sanger 1894.

Differences in means and liking operate on demand as well as on the utility of money. After presenting the tea buyer's demand schedule, Marshall adds, "Some will be richer and some poorer than the individual consumer whose demand schedule we have just written down, some will have a greater and others a smaller liking for tea than he has" (1890, 159; 1920, 99). In this chapter, Marshall deals with income variation in two different ways.

Income variation can be allowed for by "considering one class of society at a time." Marshall draws separate demand curves for green peas for the rich, the middle class, and the poor and shows how they can be summed to obtain a total demand curve. This procedure rests on no particular specification of how demand depends on income. But a specification is implicit elsewhere. In the law of elasticity of demand, the reference to price being high or low relative to means implies that demand depends on the ratio of price to means,

$$x = g(p/m),$$

where x is the quantity demanded of the commodity, g is a decreasing function, p is the price, and m is the person's means.

It may seem unreasonable to take such references strictly, but the formulation also underlies Marshall's suggestion for combining the laws for different classes to obtain a demand law valid for wide variations in price.

If the present price is very high relative to the middle or to the working classes, we may be able to infer from the laws of their demand at the present prices what would be the demand of the rich if the price were so raised as to be very high relatively even to their means. On the other hand if the present price is moderate relatively to the means of the rich, we may be able to infer from their demand what would be the demand of the working classes if the price were to fall to a level which is moderate relative to their means. It is only by thus piecing together fragmentary laws of demand that we can hope to get any approach to an accurate law relating to widely different prices. (1890, 174; 1920, 114)

It is never placed prominently but this suggestion appears in all editions of the *Principles*. In *Money, Credit and Commerce*, Marshall notes that the method, which provides "guidance" on the behavior of demand in the case of extreme price variation, cannot be applied to the demand for imports (1923, 166).

Marshall's use of this specification has gone unremarked.⁴⁹ Vilfredo Pareto (1895) used it with his law of income distribution to rationalize Gregory King's law. He cites Marshall for King's law but does not mention Marshall's use of the ratio specification. Marshall was acutely aware of the problems created by income heterogeneity in trading bodies—hence the comfortable thought that the events of economics affect all income groups equally, discussed above.

Marshall took cross-section income variation very seriously because it affected the significance of price for individuals. Of course there were other influences on the market demand schedule.

It must be remembered that the demand schedule gives the prices at which various quantities of a thing can be sold in a market during a given time and under given conditions. If the conditions vary in any respect the figures of the schedule will probably require to be changed. One condition which is especially important to watch is the price of rival commodities, that is, commodities which can be used as substitutes for it. For instance, the demand schedule for tea is drawn out on the assumption that the price of coffee is known; but a failure of the coffee harvest would raise the prices throughout the demand schedule for tea. (1890, 160; 1920, 100)

In his discussion of Gregory King's law, Marshall shows how the demand curve is altered when account is taken of other uses of a commodity (as wheat is used for animal feed) and the availability of substitutes (1890, 167; 1920, 106). But no theory of substitutes was presented.

The problem of other goods has to be faced when trying to apply the demand curve to the solution of practical problems. In *Domestic Values*, there is a list of possible changes to the circumstances of demand. The periods

are sufficiently short to exclude fundamental changes in the circumstances of demand. . . . [It] is requisite that the periods should not include (i) any very great change in the prosperity and purchasing power of the community; (ii) any important changes in the fashions which affect the use of the ware; (iii) the invention or the great cheap-

49. Pigou (1910) does not mention Marshall's proposal. The possibility of estimating demand laws from cross-section data (that is, using income variability as surrogate for price variability) was much discussed later. See Schultz 1938, chap. 3 for a review. Creedy 1986 discusses Pareto's work.

ening of any other ware which comes to be used largely as a substitute for it; (iv) the deficiency of the supply of any ware for which the ware in question may be used as a substitute. . . ; (v) a sudden large requirement for the commodity. . . ; (vi) the discovery of new means for utilising the ware, or the opening up of important markets in which it can be sold. (W, 2:206)

Despite this formidable list of requirements, Marshall seems to have accepted the possibility of obtaining the data necessary to construct at least a short segment of the demand curve.⁵⁰

This is a natural point to comment on the quest for "Marshall's demand curve." There were two pressing questions about Marshall's theory: how is it to be related to the 'facts' and how is it to be related to other theories? Marshall cared about the first question. Some of his readers cared as much about the second; for example, Edgeworth (1894) discussed Marshall's construction in relation to those of Auspitz, Lieben, and Walras. Such efforts at integration complemented Marshall's analysis and were important for its fate as economic analysis, but they were not Marshall's work. The concern with the theory connection continues in the quest of modern historians for "Marshall's demand curve." They answer the theoretical question by selecting from Marshall's answers to the factual question.

"Marshall's demand curve" is elusive because, as interpreted by Friedman 1949, Stigler 1950, or Alford 1956, it is not in Marshall. Marshall only wrote one equation for market demand,

$$y = f(x),$$

where y is "the price at which an amount x of a commodity can find purchasers in a given market" (1890, 740; 1920, 841). "Marshall's demand curve" is the equation from which this one is obtained holding "other things" equal.

In section 5, we saw that behind this market curve is the individual's demand schedule representing the outcome of balancing desire for the good against desire for money of "uniform value." Marshall considered how this balancing depends on the individual's means and liking for the

50. In the *Essay on Value* he speculated, "It is possible that at some very remote period a diligent detailed tracing of the curves for an enormous number of different commodities may lead up to approximate generalisations" (W, 1:156).

good. It does not seem to misrepresent his analysis to write down that equation as

$$y_i = f(x_i, m_i, l_i) \text{ or even } y_i/m_i = f(x_i, l_i)$$

for the i -th individual with means m_i and liking l_i . Means and liking have a claim to be the “other” variables in “Marshall’s demand curve,” but they are not emphasized in the “Marshall’s demand curve” literature.

Marshall describes numerous influences on the market demand schedule. If the “variables” from the passage in the *Domestic Values* are pressed into an equation, the result will resemble the demand equation of a richer theory or better, given the context of application, a statistical demand equation. But such a concoction lacks the status of the quantity/price equation or the quantity/price/means/liking equation in Marshall’s system.

In the *Principles* (1890, 170; 1920, 109) the discussion of “disturbing causes,” the “other things” that fail to be equal, is even more elaborate. The discussion from which the conventional variables, “money income” and “index of all prices” (see Stigler 1950, 139), can be extracted bears little resemblance to the framework of consumer theory. The setting is the business cycle with attendant changes in the price level and redistribution of income. Marshall’s observations about particular goods could also be “theorised”: for example, his point about the significance of stocks of old clothes and boots could be integrated into a theory of consumer durables. “There is a great deal of reserve wear in the coats and hats that are thrown aside in prosperous times as worn out, but not so much in the boots” (1890, 173; 1920, 112). On a more general level, Marshall emphasizes the element of time, “the full effects of a cause seldom come at once, but often spread themselves out after it has ceased to exist” (1890, 170; 1920, 109). So ‘history’ could be included as a variable in the demand curve; the result does not much resemble the demand curve of conventional consumer theory.

His review of the problems of obtaining laws from time series market data left Marshall pessimistic: “the difficulties of deducing accurate laws of demand from statistical tables are so great” (1890, 173). But an alternative, centered on the “shopkeeper in the working man’s quarter of a manufacturing town,” is central to the proposal. He has the necessary information as it is his business to be informed about the circumstances of his customers.

To illustrate what could be done, Marshall takes the case of butter. Demand depends on price, which is high in winter. But people want

more butter when it is cold, so price and seasonality are confounded. "If however in two neighbouring winters his customers have been about equally numerous, and in receipt of about the same rate of wages; and if in the one the price of butter was a good deal higher than the other, then a comparison of his books for the two winters will afford a very accurate indication of the influence of changes in price on consumption" (1890, 173; 1920, 114).

Apart from the number of customers, the only determinants of total demand mentioned for the shop's butter are price, liking (which varies seasonally), and income. Marshall studies these factors in his chapter on the law of demand. Though he contrasts the seasonal shift to butter with the shift away from vegetables, he does not discuss the price of one good as an influence on the demand for the other at all. The two foods merely satisfy different wants.

7. "The Measurement of the Utility of Wealth."

"The Measurement of the Utility of Wealth"⁵¹ is a fairer description of the chapter's contents than Marshall implies when he says, the "utility of wealth" is just the pleasure that arises from the possession of a thing (1890, 175; 1920, 124). The basic exposition of consumers' rent is taken from *Domestic Values* with some footnotes and mathematical notes added. But the reach of the doctrine is greatly extended. In the second edition, the exposition is improved and the doctrine really soars.

The only addition to the foundations is that the role of the constancy of the marginal utility of money is recognized at last.

We assume that the marginal utility of money to the individual purchaser is the same throughout. Strictly speaking we ought to take account of the fact that if he spent less on coals, the marginal utility of money would be less than it is, and he would get an element of Consumers' Rent from buying other things at prices which now yield him no such Rent. But these changes of Consumers' Rent (being of the second order of smallness) may be neglected, on the assumption, which underlies our whole reasoning, that his expenditure on any one thing, as, for instance, coals is only a small part of his whole expenditure. (1890, 740; 1920, 842)

51. For this section, see Samuelson 1990.

Marshall does not explain any further; it seems from the letter to J. N. Keynes that he had not intended to say even this much (1961, 2:260–61). It was just enough to ensure “strict mathematical accuracy” without troubling the “ordinary reader” too much—as he put it to Leslie Stephen.

Marshall refers to his account of the temporary equilibrium of demand and supply in his discussion of the constancy of the marginal utility of money. The assumption that it is “practically the same” “is justifiable with regard to most of the market dealings with which we are practically concerned. When a person buys anything for his own consumption, he generally spends on it a small part of his total resources” (1890, 393; 1920, 335).

Jevons restricted the constancy assumption to insignificant items like salt or nutmeg. It could not apply to an item like bread. “The price of bread, for instance, cannot be properly brought under the equation in question, because, when the price of bread rises much, the resources of poor persons are strained, money becomes scarce with them, and . . . the utility of money, rises” (1871, 141).

For Marshall, non-constancy characterizes “barter” rather than buying and selling.⁵² The sale of labor is a form of barter because a large part of the money at a worker’s disposal will be wages. The consequences are serious: “When a workman is in fear of hunger, the marginal utility of money to him is very high; and if at starting he gets the worst of the bargaining, and is employed at low wages, it remains high, and he may go on selling his labour at a low rate” (1890, 394; 1920, 335).

Marshall (1890, 596; 1920, 568) returns to the bargaining position of the vendor of labor, but he does not develop this hint of a low-wage equilibrium and a high-wage one. In a note dated October 1881 (W, 2:333), he mentions the case in which “labourers increase their supply of labour in consequence of a fall in wages” and draws a diagram with two equilibria. Yet in the disutility analysis in the *Principles* he seems to exclude the possibility: “Subject to [certain qualifications] it is broadly true that the exertions which any set of workers will make, rise or fall with a rise or fall of the remuneration which is offered to them” (1890, 188, 742; 1920, 142, 843). There is no mention of any qualification relating to the utility

52. He gave a historical gloss to the contrast between barter and buying and selling. He argued that any commodity for which the marginal utility is practically constant is “so far well suited to act as a medium of exchange, and discharge the simpler functions of money for the small business of a primitive community” (1890, 397). The passage, excised from the *Principles*, reappears in “Notes on the Evolution of Money” in *Money, Credit and Commerce* (1923).

of income nor indeed any derivation of this proposition. The formalization of the worker's situation parallels that of the consumer's situation: the worker considers the compensation required for working an extra hour. In the mathematical note, Marshall shows how the compensation varies with the person's endowment of money and dislike for labor. He may have realized a backward bending curve was compatible with the formalization but not been able to show it.⁵³

Practical constancy of the marginal utility of money is the only addition to the foundations, but there are other additions. In *Domestic Values*, Marshall limited the use of rent analysis to small changes from the ruling price. In the *Principles*, he mentions the same limitation but disregards it.

The new footnote and mathematical note 6 describe a new project and accompanying problem, "estimating the total utility of commodities some supply of which is necessary for life" (1890, 178; 1920, 133). "If however an amount b of the commodity is necessary for existence, $f(x)$ will be infinitely great, or at least indefinitely great, for values of x less than b . We must therefore take life for granted, and estimate separately the total utility of that part of the supply of the commodity which is in excess of absolute necessities: it is of course $\int_b^a f(x) dx$ " (1890, 740; 1920, 841).

In this case of necessary supply, Marshall discusses the possibility of several goods satisfying the same want—his only mention of other goods as an obstacle to the analysis. "If there are several commodities which will satisfy the same imperative want, as e.g. water and milk, either of which will quench thirst, we shall find that, under the ordinary conditions of life, no great error is introduced by adopting the simple plan of assuming that the necessary supply comes exclusively from that one which is cheapest" (1890, 740; 1920, 841). It is not clear how he meant to exploit the assumption. The assurance that "no great error is introduced" and the promise "we shall find" may suggest that there is a more powerful theory waiting to be deployed. None ever was.

Marshall does not only aim to estimate the total utility of a single commodity for an individual (and possibly for the totality of consumers). He sums consumers' rent across commodities. The validity of the summing was a matter on which he changed his mind again and again. An entry

53. Marshall reviewed *Mathematical Psychics*, but there is no evidence that he could derive an offer curve from utility maximization.

in his mathematical notebook (W, 2:280) shows he had once considered the summing valid. In his “Abstract Theory of a General Uniform Tax” he rejected it.

If we desire to compare the effects of an income tax with those of customs or excise duties, we cannot make the assumption which often gives a tolerably accurate approximation: viz. that the effect of the imposition of a tax on any commodity may be to this extent separately discussed—that it shall not be necessary to investigate the effects on the demand of other commodities of a certain excess or deficit, as the case may be, in the amount spent on purchasing this [the taxed] one after the imposition of the tax. For this sum account has to be taken. (W, 2:289)

Now he writes, “Subject to [certain] corrections then we may regard the aggregate of the money measures of the total utility of wealth as a fair measure of that part of happiness which is dependent on wealth” (1890, 179; 1891, 187). The procedure is described in mathematical note 7: “Thus if $a_1, a_2, a_3, . . .$ be the amounts consumed of the several commodities of which $b_1, b_2, b_3, . . .$ are necessary for existence, if $y = f_1(x), y = f_2(x), y = f_3(x) . . .$ be the equations to their demand curves and if we may neglect all inequalities in the distribution of wealth, then the total utility of his wealth, subsistence being taken for granted, is represented by, $\Sigma \int_b^a f(x) dx$ ” (1890, 741; 1920, 842).

Marshall’s mixing of the individual (“his”) and the collective (“distribution of wealth”) here is confusing—and evidently confused. There is the same mixing of individual and collective considerations in his account of the necessary “corrections.” On the one hand, differences in “physical, mental and moral health” are important for happiness; on the other, the utility of money varies across individuals because some have more of it.

A few lines after the phrase “part of happiness which is dependent on wealth,” there is a similar phrase in which income figures: “In accordance with a suggestion made by Daniel Bernoulli we may perhaps suppose that that part of a person’s happiness which he derives from his income, may be regarded as commencing when he has enough to support life and afterwards as increasing by equal amounts with every equal successive percentage that is added to his income; and *vice versa* for loss of income” (1890, 180; 1920, 135).

In the second edition, Marshall makes it clear that, despite the differ-

ent terms, the concepts are the same: “when we speak of happiness as measured by wealth, we of course refer to the flow or stream of happiness as measured by the flow or stream of incoming wealth and the consequent power of using and consuming it” (1891, 187; 1920, 134). According to Bernoulli’s law, the happiness y associated with income x is given by $y = K \log x/a$, where a is the income ‘sufficient to purchase the necessities of life.’” Marshall adds, “Of course both K and a vary with the temperament, the health, the habits and the social surroundings of each individual” (1890, 741; 1920, 842). In large groups, in which all temperaments are represented, variations in these parameters average to a common value.

Marshall never says so, but it seems a fair inference from what he does say that for the happiness U associated with spending $\Sigma af(a)$, that is, income y , $U = K \log y/y_0$ is measured by $\Sigma \int_b^a f(x) dx$. The income level y_0 covers expenditure $\Sigma bf(b)$ on absolute necessities.

This review of the core chapters on demand in the first two editions of the *Principles* reveals arguments going from different starting points without, it seems, any organizing principle. Marshall also mixes conjectures and approximations with certainties. How do they all fit together—bearing in mind that Marshall’s standard of integration is not the same as the modern theorist’s?

The analysis behind the “law of demand,” discussed in section 5 above, is based on a utility function of the additive form

$$u(x) + \mu(m), \quad (1)$$

where x is “tea” and m is what is left of the person’s means when the tea is paid for. Marshall regarded diminishing marginal utility of the good (x) and income (m) as much more firmly based than any particular functional forms.

Marshall’s first route to the “part of a person’s happiness which he derives from his income” is based on the constancy of the marginal utility of money. In the second edition, Marshall reinforced his discussion of barter with mathematical note 12 (1891, 755; 1920, 844). He uses a function of the form

$$u(x) + \alpha m, \quad (2)$$

where α is the constant marginal utility of money; and α is small for

the rich and large for the poor. Edgeworth (1891) used equation (2) in his criticism of Marshall's barter analysis and Marshall adopted it.⁵⁴ He takes it as an approximation to equation (1) when the good is a small item of expense.

What is the role of equation (2) in demand curve analysis? Most of Marshall's demand theory is laid out before constancy appears, yet his statement to Edgeworth, "in consequence [of working with quantity and price] I had to sit upon changes in the marginal utility of money" makes constancy constitutive of the demand curve (1961, 2:797). Perhaps it is, for the curve to be of any use.

Sometimes diminishing marginal utility of income suffices, as for the case that gambling is an economic loss. But the Bernoulli form is considered a good guess for the utility of income. The question of how functions of the form of equations (2) or (1) add up to Bernoulli for income as a whole is not addressed. It is tempting to speculate that the Bernoulli form underlies the ratio specification of the demand curve. The specification recommends itself as the simplest compatible with the theorem on the dependence of demand price on income. But it also follows from that theory when the Bernoulli hypothesis is applied to $\mu(m)$.

Using the notation above, from $dp/dx = du/dx \div d\mu/dm$ and $d\mu/dm \propto 1/m$, we have $du/dx \propto (dp/dx) \div m$. As du/dx is a decreasing function of x , we can invert it to obtain $x = g(dp/dx \div m)$, where g is a decreasing function. As the consumer chooses x to match dp/dx to the ruling price, the ratio form follows.

Marshall took the Bernoullian hypothesis seriously, and it is reasonable that he should try to exploit it in treating the demand function. Only one of Marshall's pupils worked on utility in the 1890s, C. P. Sanger. He showed a marked interest in Bernoulli's law and how it could be used in demand analysis.

Johnson and Sanger's⁵⁵ "On Certain Questions Connected with Demand" uses multigood analysis to treat some issues from the *Principles*. They show that, under certain assumptions, "the area of the ordinary demand curve measures in money the difference in total utility between having any given quantity and having nothing of the commodity con-

54. Newman 1990 reviews this debate.

55. For Johnson and Sanger see Baumol and Goldfeld 1968, 41–42 and Kadish 1989. In a letter quoted by Kadish, Marshall describes Sanger as being "really worth teaching" (1989, 169).

sidered (amounts of other commodities remaining unchanged)" ([1894] 1968, 45). The technique is striking. They maximize a general utility function subject to a budget constraint and develop the indirect utility function and Roy's identity. Among their assumptions is the peculiar one that the marginal utility of money is a function of money income alone and not of individual prices; as an example, they give Bernoulli's hypothesis. The paper shows how in the Cambridge of 1894 it was possible to pursue Marshall's questions using very different methods.

8. The Third Edition

Marshall thought that, as some readers had been "misled" by his account of consumers' rent, it "would be improved by fuller explanation, even at the expense of some repetition" (1893, 92). So he rewrote the chapter yet again; with a new title—"Value and Utility"—it took on more or less its final form.⁵⁶

The theorists had not been misled. Marshall (1895, 208n; 1920, 132n) noted Sanger's (1895, 126) review of results in Pareto and Barone and relayed the news that Barone had shown that Marshall's consumers' rent and Walras's consumers' rent are equal to the second order of small quantities. Marshall took this as vindicating his analysis.

The recasting of the chapter owed more to the criticisms of J. Shield Nicholson (1892) and Simon N. Patten (1893). Marshall (1893) warded Patten off with quotations from the *Principles*. Patten added together the rents from different foods for a person in a life-or-death situation, a situation that Marshall explicitly ruled out. In the new edition, adding-up is disallowed on a much wider front than that of "necessary supplies." Marshall begins a long footnote, "Some ambiguous phrases in earlier editions appear to have suggested to some readers that we can say that the total utility of two commodities that contribute to the same purpose is equal to the sum of the total utilities of each separately" (1895, 207n; 1920, 131n). In a second note, he disowned the use of the assumption of constant marginal utility of money "if we attempted to add together the total utilities of all commodities" (1895, 208n; 1920, 132n). The route to "the satisfaction which a person derives from his income" through Bernoulli's law survived these changes.

Though consumers' rent was better founded in the *Principles* than in

56. For this section, see Dooley 1983, Stigler 1950, and White 1990.

Domestic Values, it was still over-extended. Now it was to be cut down to roughly its 1879 size. Marshall refers to Nicholson in two footnotes yet his criticisms seem to have motivated the rewriting of the chapter.⁵⁷ Nicholson objected to the project as a whole and to the assumptions of the analysis. The objections still applied to the reduced doctrine of the third edition.

Marshall worked hard to emphasize the common sense nature of the analysis. In explicit reply to Nicholson, he gave two new examples to illustrate the sense in making monetary estimates of a person's environment or "conjuncture" (1895, 203n; 1920, 127n). In the first, somebody living in England on an income of three or four hundred pounds a year is better off than someone in Central Africa with an income of one thousand pounds. In the second, an individual pays a penny toll on the bridge to save a ride costing a shilling; were the bridge swept away on a day when he needed it, he would be in "at least as bad a position as if he had been deprived of eleven pence" (1895, 203n; 1920, 127n). To Hicks's trained eyes, the examples make an odd pair: were the Englishman transported to Central Africa, would he not be in "at least as bad a position as if he had been deprived" of six or seven hundred pounds?⁵⁸

We have seen how Marshall—unlike Jevons—took the constancy of the marginal utility of money as the typical case; otherwise it was not "exchange" but "barter." "But what if all exchange is barter?" asked Nicholson (pointing out that "the great mass of the people spend the bulk of their earnings on a very few commodities" ([1894] 1902, 64).⁵⁹ Marshall responded with a discussion of non-constancy, adding instruction to the mathematical note above on how to correct for changes in the marginal utility of money: "it is only necessary to multiply $f(x)$, within

57. Marshall also replied (privately) to Nicholson with quotations. In his second edition, Nicholson gives the references. Marshall also told him, "Some (American) writers have thought it possible to aggregate consumer's surplus for all things. I never have. If the necessities of life be taken for granted, and a number of arbitrary assumptions made, the surplus might conceivably be elaborated. But my own attempts (made twenty-five years ago) . . . failed so completely that I never implied it could be done" (quoted in Nicholson [1894] 1902, 65). Presumably "twenty-five years ago" refers to the "Abstract Theory" discussed above.

58. Patinkin remarks that Marshall sometimes thought in terms of Hicks's compensating variation and sometimes in terms of equivalent variation (Patinkin 1963, 190, 203). Robertson, who had earlier (1952) given a Marshallian rebuff to the new demand theory, thought Marshall's examples show that he "cannot have been far off perceiving" the need for these concepts (1962, 680).

59. Edgeworth (1894) wrote a defense of constancy though his own welfare analysis did not rest on the assumption (1894a, 433). See Aldrich 1992.

the integral given above by that function of $xf(x)$ (i.e. of the amount which he has already spent on tea) which represents the marginal utility to him of money when his stock of it has been diminished by that amount" (1895, 795; 1920, 842). Marshall had been using the area under the demand curve to represent consumers' rent for nearly thirty years. A by-product of this correction is that at last he is reasonably explicit about how the constancy of the marginal utility of income figured in this area estimate.

The enigmatic Giffen "exception" appears under the same heading of non-constancy.⁶⁰ It was exceptional in requiring a correction for a change in the marginal utility of money, presumably using the method of the note. Before considering the argument in detail we should consider its point.⁶¹ A case is brought forward where a correction should be made. The case is extraordinary for the buyers respond to the higher price by buying more. Marshall seems to be saying that corrections are only needed in such "rare" cases.

M. V. White 1990 emphasizes Marshall's general debt to Jevons here. Like Jevons above, Marshall uses bread to illustrate non-constancy. Indeed, the first part of his account—even the phrasing—is very similar:

as Mr Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families and raises so much the marginal utility of money to them, that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and, bread being still the cheapest food which they can get and will take, they consume more, and not less of it. But such cases are rare; when they are met with they must be treated separately. (1895, 208; 1920, 132)

Interpreting this passage is like reading tea leaves, most of the structure is supplied by the reader. For Friedman, the passage is pure *Value and Capital* (Hicks 1939): it "clearly offsets an income effect against a substitution effect" (Friedman 1949, 486). But Marshall does not even isolate these effects. He was not as well schooled in Hicksian analysis as Friedman.

60. Mason (1989) has a useful review. Much of the large literature considers how Marshall should argue in order to convince the modern reader.

61. For a discussion of how Marshall used the Giffen exception both in the *Principles* and in the "Memorandum on International Trade," see White 1990.

There is little hope of establishing any deep meaning in this passage. Yet the effort is a good way of probing Marshall's treatment of demand. We do not know what Giffen "pointed out." From the phrasing, he may have given the entire account regarding the effects of an increase in the price of bread on the demand for meat and bread with the theoretical interpretation or just part of it. The tradition is that Giffen contributed the fact that the poorer laboring families buy more bread when the price rises. Yet after searching his work, Stigler concluded, "it is now virtually certain that he never gave direct evidence" for this upward sloping demand curve (1948, 61).

Marshall did not link the Giffen passage to the rest of the *Principles*. But we can ask whether it fits or not. It could fit in the very strong sense that it comes from applying ideas from the *Principles* to something Giffen said. The statement quoted by A. R. Prest from Giffen's "The Real Agricultural Development of the Last 20 Years" would make a good seed: "Why do people as they grow richer consume less wheat etc instead of more? . . . Because with their increase of resources, they consume more meat, which pro tanto displaces the cereals" (1948, 58). This observation on the inferiority of wheat generates a positive quantity-price relationship if the ratio specification above is granted. But this argument does not fit the passage with its focus on the marginal utility of money. A better use of Giffen's observation follows. Consider with Jevons the case of a rising price and take this as producing a fall in resources; now replace the conclusion that this would lead to a reduction in consumption in all directions with Giffen's empirical observation applied to a decrease rather than to an increase in resources and the conclusion follows. It would be nice if the effect were from Marshall's imagination. He is a better parent than Giffen—he at least believed in the proposition and defended it.⁶²

Marshall does not address the issue that has most concerned his readers, that is, the *prima facie* incompatibility of the Giffen case with the theory of the "law of demand" (see Section 5 above). The law of diminishing marginal demand price is conditional on resources being unchanged and is not impugned when resources are changing. Nor is the law of demand necessarily impugned if the phenomenon is confined to the poorer la-

62. For Giffen's disbelief, see Stigler 1948, 61. Marshall tried hard to convince Edgeworth of the inferiority of wheat using his own data—as though this were the heart of the matter (see Pigou 1925a, 438).

boring families.⁶³ But the law of increasing marginal demand price with increasing income does seem to be impugned, or at least the version in the notes. The curtailment of expenditure on meat obeys it—when the marginal utility of money increases, consumption of meat falls; but so should the consumption of bread. Marshall may not have seen that one of his cardinal principles was contradicted. The version in the text is, “every increase in his resources increases the price which he is prepared to pay for any given pleasure” (1891, 153; 1920, 96). If a rich man wants a loaf of bread as badly as a poor man, he will be prepared to pay more for it. This conditional statement can be valid even though the premise has no application; a rich man with plenty of meat will not want the loaf as badly as the poor man. Yet, though Marshall may not have perceived an inconsistency, there is one—the Giffen case is impossible with the additive utility function he used.

If Marshall had perceived the inconsistency, he would not necessarily have been bothered. From the first edition he had emphasized the need for special treatment in cases where “necessary supplies” could be provided by different goods: if water and milk, why not bread and meat? In the third edition, there is even greater emphasis on the need for care in defining a commodity for the purposes of applying the theory. So much tact was needed already in applying the theory that a bit more would do no harm.

Let us leave the tea leaves and consider some of the other new points. He had always insisted that individuals paying the same price differ in their income and in their liking for the good. Now he entertained some further possibilities: “There might conceivably be persons of high sensibility who would suffer specially from the want of either salt or tea: or who were generally sensitive, and would suffer more from the loss of a certain part of their income than others in the same station of life” (1895, 206n; 1920, 130n). Once he thought that such peculiarities of temperament would average out, but they might not, say, if “those who laid most store by tea were a specially sensitive class of people” (1895, 206n).

Baumol and Quandt summarize the changes in this edition: “he patched up some parts of his theory to take account of interdependencies but he never fully reworked his theory with a general utility function” (1968, 41). This sounds the right note about the absence of radical change but the

63. In the “Memorandum” (Marshall 1908, 382), the proposition is extended to England’s total demand for wheat.

wrong note about the nature of the patches and the nature of the theory. The alterations were not repairs to a theory, they were more admissions that it did not apply.

The new difficulties had to be added to the old ones—associated with the diversity of consumers—on which he had made no progress. Yet the value of rent analysis was reaffirmed, and Marshall went on maintaining that “the exact measurement of the Consumers’ Rent in a market . . . may become of high practical importance” (1920, 131). The new material made such a prospect even more doubtful. Guillebaud records what was probably Marshall’s final position: “He told me on one occasion that a major disappointment in his life was the recognition, which gradually forced itself on him, that his concept of consumer’s surplus was devoid of important practical application, because it was not capable of being quantified in a meaningful way” (1971, 6).⁶⁴

9. “Marshallian Demand Theory”

Marshall went on revising his book. Rather than trace minor changes through the later editions, I consider “Demand Theory in Marshall’s Tradition.” Ekelund, Furbotn, and Gramm 1972 use this phrase for all the work that appeared between the *Principles* and *Value and Capital*. However, “Marshallian demand theory” is better restricted to the work of Marshall’s pupils and those writers—neo-Marshallians—who saw themselves as extending and modernizing his work. The neo-Marshallians have had a profound influence on how Marshall is read. For the modern reader, Marshall has usually meant their reading of Marshall.

Marshall took pains to distinguish himself from Jevons but more or less ignored all other demand theorists. So he drew up no rules of demarcation. Yet his demand theorizing, a medley of demand theory in the strict sense and welfare economics, was very distinctive. The technique was based on a quantity-into-price formulation of demand in a good-versus-money framework. Among the presuppositions was cardinal utility—for the good and for money—with meaningful interpersonal comparisons. Welfare analysis, in particular consumer surplus analysis, provided its ultimate rationale. The package did not last. His followers took different bits but all left much behind.

Of the Marshallians, Sanger and Pigou should be noted. Besides taking

64. See Albon 1989 for Marshall’s only practical consumer surplus estimate.

off from Marshall, there was a slight influence in the reverse direction. Sanger's contribution was confined to the two articles mentioned earlier. Pigou's work on demand and welfare—particularly on welfare—was on a vast scale, stretching from “Some Remarks on Utility” (1903) to “Some Aspects of Welfare Economics” (1951).

Though it was eclipsed in his later welfare economics, consumers' surplus is treated in some of Pigou's first papers. “Some Remarks on Utility” follows Cunynghame 1892 and treats the implications for consumers' surplus of the dependence of an individual's utility on the amounts consumed by others of different degrees of proximity in “place or station.” It also discusses the interpretation of utility. Pigou saw a “broad distinction between writers who do, and those who do not, treat the term as identical to pleasure” (1903, 59). Jevons belongs in the first group; Marshall, with Fisher “following” him, defines utility with reference to desire. Here Pigou was anticipating: Marshall only wholly went over to this view in the fifth edition of 1907, although it fit his long-established emphasis on the measurability of motives.⁶⁵ Pigou and Sanger were comfortable with multigood analysis; their work was not restricted to Marshall's single-good universe. Where it would be useful—as in his 1910 paper on measuring elasticities—Pigou assumed additivity.

All the while, Marshall's work was discussed by outsiders. They asked, what in it is correct and worthy of a place in the body of accepted doctrine? There is not much of Marshall or “Demand Theory in Marshall's Tradition” in Bowley 1924 or Schultz 1938. One can see why from Stigler's 1950 history, in which Pareto and Slutsky are preeminent. Samuelson (1942; 1990) has seen in Marshall's work a great fund of errors. Regarding positive contributions, he writes characteristically, “Alfred Marshall has nothing interesting to add on these subjects, a fact recognized too rarely” (1974, 1280).

The 1930s saw the end of a development in demand theory extending back to Fisher and Pareto in the 1890s, namely, the replacement of utility and disutility with ordinal preferences. The final blow came from Hicks and Allen 1934. Pigou, the old Marshallian, responded by admitting that although the desires and aversions behind schemes of preference and indifference cannot be measured by any instrument, “Nevertheless I am loath to let them go” (1933, 3). He (1951) was even more loath to let

65. Marshall began to worry about what had seemed a “verbal joke” (1893, 388), taking the technical use of pleasure as acceptance of the utilitarian theory in ethics. Disclaimers appeared in the *Principles* (1895, 78; 1920, 17).

go of interpersonal comparisons of cardinal utility in the field of welfare economics.

The modern reading of Marshall owes less to the Marshallians than to the neo-Marshallians. Hicks led the revitalization, though he rejected the basic tenets of cardinal utility and the possibility of interpersonal comparisons. First, *Value and Capital* made a revolution in esteem for Marshall's work by taking a fragment and declaring it classical. Hicks (1939, 27) showed how the assumption of the constancy of the marginal utility of money would be an inspired response to problems that could only be completely solved by the new theory. He made single-good analysis compatible with multigood analysis by interpreting "money" as a composite good. Marshall had been inclined to a different resolution, the good's own utility reflects availability of alternatives. Yet while some of Marshall's work was usefully wrong, much stayed uselessly wrong; there was no instruction in following his grappling with differences in the way individuals feel prices. This Marshall was dead.

Value and Capital (Hicks 1939) was a Paretian exercise that treated Marshall with respect. The project that culminated in the *Revision of Demand Theory* (Hicks 1956) was genuinely neo-Marshallian. Hicks reworked consumer surplus and marginal demand price using ordinal theory.⁶⁶ After the reworking, Hicks praised Marshall, "almost everything which [he] says in his Book III retains its validity and requires, in some form or other, to be kept. And the things which Marshall said were the really important things" (1956, 1). Friedman's "Marshallian Demand Curve" 1949 carried forward the revolution in esteem. It took a different fragment and argued that Marshall anticipated the full application of modern theory to the circumstances of single good analysis (466). Commentators just had not made the right connection with Slutsky.

Friedman's essay did not generate new economic theory but it established Marshall's demand theory as a topic for historians of economic thought. Its bold thesis and its detailed attention to what "Marshall really meant" were provocative. It set a pattern: the subject text is tortured until a confession is produced that can be corroborated by modern theory.

66. The work Hicks regarded as a continuation of Marshall's is commemorated in the usage "Hicksian demand function" for the function expressing the dependence of demand on prices and the level of utility. The companion term "Marshallian demand function" expresses the dependence on prices and money income. This has more to do with "Value and Capital" than with the *Principles*. The "Hicksian demand function" is Friedman's "Marshallian demand function!"

Friedman's (1949, 474) disagreement with Hicks (1939, 32) over whether Marshall "eliminated" or "neglected" the income effect is a difference over how best to approximate Marshall's analysis in modern theory. This is an exercise in modern theory: Marshall was innocent of "income and substitution effects."

The neo-Marshallians made a contemporary out of Marshall. Others worked to the same effect by exaggerating the obviousness of modern conceptions of demand and reading them into Marshall.⁶⁷ However, the prospects for a historical understanding of Marshall's work were transformed by the publication of Whitaker's edition of *The Early Writings of Alfred Marshall* (1975). Its effect in relocating the early Marshall is well described by Hicks: "it has been supposed that what Marshall was 'working up' was the 'Marginal Utility Revolution,' in which he was anticipated by Jevons. . . . It is now quite clear that this is all wrong. Marshall began, as one would expect, from Mill. . . . We can see the outline of an early Marshall who is really pre-Jevons and much more like Mill than he is like Jevons" (1976, 368).⁶⁸ If the early Marshall could be dissociated from Jevons, perhaps the later Marshall can be dissociated from Slutsky.

Marshall has been distanced in another way, one that is not reflected in this paper. Marshall had distinctive views on history and ethics; of relevance to the concerns of this paper, he held that wants are in some sense endogenous and he also found utilitarianism unduly static.⁶⁹ These views may have led him to disparage the subject matter of this paper as "elementary analysis of an almost purely formal kind" (1891, 148; 1920, 90). However, these views on the significance of his theorizing about demand do not seem to have any purchase on its content, and it was the latter that influenced generations of economists.

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67. Belief in Marshall's "remarkable theoretical intuition"—see Walker (1982, 69)—was another factor contributing to his contemporizing. under way.

68. "Dehomogenisation" in Jaffé's (1976) sense was already under way. Howey's 1960 study of Marshall's published work showed how little it had to do with any 'Marginal Revolution.'

69. Marshall expressed his dissatisfaction with utilitarian doctrine in correspondence with Edgeworth in February and March 1880. The correspondence is in the British Library of Economics and Political Science.

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