In any specific public investment project, elements of both externalities and increasing returns are likely to be present; this is certainly true of the irrigation projects which have been so important in the United States. Margolis has subjected the investment criteria of the U.S. Bureau of Reclamation to close scrutiny with regard to the appropriate measurement of benefits and costs and, in the process, clarified considerably the concrete implications of the abstract economic principles.

On the measurement of the utility of public works*

JULES DUPUIT†

Legislators have prescribed the formalities necessary for certain works to be declared of public utility; political economy has not yet defined in any precise manner the conditions which these works must fulfil in order to be really useful; at least, the ideas which have been put about on this subject appear to us to be vague, incomplete, and often inaccurate. Yet the latter question is more important than the former; enquiries—be they ever so numerous—laws and ordinances will not make a road, a railway, or a canal useful if it is not so already. The law ought merely to confirm the facts demonstrated by political economy. How is such demonstration to be made? Upon what principles, upon what formula, does it rest? How, in a word, is public utility to be measured? Such is the object of our enquiry in this chapter.[1]

* Translated in *International Economic Papers*, 2(1952): pp. 83–110. English translation by R. H. Barback from "De la Mesure de l'Utilité des Travaux Publics," *Annales des Ponts et Chaussées*, 2d series, Vol. 8, 1844. Reprinted by courtesy of *International Economic Papers*.

† Editors' note: As the forerunners of Jevons and Menger in respect of utility analysis, Cournot and Dupuit were both singled out for special mention by Marshall. While Cournot's *Recherches sur les Principes Mathématiques de la Théorie des Richesses* was translated in Marshall's lifetime, Dupuit's essay, *De la Mesure de l'Utilité des Travaux Publics*, had, so far, found no translator. Edgeworth, in his article "Application of Probabilities to Economics" (*Economic Journal*, 1904), gives a free translation of isolated passages and regards Dupuit as "the earliest, and still, I think, the highest authority on the theory of discrimination."

Twice in his paper Dupuit mentions that his article is part of a larger work entitled *Economie Politique Appliquée aux Travaux Publics*. This work was never published.

Dupuit does not give the source of his quotations from Say and McCulloch. The editors of this volume have thought it useful to include precise references, which are printed in *italic* footnotes; in so doing they are indebted to Mario de Bernardi's annotations to his edition of Dupuit's writings (Turin and Paris, 1934), and to the assistance rendered by the translator of this essay, R. H. Barnack, Lecturer in Economics, Canberra University College.

The passages Dupuit, somewhat inaccurately, quoted from Say, would appear to have been extracted from the 5th edition (1826) of *Traité d'Economie Politique*; they differ very slightly, and not in substance, from the 4th edition, which C. R. Prinsep has translated into English (Boston, 1821). One of the passages quoted comes from the Annex to Say's *Traité*, which Prinsep did not translate; the translation of the others, for convenience' sake, leans on Prinsep so far as strict accuracy permits.

[1] This article is taken from a work entitled *Political Economy Applied to Public Works*, which the author intends to publish in the near future. See editors' note, above.

Utility and its measurement lie at the foundation of political economy; they have, therefore, been the objects of rigorous definitions. Let us see whether these definitions can serve as a basis for one of public utility.

J. B. Say says:

UTILITY. In political economy, utility is the power possessed by things of being able to serve man in some manner or other.

The most useless and even the most uncomfortable thing, like a Court cloak, has what is here called its utility if the use to which it is put, no matter what that use may be, is enough for a *price* to be attached to it.

This price is the measure of the utility which men judge the thing to have, of the satisfaction which they derive from its *consumption*; for they will not seek to consume this utility if, for the same price, they could acquire another which would yield them greater satisfaction.

Utility, thus understood, is the basis of the *demand* for products and consequently of their *value*. But this value does not exceed the *costs of production*, for beyond this amount it will pay anyone who needs a *product* to make it himself; or rather, he will never be reduced to the necessity of making it himself, because at that price it pays any *entrepreneur* to take it upon himself to produce the thing.[2] (Epitome.)

If one accepts these definitions without qualification and generalizes them, one may be led into grave errors in the measurement of the utility of many things which call for a different treatment. Let us give an example:

Some very capable engineers wanted to know what was the utility of the French roads, and starting from the datum that the prices paid by society for their use amounted to 500 million *per annum*, and applying J. B. Say's principles, they said that since society consents to pay 500 million for these transport facilities, their utility is 500 million; society would not give up this sum if it did not receive an equivalent satisfaction; 500 million, therefore, is the measure of this utility. A moment's reflection will suffice to show up the error in this reasoning. Let us suppose the introduction of some improvement in the means of transport—roads or carriages—and that it results in a fall in costs by one half, so that the same services for which society paid 500 million, will now be rendered for 250. Is it to be concluded that the roads are now only half as useful, as the principles set out above would require us to do? Is it not evident, on the contrary, that the utility of the roads, far from having diminished, would have increased by 250 million?

If society is paying 500 million for the services rendered by the road, that only proves one thing—that their utility is *at least* 500 million. But it may be a hundred times or a thousand times greater; we are left in ignorance of this. If you take the above figure as the measure—and not as the lower limit—of a quantity the exact magnitude of which you do not know, you are acting like a man who, wishing to measure the height of a wall in the dark and finding that he cannot reach the top with his arm raised, says: "This wall is two

[2] J. B. Say, *Traité d'Economie Politique*, article on Utility in the annexed *Epitome des Principes Fondamentaux de l'Economie Politique*.

meters high, for if it were not, my hand would reach above it". Now, if you say that the wall is at least two meters in height, then we are agreed; but if you go so far as to say that this is the actual measurement, then we are no longer agreed. In daylight, and equipped with a ladder, you will perceive that our alleged two-meter wall is fifty meters high.

As the distinction which we are trying to establish rests upon considerations of some refinement, we must stress these preliminary notions and elucidate them by somewhat numerous quotations and examples:

J. B. Say says:

Although price is the measure of the value of things, and their value the measure of the utility imputed to them, it would be absurd to draw the inference that, by forcibly raising their price, their utility can be augmented. Exchange value, or price, is an index of the utility men recognize a thing to have only so long as human dealings are subject to no influence alien to that same utility.

In fact, when one man sells any product to another, he sells him the utility vested in that product: the buyer buys it only for the sake of its utility, for the sake of the use he can make of it. If, for any reason whatever, the buyer is obliged to pay more than this utility is worth to him, he pays for value which does not exist and which, consequently, he does not receive.

This is precisely the case when the government grants to a particular group of merchants the exclusive privilege of engaging in a certain trade, the India trade for instance; the price of the merchandise concerned is thereby raised, without any accession to its utility or intrinsic value. This excess of price is money transferred from the pockets of the consumers into those of the privileged traders, whereby the latter are enriched by exactly as much as the former are impoverished.

In like manner, when the government imposes a tax on wine, which causes to be sold for 15 sous a bottle which would otherwise have been sold for 10 sous, what does it else but transfer 5 sous per bottle from the hands of the producers or the consumers of wine to those of the tax collector? The merchandise is here only a more or less convenient means of reaching the taxpayer, and its current value is composed of two elements, *viz.* its real value based on its utility, and the value of the tax which the government thinks fit to levy upon its manufacture, transport, or consumption.[3]

It is beyond doubt that a tax can add nothing to the utility of a product; but when we look at it from the consumer's point of view we can say that its existence brings to light undeniably that the product has a utility greater than the cost of production. Why is the bottle of wine purchased at 15 sous? It is because the buyer finds at least an equivalent utility in it; for, in spite of the tax, he is at perfect liberty to but it or not to buy it. It is not within the power of the state to make him pay, by means of the tax, anything more than the utility which he derives from this purchase.

This is how we see the situation: several individuals want to buy wine; but the need of each to acquire this good is different. Thus some of them, the rich,

[3] J. B. Say, *Traité d'Economie Politique*, 5th edition, Vol. 1, pp. 7–9; Prinsep translation pp. 5–7.

attach such a utility to it that they would be willing to buy even at 30 sous a bottle if that were the current market price; others, less rich, would not go above 15 sous; the less well-off would not pay more than 10 sous; others, in narrow circumstances, would only buy at 6 sous, and the poor only at 4 sous. On entering the market, they find that the price of the wine itself is 10 sous but, the government having imposed a tax of 5 sous, the commodity can only be supplied at a price of 15 sous.[4] What happens? All those who attach to the purchase of wine a value greater than 15 sous will buy, and will derive a kind of profit which will vary according to the significance which they put upon their acquisition; all those who would have bought wine at 10, 12, 13, or 14 sous will go without on account of the tax; and lastly those to whom the significance of such a purchase was less than 10 sous will not buy and would not have bought in any case. There is, then, only one single class of individuals to whom we can be certain that the utility is just 10 sous and that is the producers or sellers of the wine: they cannot derive a greater utility from it, no matter what the tax; for those who buy, it is greater than 15 sous, and for those who do not buy, it is less.

Thus, on examining the facts more closely, we have come to see that the utility of everything which is consumed varies according to the person consuming it. Nor is this all: each consumer himself attaches a different utility to the same thing according to the quantity which he can consume. Thus, a purchaser who would have bought 100 bottles at 10 sous might only buy 50 at 15 sous, and 30 bottles at 20 sous. Let us show this by an entirely different example, in order to demonstrate that it is a general phenomenon and one which, therefore, arises in the case of public works and must be taken into account when measuring their utility.

Consider the establishment of a water system in a town which, being situated at a high altitude, could previously procure water only at considerable trouble. Water then was so valuable that the supply of 1 hectoliter per day cost 50 francs, by annual subscription. It is obvious that each hectoliter consumed in these circumstances has a utility of 50 francs. With the installation of pumps this same quantity of water costs only 30 francs. What will happen? The inhabitant who was consuming 1 hectoliter will at first continue to do so and will derive a profit of 20 francs on this first hectoliter; but it is highly probable that the fall in price will induce him to increase his consumption; instead of using the water sparingly for personal purposes he will employ it also for less urgent and less essential needs, the satisfaction of which is worth more than 30 francs to him—since that is the sacrifice he makes to obtain the water—but less than 50 francs, since at that price this consumption was foregone. Thus, of these two hectoliters supplied to the same individual by the public pumps, one has a utility greater than 50 francs, while the other has a utility of between 30 and 50 francs. Suppose that by virtue of a technical

[4] For this to be the effect of the tax, it would have to have been in existence long enough to have diminished the quantity of wine produced.

improvement in the pumps, or by the very fact of increased consumption, the price is now reduced to 20 francs; it may well be that the same individual would take 4 hectoliters in order to be able to scrub his house every day. Let him have them at 10 francs each, and he will demand ten to water his garden; at 5 francs he will demand 20 to keep up the level of his pond; at 1 franc he will want 100 to keep a fountain going, and so on. If you look at this situation and ask what is the utility of the water supplied by the public pumps to this consumer, you must not say that it is 50 francs per hectoliter, because that is the price of the one he formerly consumed, before the installation of the pumps. There is only one hectoliter for which this figure is the measure of utility. That of the second hectoliter is between 30 and 50 francs; of the next two, between 20 and 30 francs; of the next six, between 10 and 20 francs; of the next ten, between 5 and 10 francs; and of eighty more, between 1 and 5 francs.

If you want to verify this, raise the price of the water. A tax of 4 francs per hectoliter, when the price is 1 franc, will immediately reduce consumption from 100 hectoliters to 20; a tax of 9 francs, from 20 hectoliters to 10; a tax of 19 francs, from 10 hectoliters to 4; and so on, until the price is brought up to 50 francs and only 1 hectoliter is consumed. By going further, you would eventually discover the utility of this last hectoliter, which you do not at the moment know.

Thus every product has a different utility not only for each consumer but for each of the wants for the satisfaction of which he uses it: we shall see this at every turn when we come to deal with the measurement of public utility. But first we must lay stress once again on those general notions, which are fundamental to the method which we shall presently expound.

At the outset, too, we feel it needful to deal with the objection which might be raised about our use of the word utility; it might be said that we have deviated from its scientific meaning and used it in a completely new sense in order to unfold a method of mensuration which, at first sight, is rather complicated. We shall merely recall that the distinction which we are expounding is to be found in Doctor Smith, who recognizes two values in an object—its *value-in-use,* which is its utility as we understand it, the value to him who has a need to consume the product; and its *value-in-exchange,* which is the value of the same product to him who has a need to sell it. McCulloch, who has annotated Smith, sets out this important distinction in a note:

"The word *value*", he says, "has been frequently employed to express, not only the exchangeable worth of an article, or its capacity of exchanging for other things obtainable only by means of labour, but also its utility, or its fitness for satisfying our wants, and contributing to our comforts and enjoyments. But it is obvious that the utility of commodities—that the capacity of bread, for example, to appease hunger, or of water to quench thirst—is a totally different quality from their capacity of exchanging for other commodities. Dr. Smith perceived this difference, and showed the importance of distinguishing between the utility, or, as he expressed it, the *value-in-use* of

commodities, and their value in exchange. To confound such essentially different qualities must evidently lead to the most absurd conclusions. And hence, to avoid mistaking the sense of so important a word as value, it would be better not to use it except to signify exchangeable worth or value in exchange; and to use the word utility to express the power or capacity of an article to satisfy our wants or gratify our desires."[5]

We are therefore not the first to point out the importance of this distinction; and the example we quoted, of the manner in which the utility of the roads was evalued, shows that McCulloch was not mistaken in saying that without this distinction one could be led into the gravest errors.

As for the more or less complicated measure of utility which follows from this new definition, we need only point out in its defence that political economy is not a science of expediency but of positive facts, to the statement of which it confines itself. The facts must be accepted just as society supplies them. We cannot adopt one idea which is simple but inaccurate, and reject another which is complex but true. Besides, is J. B. Say's formula really so simple? If we are to say that the utility of a bottle of wine being sold at 15 sous is only 10 sous, on the grounds that costs of production are 10 sous and 5 sous are tax, how are we to measure the utility of a kilogram of tea sold in Paris for 50 francs? How are we to deduct all the taxes which have hit this product in the course of being picked in China, sold over so many different counters, carried under three or four flags, until finally it arrives at the merchant's shop? And would not justice require us, after having made such deduction—which would need a lengthy study of this branch of business—to add on the benefit afforded to tea production by the distribution of the tax? After all, if the state keeps up a naval squadron, if it has agents, consuls, and ambassadors to represent it in these distant countries, in order to protect the purchase and carriage of tea—why then, here are real expenses to be added to those of tea production. The pay of the army of the *Compagnie des Indes* must of necessity be added to the cost of production of the sugar which it brings to market. In like manner, state expenditure which favours certain lines of production in some special manner forms part of their net cost; that is patently obvious in cases where bounties are granted. It can be seen, then, that although this method of calculation appears to be simple, yet it too has its difficulties.

The variable, yea mobile, nature of the value of utility is indeed well known to business men and has long been exploited by them. That is what lies behind all transactions which are sheltered from competition, either by dint of secret manufacturing processes, or by any other means which secures a monopoly profit to the seller. If some very useful object only costs a monopolist 1 franc to produce, will the manufacturer fix its value at 100 francs,

[5] Adam Smith, *An Inquiry into the Nature and Causes of the Wealth of Nations*, edited by J. R. McCulloch, Edinburgh, 1853, p. 438, in *Supplemental Notes and Dissertations*, by J. R. McCulloch.

knowing full well that there will be buyers at that price? Not in the least, for he also knows full well that there would not be very many of them—say a hundred perhaps, which will only yield him 9,900 francs profit; and that by reducing the price to 20 francs he might have a thousand buyers, which would give him a profit of 1,000 × 19 francs = 19,000 francs. Nor is that all. He knows, too, that of these thousand purchasers many would have been willing to pay a higher price—some would have been willing to pay 25 francs, others 30, 50, 80, or 100 francs—and that these buyers thereby derive a kind of gain of 5, 10, 30, 60, and 80 francs, respectively; and therefore he has recourse to a great variety of devices to secure the payment to him by each one of the buyers of as large a part as possible of this gain, which he considers is made at his expense. The same commodity in various guises is very often sold in different shops at quite different prices to the rich, the moderately well-off, and the poor. The fine, the very fine, the superfine, and the extra fine, although drawn from the same barrel and although alike in all real respects other than the superlative on the label, sell at widely different prices. Why? Because the same thing has a widely differing utility depending on the consumer. If there were only one medium price, there would be a loss to those who did without the product because its utility to them was less than that price, and a loss to the seller who, from many buyers, would be receiving payment for only a fraction of the utility of the service rendered. God forbid that we should try to justify all the frauds that go on in business; but it is well to study them because they are founded on a close knowledge of human nature, and are often found to be more equitable and fairer than one might expect at first sight and, indeed, they might be good examples to follow. We shall return later to this subject in the article on "Tolls", because this same consideration of a varying utility for the same object is the basis of pricing for all things the production costs of which are composed of two parts—one, a large outlay, made once for all or at least for a good many times; and the other, a small outlay, incurred for each object produced. Thus when a bridge is built and the state establishes a tariff, the latter is not related to cost of production: the heavy cart is charged less than the sprung carriage even though it causes more wear to the timber of the carriageway. Why are there two different prices for the same service? Because the poor man does not attach the same value to crossing the bridge as the rich man does, and raising the charge would only prevent him from crossing. Canal and railway tariffs differentiate between various classes of goods and passengers, and lay down markedly different rates for them although the costs are more or less the same. In drawing up these tariffs in advance the legislator merely defines certain features and characteristics which seem to him to indicate a greater or lesser degree of utility in the same service rendered to different people. In business, the merchant—who is in direct contact with the purchaser—goes further, he sets traps for the buyer's vanity and his credulity; but the aim is always the same, and that is to make the payment for the service rendered equal not the cost but what the buyer thinks it is worth. If, therefore, this

variable utility for each object were unknown, none of these devices for taking in dupes would exist—if dupes there be; for no one is ever a dupe except in relation to the cost of production. The purchaser never pays more for the product than the value he places on its utility.

To sum up, political economy has to take as the measure of the utility of an object the maximum sacrifice which each consumer would be willing to make in order to acquire the object. We say political economy, because this is not, in the last analysis, a rigorous measure of the quality which things have *of being able to satisfy men's needs*;[6] it would be difficult to say whose hunger was the greater—the rich man's, who would be willing to give a million for a kilogram of bread, or the poor man's, who, having nothing else to give, would risk his life for it. But political economy, being concerned only with wealth, can take account of the intensity of a wish only through its monetary expression. Political economy only bakes bread for those who can buy it, and leaves to social economy the care of supplying it to those with nothing of value to give in exchange.

The utility which we have just been considering and measuring is the absolute utility of all things that satisfy our desires—of those which nature supplies free as well as of those which can only be bought at the cost of the most arduous labour. If, in consuming a product, someone says: "It would take 30 francs to make me give it up," then that product really has 30 francs' worth of utility for him, no matter whether he only had the trouble of picking it up from the ground, or whether he paid 20 francs for it. But the relative utility to the consumer will be very different in the two cases. In the first case it will be all of the 30 francs of absolute utility, but in the second case it will be no more than 10 francs, the difference between the absolute utility and the purchase price. In effect, to satisfy a want which seems to him to be worth 30 francs, he is obliged to sacrifice another want to the extent of 20 francs. Thus he benefits only from the difference between these two sums. For the consumer who valued the satisfaction of the same want at only 29, 28, or 21 francs, the utility would only be 9, 8, or 1 franc. It would be zero to him, who, valuing it at only 20 francs, would be undecided whether to acquire it. There would be a loss of utility to anyone forced to pay 20 francs for a satisfaction which he valued at only 19, 18, or 17. Lastly, no utility would be produced if no one were willing to give more than 15 francs for an object which cost 20; there would be a loss of utility to the seller, and production would cease. Hence the saying which we shall often repeat because it is often forgotten: the only real utility is that which people are willing to pay for. We see that in general the relative or definitive utility of a product is expressed by the difference between the sacrifice which the purchaser would be willing to make in order to get it, and the purchase price he has to pay in

[6] See above, Note 2. This is not a textual quotation.

exchange.[7] It follows that anything which raises the purchase price diminishe the utility to the same extent, and anything which depresses the price increase the utility in the same manner.

Suppose, for example, that the market price of an article is 20 francs, whicl is more or less equivalent to the costs of production. According to the cir cumstances in which it is consumed, the (absolute) utility of this article ma have any one of the following values:

30, 29, 28, 27, 26, 25, 24, 23, 22, 21, 20 francs;

and in corresponding circumstances, its utility will be

10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0.

If a tax of 5 francs is imposed, the utility of the product will diminish by francs in the case of all those who were deriving a utility of 10, 9, 8, 7, 6, or francs from it, and they will now only derive 5, 4, 3, 2, 1, or 0 francs of utility the loss is the same in each case. As for those who only derived 4, 3, 2, 1, francs of utility and who, because of the tax, cease to consume the article they lose precisely that utility which they would have derived from the con sumption of the article; their loss will therefore be different in each case an will equal 4, 3, 2, 1, 0 francs, respectively. Thus the tax affects not only thos who pay it, but all those who would have been consumers but for the tax We shall return to this consideration later.

Let us now make the opposite assumption—that the costs of production and consequently the expense of purchase, fall by 5 francs; so that what cos 20 francs now only costs 15. It is clear that those who, at 20 francs, had utility of

10, 9, 8, 7, 6, 5, 4, 3, 2, 1 franc

will in the same circumstances now have a utility of

15, 14, 13, 12, 11, 10, 9, 8, 7, 6 francs.

The effect of this fall in price is obviously to leave them 5 francs more wit which to satisfy other wants. Nor is this all. The price having fallen, th

[7] The error of the physiocrats, who claimed that industrialists and farmers produced n utility because the consumption represented by the cost of production cancelled the utilit produced by them, was founded on nothing other than the false measurement of utilit based on production costs. If the carriage of a hogshead of Burgundy to Paris has no utilit other than the 15 francs which it has cost and which you pay to the carrier, then it is rigl to conclude that the carrier has produced no utility because those 15 francs represent h consumption and that of his horses; but if it be recognized that among the purchasers of th Burgundy wine there are some who would have paid much more than 15 francs over an above its price in order to obtain it if that had been necessary, then it follows that the carrie his cart, and the road which he used have been able to produce a much greater utility.

article is now within reach of those who formerly only estimated the utility of the article at

20, 19, 18, 17, 16, 15 francs

and did not buy because this utility was less than the market price: thus there will now be some new consumers. What will the utility of the product be for them? Still the difference between the absolute utility and the purchase price,

5, 4, 3, 2, 1, 0 francs.

The fall in price therefore yields a different amount of utility to each new consumer.

In general every rise or fall in price decreases or increases utility by an amount equal to this variation for those who are consumers in both situations; for those who disappear or who appear, the utility lost or acquired is equal to the old or to the new relative utility yielded to them by the product.

This formula comprises, implicitly, the measurement of all kinds of utility, including public utility, which is no different from any other. It is the latter with which we shall now be concerned, but we feel we ought to pause for a moment to see where we are in agreement with, and where we differ from, those who have preceded us in this line of inquiry.

J. B. Say has said:

> Roads and canals are costly public amenities, even in countries where they are set up judiciously and economically. Yet probably, the benefits which they afford to the community, in most cases, far exceed the annual cost to the latter. Of this, the reader may be convinced on reference to what I have said of the creation of value, due alone to the commercial operation of transfer from one place to another, and of the principle that every saving in the cost of production is a gain to the consumer. Were we to calculate what would be the cost of carriage of all the goods and merchandise now passing annually along this road, if the road did not exist, and to compare that enormous cost with the cost under present circumstances, then the difference would show the gain to the consumers of all those goods—a real and net gain to the nation.
>
> It would be wrong to say that, if the road did not exist, the costs of transport would not be so enormous as here suggested, because the transportation would not take place at all and people would do without the goods now transported. It is not to be rich, to do without things because one cannot meet their cost. Each consumer is infinitely poor in respect to any good which is too dear for him to consume; and he becomes richer in respect to it in the measure in which its value diminishes.[8]

This method of evaluating public utility is the one which has been most widely adopted. Note that it turns aside completely from the measurement of utility based on cost of production. Here, on the contrary, utility is measured by means of a reduction in these expenses, as it should in fact be. There is thus a kind of contradiction between these two definitions which does not exist in

[8] J. B. Say, *Traité d'Economie Politique*, 5th edition, Vol. 3, pp. 136–37; Prinsep translation p. 334.

the manner in which we have looked upon utility. Moreover, if the general principle which we have just quoted is basically true, it is so incomplete in formulation and in detail that it cannot but lead to entirely erroneous results.

To show this we shall take an example from an article in *Annales des Ponts et Chaussées* (1832, 1st half year), in which M. Navier has treated the same question with literal application of J. B. Say's formula. This method of calculation has, incidentally, been widely used, and if we needed other examples they would not be lacking.

"The government", he says, "by using funds raised from the taxpayers, has spent money on a construction and, further, it will have to take from these same funds whatever is necessary to provide for the costs of maintenance. It imposes a toll with a view to reimbursing itself for the costs which it has met and the new expenses to which it is committed. It is not difficult to perceive that, in order for this operation not to be a burden on the taxpayer, the annual economy effected by the transport must be at least equal to the interest on the capital expended together with the costs of maintenance. This fact establishes a limit to the tonnage of traffic below which the enterprise could not be conducted without loss.

"To give them greater precision, we shall try to apply these notions to the building of canals, and we shall assume the following data:

> Cost of constructing one league of navigable canal, 590,000 francs; which becomes 700,000 when costs of management and loss of value to property are added; the annual interest on this is 35,000 francs.
>
> Annual upkeep, costs of management and administration for one league of the same canal, 10,000 francs.
>
> Charges paid by merchants for the transportation of one ton of merchandise per league: by road 1 franc, by canal 0 fr.13 (excluding the toll). Saving effected by latter mode of transport 0 fr.87.
>
> According to these data it is possible to work a canal without loss to the state wherever the quantity of goods carried annually by the canal could be equal to 45,000/0.87, or 52,000 tons: if the tonnage is greater than that, the state will earn annually a sum equal to the product of 0 fr.87 and the number of tons exceeding 52,000."

The error in this calculation, following as it does the terms of J. B. Say's formulation, is to attribute to all the tons carried by the canal a value of utility which is true of only a very small number of them so that the utility of the canal is vastly exaggerated. Thus one is led to completely false results which could have the most serious consequences for the public wealth.

In the first instance, there is no very clear reason why, in this measurement of utility, it should be the road which is the standard of comparison: if a canal is built alongside a river where navigation is laborious and therefore costly, is it not clear that in the case of some goods it is with the river charges that the canal charges ought to be compared in order to get to know the utility of the canal? When a railroad is built, will it be the canal which is to serve as the

standard of comparison or the road? And in the case of a road, what is to be done?

Our method provides an answer to all these difficulties. We shall illustrate it by a series of examples, comparing it with the preceding one.

A town uses 10,000 tons of stone each year for the construction and repair of its houses. Twenty francs are paid for each ton. That is the total of the costs of production, the components of which we shall set out in detail presently. A new means of communication is established, which may be a canal, if you like, or anything else;[9] as a result, the costs of production of a ton of stone are reduced from 20 to 15 francs. In this case we say that the measure of the utility of the canal is the product of 5 francs—by which the price of a ton has fallen—by 10,000, the number of tons formerly consumed; *i.e.* 50,000 francs. It will be seen that here we are no longer comparing the costs of transport by the new and the old routes, but the costs of production. Herein lies a capital difference between the two methods. Thus, in this example, it could happen that the actual cost of transporting the stone was higher by the new route than by the old, on account of the new route being longer, and that this extra cost could be compensated by other circumstances. Suppose that the components of the old price of 20 francs are as follows:

Extraction from the quarry	16 francs
Transport over a short distance (say 4 leagues)	4 francs
Total of former costs of production	20 francs

As against this the canal, passing by an easily worked quarry which had not formerly been exploited, or the product of which had not been brought to this particular town because of the great distance involved, now brings about the following costs:

Extraction	2 francs
Transport over long distance (100 leagues)	13 francs
Total present costs of production	15 francs

We see then that the transportation cost of the old stone was only 4 francs, whilst that of the new is 13 francs; so that if we were to stick to the letter of J. B. Say's words[10] on the matter and compare the costs of transport only, we should find that the canal causes a loss of 9 francs of utility; whilst if we were to use M. Navier's method we should say that a ton of stone carried 100 leagues costs 13 francs by canal and would have cost 100 francs by road, and that therefore the utility of the canal is 87 francs per ton. But actually it is only 5 francs, that is to say more than 17 times smaller.

[9] It could even be a piece of equipment, some machine or other.

[10] We say 'letter', because on going back to the principles developed elsewhere by J. B. Say it is seen that it is the costs of production and not the costs of transport that this economist is comparing.

We could assume that the stone comes a greater distance still, and so long as the canal delivers it at less than 20 francs the new stone will replace the old. Thus, when the stone is worth 19 francs, the cost of transport being 17 francs, the utility of the canal will only be 1 franc; yet M. Navier's method would make it more than 113 (0 fr.87 multiplied by the number of leagues travelled by the stone).

The foregoing is not an exceptional case which could be neglected: things almost always happen in that way. Indeed, if one considers how a centre of consumption is supplied, one finds that it is provisioned by a series of radiating routes which form a certain pattern around it. When a new and more economical route of communication is established in one direction, not only does it substitute entirely in the supply of goods carried by the parallel route but it enters into competition with the routes running in other directions because its lower charges allow it to push back the sources of supply to a much greater distance. Thus, when the canal appears after the roads, it can, the other costs of production being the same, go six or seven times as far away; it will go twenty times and a hundred times further if certain circumstances allow it to take advantage of cheaper production. It is seen, then, that in general the result of the establishment of a much more economical means of communication is to alter the sources of supply, so that a comparison of the cost of transport by the canal and by the parallel road is necessarily wrong for the vast majority of products. This will be seen also from other considerations which we shall put forward.

The ultimate aim of a means of communication must be to reduce not the costs of transport, but the costs of production.[11] It may be quite rational to build a road of 40 kilometers in order to fetch, at its far end, goods which are to be had only 10 kilometers away by another road. The utility produced for the new objects, which replace the old, is equal to the difference in price multiplied by the quantity formerly consumed.

We say formerly consumed because that is an essential qualification; if we did not make it we should be led into grave errors.

The effect of the canal in having reduced the cost of production of the stone by 5 francs and consequently in having yielded a utility of 50,000 francs on the 10,000 tons used will not stop there. This fall in price will necessarily render the stone suitable for new uses; in many buildings it will replace brick and timber; streets will now be paved which were not so before, and so on; so that consumption, instead of being 10,000 tons will become perhaps 30,000. Thus before the canal was built, 10,000 tons were consumed at 20 francs; after the canal is built, 30,000 tons will be consumed at 15 francs. Is the utility produced for these 20,000 extra tons measured by 5 francs, as it is for the first 10,000? The considerations we have expounded on utility in general show that this cannot be so. Since the new purchasers did not buy

[11] By cost of production we mean what it costs to make an article available for consumption.

at the price of 20 francs it is apparent that they did not attribute that much utility to the consumption of stone; they do not, therefore, benefit by the kind of gain which constitutes relative utility. True, they buy at 15 francs; but amongst them there are some who attach so little value to the consumption of this material that they would give it up if the price were to rise by as little as 1 franc. For them the relative utility, the gain, is therefore less than 1 franc. Others would cease to buy only after a rise of 2 francs: for these latter the utility is between 1 and 2 francs. In short, in order to know the utility of each ton consumed it would be necessary for each consumer to make known the strength of his desire in terms of the price which would make him cease consuming. Then the calculation would become very easy. Suppose that a tax of 1 franc imposed on this stone, the production costs of which are 15 francs, deprives the canal of the carriage of 7,000 tons, then we will not be far wrong in saying that the utility of this transport is 1 franc. A new tax of 2 francs reduces traffic by another 5,000 tons, for which, therefore, the utility may be estimated at 2 francs at the most. By thus relating taxes with the amounts of traffic which they cause to disappear, we can arrive at the following result for the 20,000 new tons carried by the canal:[12]

7,000 tons	at 1 franc	.....	7,000 francs
5,000 "	2 francs	.....	10,000 "
4,000 "	3 "	.....	12,000 "
3,000 "	4 "	.....	12,000 "
1,000 "	5 "	.....	5,000 "
20,000 tons		.....	46,000 "

That is to say an average utility of 2 fr.30 instead of 5 francs, which we would get from a calculation based only on the fall in the cost of production.

If to these 46,000 francs of utility we add the 50,000 francs corresponding to the 10,000 tons of initial consumption—which we could have included in the same calculation since they disappear with a tax of 5 francs—we arrive at a figure of 96,000 francs for the total utility of this type of transport. M. Navier's formula would give $30{,}000 \times 87 = 2{,}610{,}000$. Is it not necessary before starting a canal to know whether its utility is one or the other of these quantities?

So far we have been concerned with products which are already being consumed. But at the stage of civilization which European nations have now reached, there has arisen, besides the essential needs which men have felt at all times, an infinite number of new needs which vary with different lands, climates, and customs; at the same time, human industry has varied the products with which to satisfy the same needs. Because of all this the opening of a new means of communication, if it be cheap as canals are, and speedy

[12] For convenience of exposition we have used calculated differences instead of using the differential calculus. Those who are familiar with the elements of the calculus will see later how precision may be substituted for approximation. See the end of the chapter for the exact formula.

as railways are, causes altogether new products to make their appearance in the areas which it serves. Tiles come to replace thatch on all houses of some village: elsewhere slate, in its turn, comes to replace tiles; the rich will have excellent wine where the surrounding countryside only yields poor wine; the poor, who used to drink water, now find that beer is within their reach; where sea fish was salted it now comes fresh; there will be plaster instead of lime; stone instead of brick, or *vice versa,* and so on. How are we to measure the utility of these new commodities which were not in use before the new means of communication?

We have seen above how J. B. Say answered this objection. "Each consumer", he says, "is infinitely poor in respect to any good which is too dear for him to consume; and he becomes richer in respect to it in the measure in which its value diminishes".[13] And this famous economist would have us take account of the utility of these commodities in the same manner as for the others, by estimating the difference between the cost of transport in the supposed absence of the road, and the actual transport cost since it is open, no matter how large this difference might be.

Here, the exaggerated character of this method of evaluation seems to hit the eye. Slate, unknown previous to the new road or the new canal, will be worth 20 francs per thousand, whereas it might be worth perhaps 200, 300, or even 1,000 francs without the canal. The slate quarries, which the canal now skirts, may not previously have had any means of exploitation, and if one had insisted on extracting the slate, it would have had to be taken away on mule-back. Are we to say that because slate would have cost 1,000 francs when everybody did without it, and because it is worth 20 francs now that everybody uses it, that therefore the utility of the service rendered by the canal is 980 francs per thousand slates? We can easily convince ourselves that it is not so, because it might well happen that a tax of 10 francs per thousand would reduce consumption by half; half of the consumers would go back to the tile which they had abandoned; this being so we can say that one half of the transported slate has a utility not greater than 10 francs per ton. Further, if a tax of 20 francs would cause slate to disappear from the market altogether, we should say that the utility of this second half was less than 20 francs. The figure of 980 francs, based on what would have been its cost, is therefore imaginary; there is no utility other than that people are willing to pay for. That is the dictum of political economy which we must always keep in mind when dealing with all these questions. If you have put 1,000 francs' worth of work into a product, and yet can only find a buyer at 100 francs, you have lost 900 francs' worth of utility.

In the case of new commodities being transported, as in the case of those now transported in excess of the former consumption, the measure of utility is not the fall in the costs of production, but the lowest tax which it would be necessary to impose on them in order to prevent their being carried by the

[13] See above, Note 11.

new route. This latter measure could even be applied to the amounts of commodities which were formerly being carried, because the tax which they can sustain is obviously equal to the fall in the cost of production. Moreover, this method is not peculiar to means of communication, but can be applied to everything, to any working tools whatever and to their products; so that we can say in general that the measure of the utility of a product is the tax which would prevent it being consumed. In order to discover the utility of a large number of products, or of a machine which turns out a large number of products, it would be sufficient to add up the utilities of each of them. The simplest method of doing this is the following.

Suppose that all those similar commodities of which we want to discover the utilities are all subjected to a tax which rises by small steps. Each successive increase will cause a certain quantity of the commodity to disappear from consumption. This quantity, multiplied by the rate of tax, will give its utility expressed in money. By thus letting the tax go up until there are no more consumers, and by adding together all the products of this multiplication process, we will arrive at the total utility of the goods.

Let us illustrate this formula by an example. We want to know the utility of a footbridge which is being used free of charge at the rate of 2,080,000 crossings annually. Suppose that a toll of 0 fr.01 would reduce the number by 330,000, that a tax of 0 fr.02 reduces it by 294,000, and so on. We then say that for 330,000 crossings the utility is about 0 fr.01 and that for the next 294,000 crossings the utility is about 0 fr.02 and we can then draw up the following table.

330,000	crossings	at 0 fr.01	produce	a utility	of 3,300	francs
294,000	"	.02	"	"	5,880	"
260,000	"	.03	"	"	7,800	"
228,000	"	.04	"	"	9,120	"
198,000	"	.05	"	"	9,900	"
170,000	"	.06	"	"	10,200	"
144,000	"	.07	"	"	10,080	"
120,000	"	.08	"	"	9,600	"
98,000	"	.09	"	"	8,820	"
78,000	"	.10	"	"	7,800	"
60,000	"	.11	"	"	6,600	"
44,000	"	.12	"	"	5,280	"
30,000	"	.13	"	"	3,900	"
18,000	"	.14	"	"	2,520	"
8,000	"	.15	"	"	1,200	"
2,080,000					102,000	"

Thus 102,000 francs would be the absolute utility to society of the bridge. We can find the relative utility by deducting the costs of maintenance and the interest on the capital expended in construction. If this latter sum were to reach or exceed 102,000 francs, the construction would have produced no utility, the difference expressing the loss which would have been made. Such is the calculation to be made in the case where crossing is free of charge. If there is a toll, we must take only the figures below that of the charge. Thus

for a toll of 0 fr.05, for example, the absolute utility of the bridge is expressed by the sum of the ten last figures or 66,000 francs; the utility lost, by the sum of the first five, or 36,000 francs; the product of the toll would be 770,000 crossings at 0 fr.05 or 38,500 francs. With this toll, then, the possible utility of the bridge would be distributed in the following manner:

To the toll collector	38,500	francs
Derived by those crossing the bridge (66,000–38,500)	27,500	"
Loss of utility arising from the 1,310,000 crossings which would have been made but for the toll	36,000	"
Total	102,000	"

As the toll increases, so does the utility of the bridge diminish in proportion; it becomes zero when the toll equals 0 fr.15, at which price noone crosses the bridge; it is therefore possible for the loss of utility to rise to as much as 102,000 francs. Does this mean that there should only be very low tolls or even that there should be none at all? That will not be our conclusion when we come to speak of tariffs; but we hope to show that their height needs to be studied and operated according to rational principles, in order to produce the greatest possible utility and at the same time a revenue sufficient to cover the cost of upkeep and interest on capital.

If instead of a footbridge you have a bridge for carriages, all you need do is to apply an analogous calculation to each article on the tariff—to horsemen, to sprung carriages, to carts, *etc.*, and to add the utilities together.

The type of calculation that we have just described is a general one; instead of crossings of a bridge you could write in the table pairs of stockings and so discover the utility of the stocking frame. If you assume that the tax is only applied to stockings made by this process, you will obtain a figure which will cause this machine to disappear completely from use so that there will be a return to handknit stockings. The sum of the amounts below this figure will give you the utility of the stocking frame. Go further, assume that the tax falls upon stockings without distinction as to their process of manufacture, and when you have caused the last pair of stockings to disappear, the sum will represent the utility of this garment.

If it were a question of discovering the utility of transportation on the royal and departmental roads, it would be necessary in the same way to assume a traffic tax increasing little by little, which would cause the successive disappearance of several of the tons together comprising the 50 million tons being carried on these roads. Each ton multiplied by the tax which would prevent it from moving would give this utility. It is seen that the total figure would not have the slightest relation to the 500 million costs of production.

It remains for us now to show that our formula is complete, that it expresses the whole of the utility of the objects we are considering and that there is nothing to be added to it. It often happens, in effect, that when the cost of production of an article falls, competition causes the price of the same commodity produced by a different method to fall to the same level, as it does also

for similar commodities. Thus, coal is carried by canal; and the utility of the coal is given exactly by our formula. But the presence of this coal on the market might, through the effect of competition, result in a fall in the price of wood, which the canal does not carry. If there is a fall of 2 francs per cubic meter of wood and people continue to consume 100,000 cubic meters, are we not entitled to say that there are 200,000 francs of utility which are owed to the canal? At another place the opposite may happen; the presence of the canal may cause large quantities of wood to be carried away so that the local price of wood rises. Complaints against the canal will follow: "It may be very useful to some places", it will be said, "but as for us, who have to pay 2 francs more for our wood, it really costs us 200,000 francs a year". Those are 200,000 francs which should be deducted from its utility. Lastly, it is often said that means of communication increase the income from and the value of the properties which they traverse, as well as the revenue from certain taxes, and so on. Without entering into the details of these effects, it can readily be shown either that their measurement is included in the above formula or else that they are merely changes in the distribution of wealth which it is not for us to take into account, because the losses and gains counterbalance each other. When we say that we are not to take them into account, we are speaking only with respect to the calculation of utility. The state, on the contrary, must concern itself very seriously with them. A new means of communication is opened; whilst it has a utility of 10 million for society as whole, yet it causes one million to pass from Peter's pocket into Paul's. Although this may at first be merely an individual misfortune, it will have repercussions on the wealth of society which the state has an interest in preventing, redressing, or mitigating.

For an increase or decrease of utility to take place, there must be, provided there is no change in quality, a decrease or increase in the *costs of production.* When there is merely a change in the *market price,* the consumer gains what the producer loses, or *vice versa.* Thus, when an object costing 20 francs to produce is sold at 50 francs because of a monopoly or concession, the producer exacts 30 francs' worth of utility from each purchaser. If for some reason or another he is forced to cut his price by 10 francs, his profit falls by 10 francs per article and each purchaser gains by that amount. It is a question of compensation, but no utility has been produced. There would have been an increase of utility if the drop in the market price had been due to a fall in the costs of production, because the gain to consumers would not have been offset by any loss to the producer. When, therefore, coal brought by canal brings about a fall in the price of wood at the place of destination, the income of the wood owners falls by as much as that of the consumers rises. If, on the other hand, the canal, by taking wood away, causes the price of the remaining wood to rise, then the income of the wood owners rises by as much as that of the consumers falls. However, it often happens that the compensation is not as precise as we have just described it. The fall in the market price indeed brings about an increase in consumption and thereby secures

for new consumers a utility which the product did not previously possess. But if we look more closely at how things actually happen, we see that although this increase of utility is very real, it cannot be attributed to the public undertaking, which in this case has merely caused the market price to fall—a result which might equally well have been obtained by a simple legislative measure. It is possible to conceive of a canal carrying neither wood, nor stone, nor iron, and yet causing a fall in the prices of those commodities because of the possibility it prevents to buyers of procuring these things more cheaply; for this possibility forces the existing producers to lower their prices in order to retain their market. Now it is evident that the utility due to this fall of price, and thereby enlarged consumption, cannot be attributed to the canal, which, carrying nothing, is but a fiction, so to speak, and the course of which could be replaced by a line of stakes. That is the situation whenever utility is produced by competition: a bridge yields large profits to the company which collects the toll; a rival company builds another bridge alongside and forces the first to cut its tariff by half; the number of people crossing the first bridge doubles and its utility increases enormously. Is this increase due to the second bridge, which no one in fact crosses? Obviously not; it is merely a result of the cut in the tariff of the first, which could have been brought about by some other means; on the contrary, since the construction of the second bridge required a considerable amount of capital it actually diminishes the public utility. Therefore when measuring the utility of public undertakings only those commodities must be included, to the production of which the undertaking contributes directly. When the method of evaluation outlined above is applied to those commodities, one may be sure not only of omitting nothing which should be included, but also of counting in nothing which should be left out.

We have shown that the methods we refuted are fallacious in several respects. First of all we saw that it is not costs of transport which have to be compared in order to arrive at the measure of utility, but costs of production; this was the first error. Then we saw that to apply this measure to the quantity by which the consumption of some commodities increases, was a second error; and that to do so for new products was a third. It remains to be shown that the unqualified application of this measure to the rare products where the substitution of one machine for another leads to no change in the quantity consumed, is nearly always a fourth error. In fact, it does not often happen that a modification of the productive process which reduces costs does not also modify the quality of the product; the latter becomes better or worse, larger or smaller, lighter or heavier, quicker or slower, and so on. Now all these qualities have a value which must be taken into account in the calculation of utility. Thus in the example borrowed from M. Navier in which comparison is made between a canal and a road, the advantage being valued at 0 fr.87 per league in favour of the former, it would not even be correct to apply this calculation to the goods which now come by canal in the same quantities in which they formerly came by the parallel road. The fact is that

carriage by road being quicker, more reliable, and less subject to loss or damage, it possesses advantages to which business men often attach a considerable value. However, it may well be that the saving of 0 fr. 87 induces the merchant to use the canal; he can buy warehouses and increase his floating capital in order to have a sufficient supply of goods on hand to protect himself against the slowness and irregularity of the canal, and if all told the saving of 0 fr.87 in transport gives him an advantage of a few centimes, he will decide in favour of the new route. But the advantages of the new route to him will only be precisely these few centimes, and if a toll of the same amount is established on the canal, then goods will no longer be moved by this route; and there we have the true measure of the utility of the canal so far as these goods are concerned. For there is no utility other than that people are willing to pay for; thus, although there is a saving of 0 fr.87 in the cost of production, there may well be two or three centimes' worth of utility, because the method of production has changed. This method of measurement by comparing costs of production would deny any degree of utility to a railway which was built after a road and which, although it charged higher fares, was able to take passengers from the latter because of its sole advantage of speed. What is its utility? Only the toll which would dissuade passengers from stepping out of the stagecoach and into the railway carriage can give us the exact measurement.

It will be seen that this method takes account of the previous situation whatever it may have been. The measure of the utility of a machine or of a public enterprise is not an absolute measure, but a measure of progress; it is the distance between the point of departure and the point of arrival. Here we have a tun of wine which has been brought 100 leagues by a new canal, and you say that 87 francs' worth of utility has been produced because the cost of transport is only 13 francs as against 100 francs for a similar distance by road. That is a grave mistake; before the advent of the canal this wine may have gone by road from the vintner's caves to a seaport, whence a coaster took it to the mouth of a river, up which it was taken to the beginning of another canal, and after travelling a certain distance along that canal it may have been brought by goods cart to its destination: now the opening of the new canal has led to the use of a new route, which in turn involves several means of transport. A mere difference of 1 franc in total costs will suffice for the adoption of this new route. The amount of 87 francs' worth of utility for a journey of 100 leagues claimed for the canal may then be reduced to as little as 1 franc, provided there has been no change in the length or circumstances of the journey.

This shows that there may be an enormous difference between the utility of two means of communication which are used with equal frequency and which cost the same to use; this difference arises from the nature of the route previously used. Thus two railways may be equally patronized and produce the same receipts with the same tariff, yet one of them may be very useful and the other practically useless. What has happened is that the first railway has

replaced a longer and badly laid out road on which transportation was costly and slow. Each of its passengers therefore derives a great advantage in going by rail, whilst the second railway runs parallel to a steamboat service and only saves its passengers a few minutes on an already short trip. The least rise in its fares would cause the loss of all its custom and show that its utility was really very small, whilst a similar step would not lose the other railway a single passenger. And so it may happen that of two railways the one that is used less, was more costly to build, and has the more defective layout and the higher fares, may be the one with the greater utility. In order to measure the latter, in fact, it is not sufficient to count the services rendered: they must also be weighed; many small services may give a smaller result than a few large services.

So far we have been concerned solely with acquired utility, because to increase utility is the purpose of all instruments of production including public undertakings; however, it will be well to say a word about the measurement of lost utility, because a loss of utility results from anything which raises the price of commodities, and because in the management of public enterprises recourse often has to be had either to tolls—which raise the market price of those commodities using the service supplied by the public undertaking—or to taxes, which have the same effect on those not using the service. It is therefore necessary to know how to calculate the loss of utility in these circumstances. We shall need but a few words, for it is merely a case of making a calculation exactly similar to that which has been put forward for the measurement of acquired utility.

We have assumed that in a town where 10,000 tons of stone had been consumed at 20 francs, the appearance of a canal or any other piece of equipment causing costs of production to fall to 15 francs had brought about a rise in consumption to 30,000 tons. And we have calculated the utility due to this fall in costs as follows:

Utility yielded on 10,000 tons formerly consumed, at 5 francs per ton	50,000 francs
Utility yielded on 20,000 tons of new consumption at a rate varying between the limits 0 and 5 francs, found to average 2 fr.30	46,000
Total utility produced	96,000

Now suppose that for some reason or another a tax of 5 francs a ton is imposed on this stone. It is quite obvious that its effect will be to reduce consumption of the stone to 10,000 tons, since it will bring the price to purchasers back to 20 francs. Only the uses for which the stone has a utility greater than that price will be satisfied. The yield of the tax will therefore only be 50,000 francs; but is that alone the utility lost by the taxpayers? Evidently not. It might even be said that for the nation as a whole the yield of the tax is not a loss, since it must be supposed that it is put to some good use. It is merely a change in the distribution of wealth; there is no loss to society as a whole, as if an extra amount of work had been required which would have raised the

costs of production to 20 francs; the 5 francs per ton would then have represented something which had been consumed. But there is a real loss to those who would have bought the stone at 16 francs and who, when buying it at 15 francs, benefited to the extent of 1 franc, of which they are now deprived by the tax of 5 francs even though they do not pay the latter; similarly there is a loss of 2 francs to those who would have been willing to pay 17 francs, and so on. In order to estimate the total amount of this loss the only data required are the quantities by which consumption falls with each increase in the tax; we find ourselves back at the table we drew up earlier, now showing 46,000 francs' worth of utility lost for the 20,000 tons of stone which are not consumed because of the tax; here, then, is a tax which yields but 50,000 francs, and which, besides, causes a loss to society of 46,000 francs of utility. We have chosen an example where the rate of the tax is fairly moderate relative to the cost of production—one third, in fact (5 francs in 15); there are some taxes on articles of consumption which double, treble, or quadruple the price of the commodities concerned, with the result that their consumption drops off enormously and there is a loss of utility to society incomparably larger than the yield of the tax. One can gain an idea of this effect by supposing that the government is so ill-advised as to treble, quadruple, quintuple . . ., indeed to raise the postal rate for letters to such a level that the public, which is at present paying 50 million for postal services, will expend no more than 25 million. A moment's reflection will show that this result is quite possible; in the measure in which the charge increases, the number of letters falls, eventually becoming zero when the postage is such that no one thinks the carriage of a letter worth that price. At a slightly lower charge there will be some letters and a very small yield. Therefore there actually is some charge higher than the present one which would reduce the yield to 25 million. Let us look at this state of affairs: is it not obvious that although the public is in some sense relieved of a burden, since it is paying 25 million less than before, yet it is losing a very considerable amount of utility in respect of the letters which are not carried because of the impost? From this may be seen how false are the comparisons which are sometimes made between the budgets of different countries; the people of one country say: we are paying 25 million, you are paying 50 million—therefore our government is twice as good as yours. Now, it may well be that the contrary is true, quite apart from the question of how the 25 and the 50 million are employed; it may well be that the harm done in levying the 25 is a hundred times more considerable than that done in levying the 50 million. The fallacy lies in taking account of only one class of citizens—those who pay the charge: account must also be taken of the much greater number who do not pay it because they cannot afford to, and who therefore are no longer consumers. In many cases, therefore, the rate or basis of a tax has a greater effect on the general well-being than does the amount which it raises.

Let us indicate some of the general properties of taxes which it is well to bear in mind in questions concerning public undertakings, since the latter always and necessarily give rise to a tax or a toll.

Suppose that we have two columns of figures showing the number of articles consumed corresponding to each market price from zero, at which consumption is largest, right up to the price which causes all consumption to cease. This series of relationships is not known for any commodity, and it can even be said that it will never be known since it depends on the volatile will of human beings; it is today no longer what it was yesterday. It is thus of no avail to try to determine this relationship exactly by experience or groping experiment, but there do exist certain general laws to which the relationship, in its very mobility, remains constantly subject, and out of these general laws there arise certain immutable general principles. One of these laws is that consumption expands when price falls; another, that the increase in consumption due to a price fall will be the greater, the lower the initial price. If a fall in the price of an article from 100 to 95 francs brings in another thousand consumers, a further fall from 95 to 90 will bring in more than a thousand. This property reflects the structure of society which, if it is divided into groups according to income, and these groups are placed one on top of the other starting with the poorest, has a shape similar to one of those pyramids of cannonballs which are to be seen in parks of artillery—the lower the layer, the more balls it contains. Thus, as the price of an article falls, its use spreads to more and more consumers, quite apart from the fact that existing consumers purchase it in greater quantities, as we have seen. All this is a fact of experience which has been verified statistically too often to need labouring here.

It follows that when the change in consumption brought about by a tax is known, it is possible to find an upper limit to the amount of utility lost by multiplying the change in consumption by half the tax. The same holds for the utility produced by a piece of equipment. Thus in the example of the consumption of stone which we took as the basis for our calculations, we found that a fall of 5 francs in the price only gave an average utility of 2 fr.30 per ton. The figures we worked on are fictitious, it is true: but no matter what they might be, so long as they obey the laws which we have just invoked, they would always give a result less than 2 fr.50; to arrive at that figure we would have to assume that the 20,000 tons by which consumption has increased have arisen in a uniform manner, that is to say 2,000 by the drop from 20 francs to 19, 2,000 by that from 19 to 18, and so on down to the drop from 16 to 15 francs. Now that is not possible, since each successive fall in price brings in more and more numerous consumers. Thus it is possible to lay down the principle that the utility lost or gained through a change in price has for its upper limit the amount by which the quantity consumed changes, multiplied by half the change in price. If a tax of 5 francs reduces the number of consumers from 30,000 to 10,000, the utility lost by the community is below $20{,}000 \times \frac{1}{2}\,5 = 50{,}000$ francs. Further, it is easily seen that the smaller the tax the nearer does this limit approach the actual figure.

Although consumption diminishes less and less rapidly as the tax rises, it is permissible, where a tax is small relative to the cost of manufacture, to suppose a uniform rate of decrease. Thus a tax of 1 franc on a thing worth 100

francs will cause the number of consumers to fall to an extent not markedly different from a tax of 2, 3, 4, 5, or 6 francs; for the relations between the numbers 100, 101, 102, 103, 104, 105, and 106 are little different. Now, the utility lost as a result of a tax of 1 franc is this unknown number multiplied by $\frac{1}{2}$ of 1; the utility lost through a tax of 2 francs will be twice this number multiplied by $\frac{1}{2}$ of 2; for 3 francs, $\frac{1}{2}$ 3 × 3. It may thus be said that the loss of utility is proportional to the square of the tax; so that a tax of 10 francs will lead to the loss of 100 times more utility than a tax of 1 franc. The enormous advantage of spreading taxes out is apparent; instead of putting a tax of 10 francs on one article, taxing 10 articles at 1 franc each may reduce the loss of utility by 90 per cent. Let us note further that the product of a tax is not proportional to its rate. A tax of 10 francs will not yield ten times as much as one of 1 franc.

If a tax is gradually increased from zero up to the point where it becomes prohibitive, its yield is at first nil, then increases by small stages until it reaches a maximum, after which it gradually declines until it becomes zero again. It follows that when the state requires to raise a given sum by means of taxation, there are always two rates of tax which will fulfil the requirement, one above and one below that which would yield the maximum. There may be a very great difference between the amounts of utility lost through these two taxes which yield the same revenue. This even applies in the case of a tax which yields the maximum revenue, for appreciably different rates of tax may yield more or less the same revenue while bringing about quite considerably different losses of utility; even in this situation there is much to choose.

Loss of utility resulting from a rise in price is not peculiar to tolls and taxes; it applies to the very price, representing costs of production, which could be considered as a kind of tax upon natural resources. Thus the price of a thing is not only a burden to him who pays it, but also to him who cannot acquire the thing because of its price. The effect of mechanical equipment in reducing prices, and of taxes in raising them, is thus merely to increase or diminish an already existing disadvantage; which latter may be calculated by the same method, for it is but the utility of a machine which would reduce the costs of production to nothing. This calculation only requires the measurement of the utility between a price of zero and the actual price, and is one which we have already performed above for the case of a bridge which it cost nothing to cross. Instead of a charge for crossing a bridge we can consider the price of some object or other and arrive at exactly the same result. This loss of utility due to a price which is not a payment for labour expended plays in political economy the part which friction plays in mechanics. No doubt the Pont des Arts does take 5 centimes' worth of utility from all those who cross over it, but insofar as this is merely a repayment of capital advanced, it is a law of human nature and of the present state of progress of the human mind to which we must needs resign ourselves; insofar as it is a profit for him who has built the bridge, it is but a change in the distribution of public wealth which does not appreciably affect its total. But this is not all: this toll of 5 centimes greatly

detracts from the utility of the bridge (we could say that this price of 5 centimes greatly detracts from the utility of such and such a commodity), because it forces many people who only attach a utility of 4, 3, 2 centimes, or 1 centime to the crossing, to go round by the Pont Neuf. Here the loss is complete and uncompensated: it is the useless friction of mechanics. You want to raise a weight of two kilogrammes to a height of one meter: don't complain that you must make an effort of one kilogramme, so long as you need not sustain it for more than two meters—this must be so unless the laws of nature were changed; but if your one-kilogramme effort has to hold out for three or four meters, then there is useless friction which mechanics will teach you how to reduce or avoid. In the same way political economy can show how to reduce those losses of utility which result from changes in price. We have, here, only inquired into the principles which can be used to measure them, and the following chapters will seek to apply those principles.

It may perhaps be objected that our formula depends upon certain data which no statistical method can furnish and that therefore we shall never be able to express by an exact figure the utility yielded by a machine, a road, or any undertaking, nor the utility lost as the result of a tax or a toll.

We might content ourselves with replying that when something cannot be known, it is a great deal, already, to realize that one knows nothing. If those who first occupied themselves in the attempt at measuring the wealth of nations had limited themselves to declaring that the question was beyond their powers, instead of putting forward the doctrine of the balance of trade, they would perhaps have rendered a greater service than those who came later and demonstrated their error. In fact, the barriers to international trade which have been erected under the influence of this doctrine have resisted, and in all likelihood will continue to resist for several generations yet, all the arguments of the true principle. This question of the measurement of utility is in like case with all other problems in political economy in that a rigorous solution is impossible in practice; yet this science alone can furnish the means to approach such a solution. It may be impossible to say that the utility of a canal would be more than five million, yet possible to say that it would be less than six, which is sufficient information upon which to base a decision not to build; it may be impossible to say that the utility of a bridge would be as much as 120 thousand francs, yet possible to say that it would be more than 80 thousand, which is sufficient to show that the bridge is worthwhile. In political economy the data for reaching a complete solution are often lacking; but this disadvantage makes a knowledge of the basic laws and general principles all the more necessary. They alone can show how to turn to account what is known so as to discover what is not known, point out what is lacking and thereby provide the means of seeking it, of finding it if that is possible, and if it is not, provide a substitute. Political economy is like geometry which, although stating its principles in terms of squares, triangles, circles, and other regular figures, yet shows how to measure the area of a field bordered by the sinuous course of a stream and a lane of which only a few

points are known. Are the known points sufficient? What are the missing ones? How find them? What will be the margin of error if we have to do without them? Those are questions which require a closer and deeper knowledge of geometry than those where all the elements of the calculation are given exactly. In like manner, the less complete and accurate are the available data in problems of political economy, the more needful is it that the rigor of fundamental scientific principles be applied to them if they are to be handled skilfully and effectively in practice.

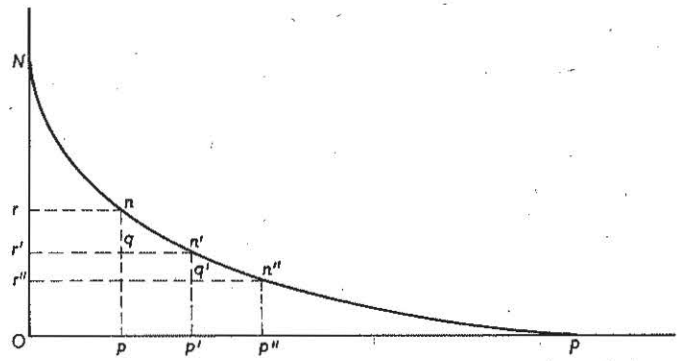
NOTE

The various points about utility which have been developed above may be presented geometrically in a very simple manner.

If it be supposed, as in Figure 1, that along a line OP the lengths Op, Op', Op'' . . . represent various prices for an article, and that the verticals pn, $p'n'$, $p''n''$ . . . represent the number of articles consumed corresponding to these prices, then it is possible to construct a curve $Nnn'n''P$ which we shall call the curve of consumption. ON represents the quantity consumed when the price is zero, and OP the price at which consumption falls to zero.

Since pn represents the number of articles consumed at price Op, the area of the rectangle $Ornp$ expresses the cost of production of the np articles, and, according to J. B. Say, also their utility. We trust we have demonstrated that the utility of each of these np articles is at least Op and that for almost all of them the utility is greater than Op. Indeed, by raising a perpendicular from p' it can be seen that for each of $n'p'$ articles the utility is at least Op', since they are bought at that price. Of the np articles there are therefore only $np - n'p' = nq$ for which utility is really only Op (or rather the average between Op and Op'); for the others it is at least Op'. We are thus led to the conclusion that for nq articles the utility is represented by the area $rnn'r'$, and that for the remainder, qp or $n'p'$, it is greater than the rectangle $r'n'p'O$; by supposing a further rise in price $p'p''$ we could show that for $n'p' - n''p'' = n'q'$ articles the utility is an average between Op' and Op'', and is measured by the area $r'n'n''r''$ and so on. By continuing this process it can be shown that the

Figure 1



absolute utility of the np articles to the consumer is the mixtilinear trapezium $OrnP$. The relative utility is arrived at by subtracting the costs of production, shown as the rectangle $rnpO$, which leaves the triangle npP; this, according to our view, is the utility remaining to the consumers of the np articles after they have paid for them. It is seen that the area of this triangle on one side of the line np has no relation to that of the rectangle on the other side.

The utility of a natural product the acquisition of which requires no expense, is expressed by the large triangle NOP.

It may be noticed that as the price of an article rises, the utility diminishes, but less and less rapidly: and that, on the other hand, as the price falls, the utility increases more and more rapidly; for it is expressed by a triangle which shortens or stretches as the case may be.

Figure 2 shows the effect of an improvement in the methods of manufacture which reduces the costs of production from Op to Op' without any change in quality; utility is increased by the difference between the two triangles $n'p'P$ and npP, or the mixtilinear trapezium $n'p'pn$. The error of which we have accused M. Navier was to take, instead of this area, the rectangle $n'p'pq$. If a change in quality occurred, say for the worse, the utility would only be the difference between the triangles $mp'S$ and npP, which could be quite small and even zero, according to the shape of the new curve of consumption.

Let Op, in Figure 3, be the price of an article which is cheap and consumed in large quantities. A small tax of pp' will yield the rectangle $pp'n'q$ and the utility lost both to the taxpayers and the fisc is the small triangle nqn'. If the tax is doubled, its yield of $pp''n''q'$ is not double the rectangle $pp'n'q$; yet the loss of utility $nq'n''$ is four times the loss represented by the triangle nqn', since both its base and its height have doubled. Similarly, if the tax is trebled, the loss of utility increases ninefold, and so on. From this may be derived the following propositions developed in the text: *The heavier the tax, the less it yields relatively. The loss of utility increases as the square of the tax.*

By thus gradually increasing the tax it will reach a level pM at which the yield is at a maximum, $pMTQ$, and the utility lost is fairly considerable, being represented by the triangle TQn. Beyond pM the yield of the tax diminishes

Figure 2

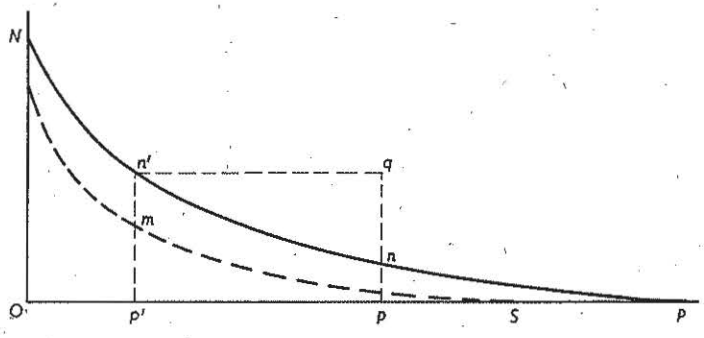
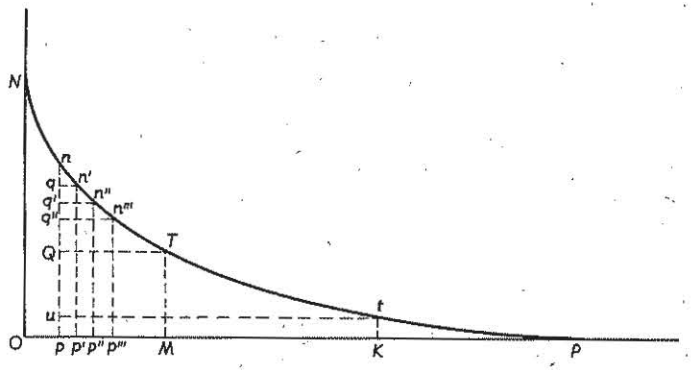


Figure 3

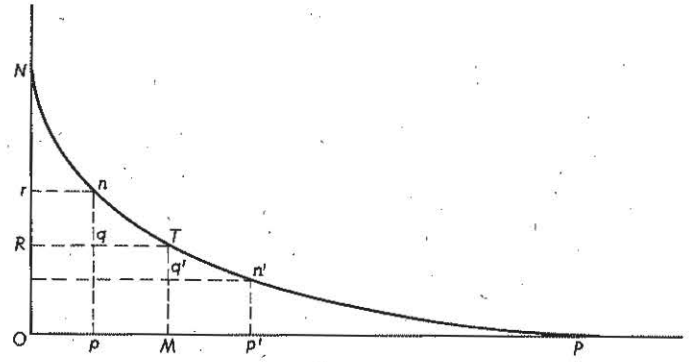


and equals that given by a lower rate of tax. For example the high tax pK would only yield $pKtu$ which is equal to or even smaller than the yield of the tax pp'. In the former instance the loss of utility is the large triangle tun, which may be ten times as large as the yield of the tax. Thus a tax of pM which yields 10 million will do less harm than a tax of pK which only yields 2 or 3 million. Lastly, a tax of pP will yield nothing; it is the one which does society the greatest harm, though it brings nothing into the treasury. Therefore it must be recognized that the yield of a tax is no measure of the loss which it causes society to suffer. It all depends on the way in which taxes are combined.

Tolls lead to similar results, for they are either taxes or increases in price. If it is wished by means of a toll on a bridge to raise a sum A representing the interest on capital expended, then, given the curve of consumption $y = f(x)$, we must solve the equation $xy = A$. If it is wished to raise the greatest revenue, we must solve the equation $dyx/dx = 0$. If, in Figure 4, Op is the value of x derived from the first equation, the product of the toll will be $Ornp$, the utility of the bridge to those who use it will be the triangle npP, and the loss of utility the triangle Nrn. If OM is the value which will raise the greatest product $ORTM$, the utility of the bridge to those who use it is no greater than the triangle TMP, and the loss of utility becomes the triangle RTN. When the consumers can be placed in several categories each of which attributes a different utility to the same service, it is possible by a certain combination of taxes, to increase the product of the toll and to diminish the loss of utility. If from among the pn consumers at price Op you can distinguish the number pq who would consume at the price OM, and from among these latter the number Mq' who would consume at price Op', and can oblige them by various combinations to pay those prices, then the yield of the tax will be the sum of the three rectangles $Ornp + pqTM + Mq'n'p'$; the utility to consumers will be the three triangles $nqT + Tq'n' + n'p'P$; while the loss of utility is merely that due to the lowest tax, the triangle Nrn.

We shall not pursue the application of geometry to political economy any

Figure 4



further here, for that would need developments which will be found in the following chapters.

In presenting, in this note, some of the principles of our science in this particular form it was our wish to try and make clear how great would be the advantages of an alliance with mathematics, despite the anathema which economists of all times have pronounced against the latter. So soon as it is realized, with J. B. Say, that political economy is concerned with quantities susceptible of a more or a less, it must also be recognized that it is in the realm of mathematics. If one has gone astray in political economy every time one has relied on mathematical calculations, it is because there are mathematicians who make false calculations, just as there are logicians who produce false arguments: the former no more invalidate mathematics than the latter invalidate logic, which alone is sometimes regarded as a science. Not only do the symbols and drawings of mathematics give body and form to abstract ideas and thereby call the senses to the aid of man's intellectual power, but its formulae take hold of these ideas, modify them, and transform them, and bring to light everything that is true, right, and precise in them, without forcing the mind to follow all the motions of a wheelwork the course of which has been established once for all. They are machines which, at a certain stage, can think for us, and there is as much advantage in using them as there is in using those which, in industry, labour for us.