Francis Ysidro Edgeworth

Arthur L. Bowley

Econometrica, Vol. 2, No. 2 (Apr., 1934), 113-124.

Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28193404%292%3A2%3C113%3AFYE%3E2.0.CO%3B2-F

Econometrica is currently published by The Econometric Society.

FRANCIS YSIDRO EDGEWORTH

By ARTHUR L. BOWLEY

AMONG the pioneers of econometrics, Francis Ysidro Edgeworth must be given a very high place. Especially important are his contributions with regard to the study of the theory of economics and the theory of statistics by the help of mathematics. For actual measurement he would give place to Jevons, for the theory of measurement to no economist.

With the recent publication of J. M. Keynes's *Essays in Biography*,[1] the facts of Edgeworth's life are familiar. He was born in 1845 at Edgeworthstown, County Longford, Ireland, on the estate where his ancestors established themselves in the sixteenth century—reduced to very small value by the time he inherited it in 1911. His father (1809–1847), who appears to have spent his life in the peripatetic pursuit of knowledge, is described by Thomas Carlyle[2] as he appeared about the year 1836; Maria Edgeworth (1767–1847), the celebrated authoress, well-known in the literary circles of the early nineteenth century, was his aunt. His mother was a Spanish refugee, married after a brief acquaintance in London. One of his great-great-grandfathers was a Huguenot refugee. Francis Ysidro was the youngest of five sons and survived all his brothers. His features suggested his Spanish descent; in his knowledge of French, German, Italian, and Spanish, and in his ready acquaintance with economists of all nations, there is a suggestion of his international origin.

In 1862 he entered at Trinity College, Dublin, where it may be presumed he studied both mathematics and classics. He was never at a loss for a classical quotation. I have found no record of his graduation at Dublin, but in 1867, in his twenty-second year, he went to Oxford, and in 1868 was admitted to Balliol College. In 1869 he was awarded first class honors in Literis Humanioribus, the great school of Philosophy; but he did not actually take his degree (B.A.) till 1873.[3] He was admitted to the Bar in 1877. During the ten years after he reached Oxford his interests appear to have been philosophy, ethics and, subsequently, economics. He was immersed in the writings of Bentham and others of the utilitarian school, while in economics he was educated in the works of John Stuart Mill. In this period he probably neglected

[1] Keynes's study of Edgeworth is essentially the same as the obituary that appeared immediately after his death, *Economic Journal*, 1926.

[2] *Life of Sterling*, Part II, Chapter IV.

[3] Subsequently he took the M.A. degree in 1877 (a matter of formality), and at a later date was given an honorary D.C.L. of the University of Durham.

mathematics, for his mathematical writing indicates a want of systematic training. Though he shows great insight into the principles of mathematics, there is a want of facility and neatness in his handling of problems. Familiar as he was with the work of Laplace, Todhunter, and Clerk Maxwell, he had difficulty in elementary applications. His line of thought is often a little obscure; sometimes he labors the obvious, and at others is so brief as to be difficult to follow. He was always the victim of numerical mistakes and errors in writing and printing. Apparently he settled in London immediately after he left Oxford, and had no definite occupation.

PHILOSOPHY AND THE PHILOSOPHICAL ASPECT OF ECONOMICS

It is evident that in the first part of his life he turned his mind principally to philosophic questions and especially to the relation between ethics and economics. His first known publication is a paper on "New and Old Methods of Ethics" in 1877. He began to interpret utilitarian and economic ideas by mathematical symbols, and very early he must have realized the importance of the conception of probability. The result of this period of study and incubation was the publication of his *Mathematical Psychics* in 1881. The mathematical economics in this book he owes primarily to Cournot, Jevons, and Gossen, something to his great contemporary Marshall;[4] the ethics is developed from Mill and Sidgwick; but the whole conception and treatment are original in the highest degree. It was so original that its importance has been only very gradually realized. The circulation was limited and it was little known;[5] the number of philosophers or economists qualified to understand it was very small, but among them were Jevons and Marshall. The following extracts from *Memorials of Alfred Marshall,* edited by A. C. Pigou, are interesting from many points of view.[6]

> Alfred Marshall first became for me a notable name when Jevons [in 1879 or 1880], conversing about mathematical economics, recommended as the latest contribution to that subject the now celebrated papers on the *Pure Theory of Foreign Trade and Domestic Values.* At the same time Jevons highly praised the then recently published *Economics of Industry.* Eagerly studying these writings, I discerned a new power of mathematical reasoning, not only in the Papers

[4] Cournot 1801–77, Gossen 1810–58, Jevons 1835–82, Marshall 1842–1924, Edgeworth 1845–1926, Wicksell 1851–1926.

[5] "Three days after Part II was finished, I received and saw for the first time the *Mathematical Psychics* of Professor Edgeworth." Irving Fisher in Preface to *Mathematical Researches in the Theory of Value and Prices,* dated 1892. (The phrase is re-translated from the French edition.)

[6] P. 66, *Reminiscences* by Professor F. Y. Edgeworth.

bristling with curves and symbols, but also in certain portions of the seemingly simple textbook. With reference to such passages, writing in the year 1881, I characterized the author by a phrase which he himself afterwards acknowledged to be appropriate, "bearing under the garb of literature the armour of mathematics." The phrase might be applied to many passages in the text of the *Principles of Economics*.

J. M. Keynes on Marshall (pp. 25–26):

In 1881 [Marshall], reviewing Edgeworth's *Mathematical Psychics*, after beginning "This book shows clear signs of genius, and is a promise of great things to come," adds "It will be interesting, in particular, to see how far he succeeds in preventing his mathematics from running away with him, and carrying him out of sight of the actual facts of economics."

Perhaps Marshall did not appreciate the path-breaking quality of the book, since he was already averse from the mathematical exposition of economics. The reading of it is at least essential to all who wish to understand the concepts that underly Edgeworth's later writings, and it is fortunately now easily accessible, since it has been reprinted at the London School of Economics (price 5s.).

PROBABILITY AND STATISTICS

We next find Edgeworth appointed as Lecturer in Logic at King's College, London, in 1880; in 1890 he succeeded Thorold Rogers there as Tooke Professor of Economic Science and Statistics, a chair more noticeable for the distinguished men who have held it than for its emoluments.[7]

Now begins the period of the publications on the theory of probability. In 1883–84, at least six papers appeared in this sphere, of which the first was "The Law of Error" (*Phil. Mag.* 1883). While *Mathematical Psychics* laid the foundation and showed part of the construction of the edifice of his economic theory, the paper (only 36 pages) on *Methods of Statistics*, read at the Jubilee Meeting of the Statistical Society in 1885, exhibits at once the whole plan of his statistical work. To use the kind of metaphor in which he delighted, the foundation, based on the work of Laplace, Lexis, and Venn, was laid, the first story was completed, and the frame work of the second was partly set up, partly in the process of execution. In the next forty years more stories were added, additional buttresses were erected where the structure was weak, and innumerable decorations within and without added to the amenities of the edifice in the best classical and italianate styles, while secret rooms were provided for initiates. There is little in the

[7] The professors have been Thorold Rogers, Edgeworth, Cunningham, Urwick. After the war its annual value was about £50, and it was merged in another professorship, now held by F. A. von Hayek under the original title.

subsequent work whose origin cannot be traced through this paper, but, as with *Mathematical Psychics*, it was appreciated by very few, and the development and the use of the important principles which Edgeworth was the first to introduce to English statisticians was left almost entirely to him for many years.

Edgeworth was the philosopher of statistics rather than the practitioner. We may give prominence to two of his main subjects, those in which his point of view differs from most modern statisticians.—Every judgment based on mathematical chance is related to *a priori* probability. There must be some presumption about the field in which the events take place. If an event occurs of which the chance is small, e.g., five successive throws of double-six with a pair of dice, we must choose between the alternatives that the dice were loaded and that an improbable event has taken place. If the difference between the average of two samples is greater than the three times the computed standard deviation, we have the alternatives that the samples were badly selected, that an unusual event has taken place, or that the populations from which they were drawn were essentially different. Or again we may need to assume, as in field samples in agricultural tests, the existence of an underlying normal curve of variation. Edgeworth frequently used the principle of maximum probability and deviations from it—his method of "genuine inverse probability"—but his treatment of it was essentially different from that of Professor R. A. Fisher. I understand the latter to deny the applicability of the conception of *a priori* probability and to make strenuous efforts to evade its use. Edgeworth was convinced that there was always an element of the unknown, that in the end we must fall back on unmeasured experience. But in his hands this did not mean that we could not obtain useful results, for in appropriate cases the influence of the unknown could be reduced to such small dimensions as to leave the result almost unaffected on any reasonable hypothesis. It is not certain, however, that this judgment, based principally on economic statistics, would be applicable to all physical experiments, though he traced it in the theory of atomic motion.

The second subject which Edgeworth developed from a specially distinctive view is the Law of Error. His Law of Great Numbers is usually erroneously classed as one of a species which contains the formulae of Thiele, Charlier, Karl Pearson, and others. It is true that the mathematical expression is very similar, but the fundamental conception is not the appropriateness for representing experimental results, but the determination of a form that would be derived from given hypotheses; the experimental verification is secondary. The hypotheses, based on Laplace's work, are the existence of numerous independent causes and their interplay in producing aggregates or

averages. According to the number and strength of these causes, the successive terms of his law of great numbers have greater or less importance. This law is

$$z = e^{-(-1/3!k_1D^3+1/4!k_2D^4-)}\ (e^{-x^2/c^2}/c\sqrt{\pi})$$

where z is the frequency with which a deviation of x from the average of measurements occurs, and c, k_1, k_2, are constants depending on the successive moments of the curve. Qualifications are introduced when the causes are not completely independent and when successive experiments are inter-related. A development to the 'method of translation' is made, when the observed quantity is not itself distributed according to the law but is some function of a variable so distributed. The formula is extended also to two or more variables.

It was a favorite theme for Edgeworth that the normal law of error, or its generalized expression, is prevalent in nature and in the subject matter of economics, however much disguised; or, if not in the raw material, then in the play of averages. It is hardly yet realized how nearly normal is the distribution of averages, even when the number of their constituents is small. The law of small numbers is practically indistinguishable from the normal law, unless the numbers are indeed very small. This was pointed out very clearly by Edgeworth. He said, "if the Greeks had been acquainted with the Law of Error, they would have erected an altar to it," presumably alongside that to the Unknown God. His interest was rather in the fundamental prevalence of the law than in its applicability to the representation of groups; his long series of papers on the Mathematical Representation of Statistical Data (*Statistical Journal*, 1913–18) was, I think, more valuable for its by-products than for its major thesis, and Edgeworth himself expressed doubt of their use. The important applications were in measuring the accuracy of averages, and the significance of the differences between them. His study of fluctuations (*Statistical Journal*, 1885) may be compared with Professor R. A. Fisher's method of variance; the one in a cross table of death-rates in years and districts, the other in a Roman square of agricultural experiments.

A general view of the several facets of Edgeworth's intellectual interests is obtained by studying his work on Index-Numbers of Prices, beginning with his reports to a British Association Committee, 1887–1890. On the statistical side, we find the examination of the merits of various types of averages, of the effect of weights and their small importance in this problem, and of the application of the law of error for determination of precision.[8] On the economic side, there is a very care-

[8] "By rejecting the Calculus of Probabilities (Mr. Walsh) has . . . thrown away an instrument necessary for the performance of that measurement" (i.e. of the value of money). *Papers*, II, 376. (Written in 1901.)

ful and systematic analysis of the quaesitum, the quantity to be measured for particular economic purposes, and of the relation to currency problems. "Beneath the apparent unity of a single question there is discoverable upon a close view a plurality of distinct problems."[9] The logical basis of the investigation is further developed in later papers, especially those which deal with Mr. Correa Walsh's work. It may be recommended to anyone who wishes to do serious work on Index Numbers to read Professor Irving Fisher's *The Making of Index Numbers*, Mr. Walsh's *The Problem of Estimation*, and Edgeworth's series of papers, reports, and reviews. If, in consequence of inability to bring all the views expressed into a coherent body of thought, our aspirant gives up the problem, he will at least have obtained an insight into the psychology of the writers.

In 1892 Edgeworth published his first paper on Correlation. It is idle to try to determine whether he or Professor Karl Pearson can claim priority in arriving at the main ideas which have led to the now well-known methods. Both attacked the problem of joint variation at the point which Galton had reached. Edgeworth gave in 1892 the general formula for multiple correlation in essentially the same form as is now used, viz.,

$$z = Ce^{-1/D.(x_1^2 R_{11} + x_2^2 R_{22} + \cdots + 2x_1 x_2 R_{12} + \cdots)}$$

where R_{11}, R_{12} $\cdots$ are the minors of the determinant

$$D = \begin{vmatrix} 1 & \rho_{12} & \rho_{13} & \cdot\cdot \\ \rho_{12} & 1 & \rho_{23} & \cdot\cdot \\ \cdot & \cdot & \cdot & \cdot\cdot \\ \cdot & \cdot & \cdot & \cdot\cdot \\ \cdot & \cdot & \cdot & \cdot\cdot \end{vmatrix}.$$

Here ρ_{12} is an average of the inverse ratios of an x_1, to that value of x_2 which is most frequently associated with it. This average was more closely investigated and defined in 1893, and an expression for it was obtained which was subsequently identified with Pearson's sum-product formula.

The numerous statistical studies published between 1893 and 1926 are to a very large extent the working out of ideas expressed or latent in the papers already named, with numerous applications to a great variety of problems and with critical and explanatory references to the work of other writers. Throughout the twoscore papers listed for these years runs the thread of the importance of sound fundamental ideas on probability in all mathematical statistics as opposed to purely empirical work. The ground deliberately chosen as central to his posi-

[9] *Papers*, I, 199.

tion is described in his Presidential Address to the Royal Statistical Society in 1912 "On the Use of the Theory of Probabilities in Statistics Relating to Society."

In writing of Edgeworth's attitude to statistics I feel that I am on fairly safe ground. When, in 1895, I was appointed to lecture on Statistics at the newly founded School of Economics, on Marshall's introduction I wrote to him for advice on the nature and literature of the subject, and he recommended principally Venn's *Logic of Chance*, Todhunter's *History of Probability* and Lexis' *Zur Theorie der Massenerscheinungen*, to which I naturally added his 1885 paper at the Jubilee meeting and his reports on Index-Numbers. From that time till his death I constantly learned from him, worked with him, and met him frequently in London and Oxford. It was with difficulty that I could turn the conversation from the nature of probabilities and the applications of the Law of Error. A trivial anecdote illustrates this. In 1904, a party of economists was bicycling out of Cambridge, and, with some danger to the traffic, Edgeworth began to discuss the method of translation or some similar topic; Professor Cannon drew up alongside and said, "Put on the pace, Bowley, he can't talk mathematics at more than 12 miles an hour."

GENERAL ECONOMICS

In Economics proper, to which I now turn, I cannot claim the same intimacy, and depend rather on the judgment of economists as shown in their quotations from his works, and especially on Pigou's very sympathetic review of Edgeworth's collected papers in the *Economic Journal*, June 1925.

Wherever economic theory called for mathematical treatment Edgeworth's interest was specially aroused, and, though this test by no means admits all the topics he treated, it is of special importance to econometricians.

It is the original analysis in *Mathematical Psychics* that has found the principal place in economic literature, namely the invention or discovery of Indifference Curves and their relation to the Contract Curve. The same analysis was applied in 1893 to the Pure Theory of International Values, and the findings of this study have been incorporated widely in treatment by later authors. Edgeworth did not seek after exceptions or paradoxes for their own sake; but in conscientious analysis he found that accepted rules were only true approximately and within limits. Thus, though convinced that free-trade was the best policy, at least for England, his analysis led to determination of the cases where an import or export duty was in part borne by the foreigner. He was greatly interested in Mr. Bickerdike's view of "in-

cipient taxes" which were advocated as benefiting an importing country, but his final judgment was adverse, as is seen in the concluding sentences of his criticism, which may be quoted in full as typical of his style and outlook.

> Thus, the direct use of the theory is likely to be small. But it is to be feared that its abuse will be considerable. It affords to unscrupulous advocates of vulgar Protection a peculiarly specious pretext for introducing the thin edge of the fiscal wedge. Mr. Bickerdike may be compared to a scientist who, by a new analysis, has discovered that strychnine may be administered in small doses with prospect of advantage in one or two more cases than was previously known; the result of this discovery may be to render the drug more easily procurable by those whose intention, or at least whose practice, is not medicinal. It was thus that the "drama of poison" perpetrated in the reign of Louis XIV was initiated by one whose baleful receipt was obtained from Glaser, a chemist of eminence, the discoverer of a new substance. Let us admire the skill of the analyist, but label the subject of his investigation *Poison*.[10]

From Cournot onwards, it has been recognized that the theory of monopoly demands essentially mathematical treatment. In this connection we find numerous studies relating to taxation, to railway rates, and other topics. The possible advantages of monopoly, in every stage from its establishment in one industry to complete socialism, are brought to light, in some cases to be condemned, in others accepted. In particular, Edgeworth discusses the possibility of discrimination, that is, rates or charges differentiated by place or class of customer, which is not inherent in pure competition but may result in net benefit to all concerned. There is very much that is important in these studies besides the often-quoted proof that in a specially devised and exceptional case a tax on one of two rival commodities may result in a lowering of the price of both. The mathematical results need for their application statistical data, such as are the proper study of econometricians—for example, questions on whether certain functions are positive or negative—and depend on the actual measurement of their elasticity.

We may agree with Pigou that the papers on the "Application of Probabilities to Economics" (*Economic Journal*, 1910, pp. 286 and 441 *seq.*) are most noticeable, and we may add, most characteristic. One of the earlier sentences in the first paper is "The theory of Probabilities lends to Economics, as to other sciences, certain premises which are evidenced, neither by pure intuition nor by formal induction, but by general impressions and what may be called mathematical common sense." We can often proceed a considerable distance with certainty by the help of accepted postulates, such that demand in

[10] *Papers*, II, 365.

general falls with an increase of price. Presently the results depend on the neglect of quantities which may be presumed to be small; such results are probably true in the light of general experience of the behavior of continuous functions. Though no numerical measurement of chance can be obtained, it may be affirmed that the probability of the failure of a theorem is very small. We may even go further and assume that in the absence of specific information a positive value of a variable is as likely to occur as a negative value, and even this meagre datum may afford definite guidance. It is to be remarked that in these papers are included not only instances of mathematical chance, but a number of applications of the conception of continuity of functions, where arguments are commonly based on unverified assumptions, not proved to be valid, but held to be reasonable or probable. Closely allied to this is the neglect of quantities presumed to be small.

Though there seems to be a cleavage between the studies on the theory of probability and those on international trade and monopoly, to Edgeworth there was an underlying unity in the fundamental conceptions and in much of the method of Economics and of Statistics; this unity found its expression in the mathematical treatment of both, and his interest was most easily aroused where mathematical ideas were involved, whether in ethics, economics, or statistics. It is in this region of fundamental similarities that we find what is most characteristic of his genius, that which distinguished him from almost all other economists and statisticians.

Neither subjective happiness nor belief can be measured, but indicators can be found in allied measurable quantities. Wealth has a relationship to welfare, belief can be connected with mathematical probability. There are scales of greater and less for incommensurable objects. The utilitarian's ideal of the greatest possible happiness is related to the mathematical economic conception of maximum utility. The best judgment or belief to be obtained from statistical data can be founded on algebraic maxima. From the fundamental expression for maximum utility and maximum probability are derived the detailed equations of exchange on the one side, and such formulae as those for 'least squares' on the other, as in Mechanics the principle of minimum potential energy leads to equations of equilibrium. While the foundations are similar, there is also interaction in the developments. Utilitarian ideas are involved in the choice of the 'best mean,' and probability justifies the conception of the representative firm, of the average man, and of the similarity of groups in relation to utility. In particular in many cases, where the solution of an economic problem depends on the determination of the direction of a curve, *a priori* probability will supply the most plausible answer.

EDITORIAL WORK

In 1891 Edgeworth succeeded Thorold Rogers[11] as Drummond Professor of Political Economy at Oxford and held that chair till he resigned in 1922 and became Emeritus Professor. During the whole of this period and till his death in 1926 he was Editor, or one of the Editors, of the *Economic Journal*, the first issue of which was in March 1891.

So far as he had a home it was, according to the time of year, two rooms at Mount Vernon, Hampstead, on the outskirts of and overlooking London, or in his Fellow's room at All Souls College, Oxford. Characteristically he had his own means of transit; from Oxford he bicycled by a route known to his friends as "Edgeworth's way" 30 miles to Great Missenden, whence he took train to Hampstead. Very few of his friends ever penetrated to his sanctum at Oxford—bachelor's rooms, for he never married—very many were entertained by him at All Souls College and at the Savile Club, London. He made no collection of books. In London he was to be found working in the rooms of the Statistical Society, and there or at Oxford he studied the books sent to the *Economic Journal* to review. He was also to be met at the sessions of the Statistical Society (President in 1912–14) at the Economics Society (that used to meet at University College, London where H. S. Foxwell was professor), and at the historic Political Economy Club, to which he was elected in 1891;[12] these three societies meet in regular sequence in the first three weeks of each winter month. He was for many years regular in attendance at the annual meetings of the British Association for the Advancement of Science, being President of the Economic Section in 1889. His name was in the first list of elected members of the British Academy in 1903. Edgeworth was thus the most accessible of the English economists. He usually spent his vacations in the Alps or near Dublin, but otherwise was not a great traveller.

The professional duties at Oxford were during his tenure of office very light. Economics till after his retirement was not a major subject in any 'School,' and in any case the custom at Oxford is to depend

[11] Thorold Rogers was Professor at Oxford 1862–68 and again from 1888 till his death in 1890. He was Tooke Professor in London from 1859 till 1890, occupying the two chairs simultaneously. Edgeworth gave up the Tooke Professorship on his appointment at Oxford. He was succeeded in London by Dr. W. Cunningham, and at Oxford by D. H. Macgregor in 1922.

[12] The first paper he read there was entitled "Under what conditions, if any, is the burden of a customs duty not borne by the consumers of the imported commodity," in 1893. The date is important in connection with his later work on tariff problems.

rather on College lecturers and tutors than on professorial teaching. Though he had great influence on many individuals, he did not organize any corporate teaching; the few courses of lectures he gave were not well attended, for indeed he had no faculty for that method of teaching. His work was in fact almost independent of his position at Oxford.

Mr. Keynes has testified to the great importance of his editorial work. It was specially marked in the organization of reviews of books. Though Edgeworth's direct writings turn on a limited number of subjects, his acquaintance with economic theory and with economists was very extensive in place and time and his memory was remarkable. In the reviews written by himself and republished, his encyclopaedic knowledge is as evident as the acuteness of his critical powers; but even these reviews are selective, for he appears to have looked critically at every book that reached the *Journal's* office. A request for a review would be accompanied by some apposite remarks on particular points in the text. Because he never developed a system of economics and never published a book except *Mathematical Psychics*, there is an erroneous impression that his sole interest was in refinements and exceptions, in mathematical curiosa rather than in the broad stream of economic thought; this impression is completely dispelled by reading his volume of reviews. Since nearly the whole edition of his *Papers* has been sold, it is to be hoped that a just view of his range has now been reached by all competent judges.

*

Readers of Edgeworth's writings[13] are often deterred by what appears to be deliberate obscurantism in the arguments. Many by-paths are followed and left with a quotation from the classics; mathematical curiosa are interspersed with poetry; elaborate metaphors are developed, where one would expect rigid deductions. But if one studies carefully a treatise as a whole, with some knowledge of Edgeworth's general lines of thought, and then reads it a second time, one finds that the whole is coherent, the arguments valid and consecutive, the theme

[13] A nearly complete study of Edgeworth's work can be made from the sources here listed:

Papers Relating to Political Economy. Three volumes. Published on behalf of the Royal Economic Society by Macmillan and Co., 1925.

Review of the above by A. C. Pigou, *Economic Journal*, June 1925.

Mathematical Psychics, 1881, reprinted by the London School of Economics, 1932.

F. Y. Edgeworth's Contributions to Mathematical Statistics. Published as a separate pamphlet by the Royal Statistical Society, 1928. This contains an annotated bibliography of seventy-four papers by him on mathematical statistics.

is made clearer and more vivid by the variations. "By steps that are neither violently abrupt nor tediously circuitous, he reaches the heights from which the mutual dependence of all economic quantities can best be contemplated. At those heights too, are observed some curiosities of theory, like Alpine flowers, found only at great altitudes." These words may surely be transferred from his review of a book by another writer and applied tc himself.

London School of Economics